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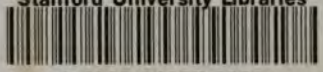

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MATHEMATICS FOR COMMON SCHOOLS

PART III

HIGHER ARITHMETIC

INCLUDING

EASY ALGEBRAIC EQUATIONS AND SIMPLE GEOMETRICAL PROBLEMS

BY

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PREFACE.

MATHEMATICS FOR COMMON SCHOOLS is a one-book arithmetic in three parts. Part I., an ELEMENTARY ARITHMETIC, contains those portions of the subject needed by all pupils of the common schools: addition, subtraction, multiplication, and division of whole numbers; simple fractions; and the most commonly used denominations of compound numbers.

Part II., an INTERMEDIATE ARITHMETIC, intended for pupils of the fifth and sixth school years, gives a sufficiently full treatment of common and decimal fractions, and of compound numbers, and takes up the simpler and more practical parts of percentage and interest. A short chapter on easy algebraic equations of one unknown quantity is added, to be used whenever it is deemed advisable.

Part III., a HIGHER ARITHMETIC, completes the ordinary grammar school course in this subject, and contains, in addition thereto, a chapter on algebraic equations and one on elementary constructive geometry, with applications. These two chapters are placed, for convenience, at the end of the work; but it is intended that suitable parts of each be taught from time to time throughout the last two years of the grammar school.

The special features of the work are its division of the arithmetical portion into half-yearly chapters, instead of the ordinary arrangement by topics; the omission, as far as possible, of rules and definitions; the very great number and variety of the examples; the use of the equation in the solution of arithmetical problems, especially in those of percentage and interest; and the introduction of the elements of algebra and geometry.

Believing that there is some foundation for the complaints frequently made by business men and high-school teachers that grammar-school graduates are too often slow and inaccurate in ordinary computations, the author

has furnished throughout the entire work systematic drills and reviews in the addition, subtraction, multiplication, and division of ordinary numbers and of fractions.

In this endeavor to enrich the grammar school course in mathematics the attempt has not been made to shorten it so much as some may desire. The intelligent teacher can and should do the remainder for himself, by rigorously omitting all such topics as he finds unnecessary.

J. H. W.

BROOKLYN, N.Y., December, 1893.

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HIGHER ARITHMETIC.

CHAPTER X.

ALGEBRAIC EQUATIONS.

ONE UNKNOWN QUANTITY.

848. Oral Exercises.

1. What number increased by 12 equals 16?

2. 9 added to a number equals 14. Find the number.

3. What number diminished by 7 equals 8?

4. 18 diminished by what number has 10 for remainder?

5. Eight times what number equals 64?

6. What number multiplied by 9 gives 63 for product?

7. Three times a certain number added to twice the same number equals 40. What is the number?

8. 36 is equal to 10 times what number added to 8 times the same number?

849. Sight Exercises.

Give values of x, y, z, etc.:

1. $?+12=16$
2. $9+?=14$
3. $?-7=8$
4. $18-?=10$
5. $8x=64$
6. $9y=63$

7. The difference between three times a certain number and nine times the same number is 66. What is the number?

8. $27,000 is divided among three children, the second of whom receives twice as much as the first, and the third of whom receives three times as much as the second. What is the share of each?

9. The sum of two numbers is 72, and the greater is 5 times the other. What are the numbers?

10. John, Henry, and James have 54 marbles. Henry has twice as many as John, and James has as many as the other two. How many has each?

11. The sum of the ages of mother and daughter is 42 years. What is the age of each, if the mother's age is six times that of her daughter?

12. A man paid $96 for an equal number of hats and coats, paying $2 apiece for the former and $10 apiece for the latter. How many of each did he buy?

13. Divide 41 into four parts, the first being twice the second, the second three times the third, and the third four times the fourth.

(Let x = the fourth.)

14. The sum of three numbers is 180. The first is double the second, and the third is three times as large as the sum of the other two. What are the numbers?

15. Mr. Smith paid 81 cents for sugar and flour, the same quantity of each. For the sugar he gave 5¢ per pound, and for the flour 4¢ per pound. How many pounds of each did he buy?

16. The length of a rectangular field is 24 rods, its breadth is x rods, its area is 456 square rods. Find the value of x.

17. It takes 340 feet of fence to enclose a square lot. What are the dimensions of the lot?

18. Mrs. B. divides $120 between her son and her daughter. She gives the latter twice as much as she gives the former. What is the share of each?

19. The earnings of a man and his son during January amounted to $175, both having worked the same number of days. The father's wages were $4 per day, and the son's wages were $3 per day. How many days did they work?

20. The sum of $240 is divided among four children, two boys and two girls. Find the share of each, if each girl's share is double that of each boy.

21. A man worked twice as many days as his son. Their combined earnings amounted to $165. Find the number of days each worked, if the father earned $4 per day and the son three-fourths as much per day.

22. A boy's bank contains 78¢ in dimes, nickels, and cents. There are twice as many nickels as there are dimes, and three times as many cents as there are nickels. How many are there of each?

23. I paid 75¢ more for a roll of 15-cent ribbon than I did for a roll of 12-cent ribbon of the same length. How many yards did each roll contain?

24. A rectangular field whose length is four times its breadth requires 250 rods of fence to enclose it. What are the dimensions of the field? (Make diagram.)

25. A girl paid 60 cents for a speller and a reader, the cost of the former being one-third that of the latter. Find the cost of each.

26. The sum of two numbers is 72, and the smaller is one-fifth of the other. What are the numbers?

27. Mary, Susan, and Jane have 54 hickory nuts. Susan has one-half as many as Mary, and Jane has as many as the other two. How many has each?

CLEARING OF FRACTIONS.

851. Oral Exercises.

1. One-fifth of a number is 4. What is the number?

2. $\frac{1}{5}$ of a number is 8. What is $\frac{2}{5}$ of the number?

3. $\frac{1}{4}$ of a number is 12. What is the number?

4. $\frac{1}{4}$ of a number is 10. What is $\frac{3}{4}$ of the number?

5. If $\frac{3}{4}$ of a number is 30, what is the number?

6. One-half a number added to $\frac{1}{4}$ of the same number equals what fraction of the number?

7. One-half a number added to $\frac{1}{4}$ of the same number equals 30. What is the number?

8. One-third of a number + one-sixth of the number = what fraction of the number?

9. One-third of a number added to $\frac{1}{4}$ of the number = what fraction of the number?

10. $\frac{1}{3}x + \frac{1}{4}x =$ what fraction of x? $\frac{x}{3} + \frac{x}{4} = ?$

852. When $x = 32$, find the value of three-fourths of x; *i.e.*, $\frac{3x}{4}$.

When $\frac{3x}{4}$, ($3x$ divided by 4) $= 24$, what is the value of $3x$? Of x?

Find the value of y, when $\frac{y}{3} = 12$. Of $2y$, when $\frac{2y}{3} = 24$.

Given the equation $\frac{4z}{5} = 20$; by what whole number can we multiply the first member to get rid of the fraction? If we multiply one member of an equation by any number, what must we do to the second member in order to preserve the equality?

853. Sight Exercises.

Give values of x, y, z, etc.:

1. $\frac{x}{5}=4$
2. $\frac{2y}{5}=8$
3. $\frac{z}{4}=7$
4. $\frac{3v}{4}=21$
5. $\frac{w}{2}+\frac{w}{4}=12$
6. $\frac{x}{2}+\frac{x}{3}=5$
7. $\frac{y}{3}+\frac{y}{6}=10$
8. $\frac{z}{3}+\frac{z}{4}=7$
9. $\frac{v}{5}+\frac{v}{5}=8$
10. $\frac{w}{3}+\frac{2w}{3}=32$
11. $\frac{x}{4}+\frac{x}{5}=9$
12. $\frac{x}{2}+\frac{2x}{5}=9$

854. Slate Exercises.

Find the value of the unknown quantity (x).

In each of the following equations, multiply both members by the least common denominator of the fractions.

1. $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=26$

Multiplying by 12, we have $6x+4x+3x=312$

2. $x+\frac{x}{2}+\frac{x}{3}=44$

Multiply by 6. $6x+3x+2x=264$

3. $\frac{x}{2}+\frac{x}{3}=35$
4. $\frac{x}{3}+\frac{x}{4}=49$
5. $\frac{x}{2}+\frac{2x}{3}=28$
6. $\frac{3}{5}x+\frac{5}{7}x=92$
7. $\frac{2x}{3}+\frac{3x}{4}=102$
8. $2\frac{7}{8}x=115$
9. $\frac{4x}{5}-\frac{2x}{3}=48$
10. $x-\frac{x}{40}=156$
11. $\frac{3x}{2}=27$
12. $1\frac{1}{2}x=27$
13. $\frac{11x}{4}=22$
14. $2\frac{3}{4}x=44$
15. $2x+\frac{3x}{4}=33$
16. $x+\frac{x}{5}=24$
17. $\frac{75x}{100}-\frac{33x}{50}=81$
18. $3\frac{1}{2}x-2\frac{3}{4}x=45$
19. $\frac{8x}{3}-\frac{2x}{5}=136$
20. $3\frac{5}{8}x=116$

21. $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=39$

22. $x-\frac{x}{2}-\frac{x}{3}=37$

23. $\frac{4x}{5}-\frac{2x}{9}+\frac{3x}{4}=239$

24. $\frac{5x}{9}+\frac{2x}{3}-\frac{x}{2}=52$

25. $x-\frac{3x}{4}=80$

26. $x+2x+\frac{3x}{7}=24$

855. Slate Problems.

1. The sum of two numbers is 90, and the smaller number is one-fifth of the larger one. What are the numbers?

$$\left(x+\frac{x}{5}=90.\right)$$

2. Divide 100 into two parts, one of which shall be $2\frac{1}{2}$ times the other.

3. After losing $\frac{1}{8}$ of his money, a man has \$714. How many dollars had he at first?

$$\left(x-\frac{x}{8}=714.\right)$$

4. A horse was sold for \$240, the seller thereby gaining one-third of what he originally paid for it. How much did he pay for it?

$$\left(x+\frac{x}{3}.\right)$$

5. One-half of a number added to one-fourth of the same number equals $66\frac{2}{3}$. What is the number?

6. The difference between $\frac{3}{4}$ of a number and $\frac{2}{5}$ of the same number is 15. Find the number.

7. One number is $\frac{3}{5}$ of another. Their sum is 55. What are the numbers?

8. Find a fraction equivalent to $\frac{7}{8}$, the sum of its numerator and its denominator being 60.

(Let $7x$ = numerator and $8x$ = denominator.)

9. Find a fraction equivalent to $\frac{5}{7}$, the difference between its numerator and its denominator being 24.

10. The sum of two numbers is 480, and the quotient obtained by dividing the greater by the less is 7. What are the numbers?

11. Find two numbers whose difference is 522 and whose quotient is 30.

12. A boy buys apples at 2¢, pears at 3¢, and oranges at 4¢, the same number of each. How many of each does he buy, if he pays 81¢ for all?

13. A girl bought 70 cents' worth of peaches and plums. She paid 3¢ each for the peaches and 2¢ each for the plums, buying four times as many of the former as of the latter. How many of each did she buy?

14. $1,500 is divided among three persons, the second of whom receives three times as much as the first, and the third three and one-half times as much as the first. Find the share of each.

15. A farmer paid for a cow three-sevenths as much as he paid for a horse. How much did he pay for each, if the latter cost $80 more than the former?

16. Three times a man's money increased by two-thirds of his money is equal to $1,100. How much money has he?

17. After giving away $\frac{3}{8}$ of his marbles and losing $\frac{1}{4}$ of them, Joseph has 24 left. How many had he at first?

18. Bought a coat, a hat, and an umbrella for $15, paying for the hat $1\frac{1}{2}$ times as much as for the umbrella, and for the coat $3\frac{1}{3}$ times as much as for the hat. Find the price of each.

19. A merchant purchased two pieces of cloth for $240, paying for one piece twice as much per yard as for the other. The former contains 36 yards and the latter 48 yards. How much does he pay per yard for each?

20. A farmer sold 4 times as many cows as horses, receiving for all $840, at the rate of $40 for a cow and $120 for a horse. How many of each did he sell?

TRANSPOSING.

856. Sight Exercises.

Give values of x, y, z, etc.:

1. $x + 15 = 21$
2. $2y + 15 = 21$
3. $z - 7 = 21$
4. $4w - 7 = 21$
5. $\frac{v}{2} + 3 = 8$
6. $\frac{x}{2} - 3 = 12$
7. $3y + 6 = 15$
8. $7y - 13 = 15$
9. $9y + 13 = 58$
10. $3y - 10 = 56$
11. $\frac{3v}{4} + 1 = 7$
12. $\frac{4w}{5} - 1 = 11$

857. If $x + 15 = 21$, $x = 21 -$ what?

When $x - 7 = 21$, $x = 21 +$ what?

If in the equation $2x + 15 = 21$, we take away 15 from the first member, what must we do to the second member to preserve the equality?

By *transposing* we mean bringing the unknown quantities (x, y, z, etc.) to one side of the equation, and the known quantities to the other.

NOTE. — In bringing a quantity from one side of the equation to the other, the *sign* of the quantity is changed.

858. Slate Exercises.

Find values of the unknown quantities.

NOTE. — Clear of fractions when necessary; then transpose.

1. $x + 37 = 56$
2. $4x - 5 = 83$
3. $3x - 43 = 98$
4. $7x + 13 = 111$
5. $x + 3x = 25 + 11$
6. $5x = x + 40$
7. $3x - 20 = x - 8$
8. $12 - 3x = 45 - 4x$

9. $3x - 6 = 48 + x$

10. $3x + 6 = 9 - 2x + 12$

11. $2x - 2 - 16 = x + 10$

12. $\frac{x}{3} - 8 = 24$

13. $\frac{x}{6} + 4 - 7 = 21$

14. $\frac{x}{2} + \frac{x}{3} = 10 + 5$

15. $7x - 5x = 20 + x + 4$

16. $6x - 14 = 16 + x$

17. $2x - 11 + 6x - 60 = 5x + 25$

18. $\frac{x}{2} + \frac{x}{3} - 5 = 10$

19. $2x - 6 = 16 + \frac{x}{2} - \frac{x}{3}$

20. $2x + \frac{3x}{5} - \frac{x}{2} = \frac{3x}{4} + 27$

859. Slate Problems.

1. The sum of three numbers is 51. The second is 5 less than the first, and the third is 10 less than the first. What are the numbers?

Let x = first number,

$x - 5$ = second number,

$x - 10$ = third number;

$x + x - 5 + x - 10 = 51.$

Transposing $x + x + x = 51 + 5 + 10,$

$3x = 66,$

$x = 22$, first number,

$x - 5 = 17$, second number,

$x - 10 = 12$, third number.

2. Add 45 to four times a number, and you will have seven times that number. What is the number?

$(7x = 45 + 4x.)$

3. Nine times a number less 27 equals six times the number. Find the number.

4. Two boys have together 48 marbles. One has 18 more than the other. How many has each?

$(x, x + 18.)$

5. The length of a rectangular lot is 75 feet more than the breadth. The distance around it is 250 feet. What are its dimensions?

6. A piece of land containing 86 acres is to be divided into two fields, one of which shall be 8 acres larger than the other. How many acres in each field?

7. At a certain election 2,436 votes were cast for two candidates, the successful one receiving 318 more votes than his opponent. How many votes did each receive?

8. A man, being asked his age, replied that if he were half as old again and 7 years more he would be 100. What was his age?

9. The sum of two numbers is 96, and their difference is 72. Find the numbers.

(Let x = less, $x + 72$ = greater.)

10. After paying $\frac{1}{3}$ and $\frac{1}{4}$ of my debts, I still owe \$45. How much did I owe originally?

11. Divide 45 into two parts, one of which shall be 6 less than twice the other.

12. William has \$5 more than John, and three times William's money added to five times John's would be \$103. How many dollars has each?

13. I bought 3 cows and 4 horses for \$635, paying \$80 apiece less for the cows than for the horses. How many dollars apiece did I pay for each?

14. Mary has a dollar in dimes and five-cent pieces. She has 11 more of the latter than of the former. Find the number of pieces of each denomination.

15. Divide 100 into two parts whose difference shall be 48.

16. In a class of 54 pupils, the girls outnumber the boys by 12. How many are there of each?

17. $18,000 is divided among three children, the second of whom receives $2,400 more than the first, and the third of whom receives $2,400 more than the second. Find the share of each.

18. The greater of two numbers is 11 more than 3 times the less. Their difference is 33. What are the numbers?

19. A boy spent a dollar for postal cards, 2-cent stamps, and 5-cent stamps. He bought 15 more 2-cent stamps than 5-cent stamps, and 15 more postal cards than 2-cent stamps. How many of each did he buy?

20. A farmer has 88 head of stock — horses, cows, and sheep. He has 17 more cows than horses, and the number of sheep is 22 greater than that of the cows and horses together. How many are there of each?

CHAPTER XI.

PERCENTAGE. — INTEREST. — DISCOUNT. — SURFACES AND VOLUMES.

PERCENTAGE.

860. Preliminary Exercises.

Per cent means hundredths. Seven per cent means seven hundredths, $\frac{7}{100}$, or .07. It is written 7%.

How many hundredths of a number is its half? $\frac{1}{4}$ = how many hundredths? $\frac{1}{5}$? $\frac{1}{10}$? $\frac{1}{25}$? $\frac{1}{50}$?, $\frac{3}{4}$? $\frac{2}{5}$?

What per cent of a number is the half of it? $\frac{1}{4}$? $\frac{1}{8}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{11}$? $\frac{1}{12}$? $\frac{1}{15}$? $\frac{1}{20}$? $\frac{1}{25}$? $\frac{1}{50}$? $\frac{1}{100}$? $\frac{1}{200}$? $\frac{1}{400}$?

861. What per cent of a number is $\frac{1}{8}$ of it? $\frac{3}{4}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{3}{10}$? $\frac{7}{10}$? $\frac{9}{10}$? $\frac{3}{20}$? $\frac{7}{20}$? $\frac{9}{20}$? $\frac{2}{25}$? $\frac{3}{25}$? $\frac{4}{25}$? $\frac{6}{25}$? $\frac{7}{25}$? $\frac{3}{50}$? $\frac{7}{50}$? $\frac{3}{100}$?

862. 1 per cent of a number is equal to what fraction of it? 2%? 3%? 4%? 5%? 6%? 7%? 8%? 9%? 10%? 12%? 15%? 20%? 25%? 30%? 36%? 40%? 50%? 60%? 75%? 80%? 90%?

863. What fractions are equal to the following:

$12\frac{1}{2}\%$? $16\frac{2}{3}\%$? $33\frac{1}{3}\%$? $37\frac{1}{2}\%$? $6\frac{1}{4}\%$? $6\frac{2}{3}\%$? $3\frac{1}{3}\%$? $62\frac{1}{2}\%$? $66\frac{2}{3}\%$? $87\frac{1}{2}\%$? $\frac{1}{2}\%$? $\frac{1}{4}\%$? $2\frac{1}{2}\%$? $\frac{1}{8}\%$?

864. 3 times a number is what per cent of it? $2\frac{1}{2}$ times? $1\frac{1}{4}$ times? $16\frac{1}{3}$ times? $4\frac{1}{5}$ times? $9\frac{1}{10}$ times?

865. Oral Exercises.

1. Find $12\frac{1}{2}\%$ of 1 gal.
2. $37\frac{1}{2}\%$ of \$24
3. $33\frac{1}{3}\%$ of 81 cows
4. 6% of 150 lb.
5. 4% of 125 bu.
6. $62\frac{1}{2}\%$ of 1 pk.
7. $4\frac{1}{2}\%$ of \$200
8. 99% of 200 gal.
9. $\frac{1}{2}\%$ of \$640
10. $\frac{1}{4}\%$ of 800 yd.
11. $66\frac{2}{3}\%$ of 66 horses
12. $16\frac{2}{3}\%$ of 1 yd.
13. 81% of \$300
14. $2\frac{1}{2}\%$ of 80 sheep
15. 40% of \$2.50
16. 20% of 65 rd.
17. 10% of 15 lb.
18. $3\frac{1}{3}\%$ of \$60
19. $\frac{1}{8}\%$ of \$72
20. $1\frac{1}{4}\%$ of \$96

866. Slate Exercises.

1. Find $6\frac{2}{3}\%$ of \$95.10
2. 12% of \$37.50
3. $33\frac{1}{3}\%$ of \$28.80
4. $\frac{1}{8}\%$ of \$1,240
5. $4\frac{1}{2}\%$ of \$92.40
6. 450% of \$92.40
7. 20% of \$51.60
8. $1{,}400\%$ of \$89.70
9. $12\frac{1}{2}\%$ of \$73.28
10. $13\frac{1}{3}\%$ of \$27.60
11. $6\frac{1}{4}\%$ of \$25.60
12. $3\frac{1}{3}\%$ of \$47.40
13. $5\frac{1}{2}\%$ of \$29.50
14. 860% of \$38
15. $\frac{1}{8}\%$ of \$24
16. 25% of \$52.36
17. 60% of \$33.30
18. 6% of \$19.50
19. $6\frac{2}{3}\%$ of \$47.40
20. 12% of \$62.50
21. $4\frac{1}{2}\%$ of \$71.50
22. 40% of \$28.30
23. 160% of \$39.40
24. 84% of \$23.75
25. $66\frac{2}{3}\%$ of \$825
26. 75% of \$59.20
27. $82\frac{1}{2}\%$ of \$392
28. $93\frac{3}{4}\%$ of \$4.96

TO FIND THE BASE OR THE RATE.

867. Slate Problems.

29. Find 25% of 280.

280 is called the *base*, 25 is called the *rate*.

Multiplying 280 by $\frac{25}{100}$, we get 70, which is called the *percentage;* that is,

868. $Base \times \frac{Rate}{100} = Percentage.$

30. Find x per cent of 65.

31. If x per cent of 65 is 26, find the value of x.

32. Find 25% of x.

33. If 25 per cent of x is 42, what is the value of x?

34. Increase x by 20% of itself.

35. If x increased by 20 per cent of x equals 132, what is the value of x?

36. Diminish x by $33\frac{1}{3}\%$ of itself.

37. Find the value of x, when x diminished by $33\frac{1}{3}$ per cent of itself equals 78.

38. Find $x\%$ of $\frac{2}{3}$.

39. What is the value of x, if $x\%$ of $\frac{2}{3} = 3\frac{1}{3}$?

40. Find $\frac{1}{3}\%$ of x.

41. Find the value of x, when $\frac{1}{3}\%$ of $x = 23$.

42. What (x) per cent of 65 is 26?

43. 24 is 18 per cent of what number (x)?

44. 250% of what number (x) = 180?

45. What number (x) increased by 25% of itself equals 85?

46. $\frac{3}{5}$ = what per cent of $\frac{4}{5}$?

47. $\frac{3}{4}$ is what per cent of $\frac{4}{5}$?

$\frac{3}{4} = \frac{x}{100}$ of $\frac{4}{5}$; *i.e.*, $\frac{x}{125} = \frac{3}{4}$. Clear of fractions and solve.

48. Find the interest on x dollars for 1 year at $5\frac{1}{2}\%$.

49. What sum at $5\frac{1}{2}\%$ gives a yearly interest of \$44?

50. What is $66\frac{2}{3}\%$ of $\frac{3}{5}$?

51. What per cent of 88 is 33?

52. Find the difference between $\frac{1}{8}$ of 800 and $\frac{1}{8}$ per cent of 800.

53. Find the selling price of a horse that cost \$175 and was sold at 25% profit.

54. How much insurance does a man receive for \$12.50 when the rate is $2\frac{1}{2}\%$?

55. What is 16% of $6\frac{1}{4}$?

56. $3\frac{1}{3}$ is what per cent of $\frac{2}{3}$?

57. What per cent of \$389.50 is \$124.64?

58. \$174.04 is 95% of what sum of money?

59. What number increased by 16% of itself equals 1,276?

60. 984 is $133\frac{1}{3}\%$ of what number?

61. $\frac{3}{4}\%$ of a number is 81. What is the number?

$$\frac{3}{400} \text{ of } x = \frac{3x}{400} = 81.$$

62. An importer paid duties amounting to \$386.75. If the duty was 25% of the value of the goods, what was their value?

63. A collector deducts $2\frac{1}{2}\%$ commission, and returns to his employer \$745.68. How much did he collect?

$$x - \frac{2\frac{1}{2}}{100}x = 745.68; \text{ or, } x - \frac{x}{40} = 745.68; \text{ or, } \frac{39x}{40} = 745.68.$$

64. A commission merchant receives $2\frac{1}{2}\%$ commission for buying grain for a customer. The cost of the grain and his commission amount to \$4,223. How much does the grain cost?

x = cost of the grain; $\frac{x}{40}$ = commission.

65. A capitalist sends a commission merchant \$8,670 to invest in cotton and to include commission at 2%. How much does the commission amount to?

66. A commission merchant receives \$1,071 to invest in oats after deducting 2% commission. How many bushels of oats at $30¢$ per bushel does he purchase?

Should the commission merchant deduct 2% of \$1,071, or 2% of the cost of the oats?

67. A house is insured for $\frac{2}{3}$ of its value at $\frac{7}{8}\%$. The annual cost (premium) is \$8.40. What is the value of the house?

Let x = value. Then $\frac{2x}{3} \times \frac{7}{800} = \frac{7x}{1200} = 8.40$.

68. What will be the taxes on a house worth \$48,000 and assessed at $\frac{2}{3}$ of its value, the tax rate being \$18.50 per \$1,000 of assessed value?

869. Oral Exercises.

1. 3 is what part of 6?
2. 3 is what decimal of 6?
3. 3 is how many hundredths of 6?
4. 3 is what per cent of 6?
5. 6 is what per cent of 3?
6. What number is 50% of 6?
7. 3 is 50% of what number?
8. 2 is what $\%$ of 100?
9. 2 is what $\%$ of 200?

10. What number is 5% of 100?

11. What % of 20 is 1?

12. 4 is what % of 200?

13. 3 is $\frac{1}{2}$% of what number?

14. What per cent of 9 is $20\frac{1}{4}$?

15. What number, increased by $\frac{1}{4}$ of itself, equals 10?

16. What number, increased by 25% of itself, equals 20?

17. 65 diminished by 5% of itself, equals what?

18. Buying price \$100, selling price \$112.50. Gain %?

19. Cost \$80, profit 20%. Selling price?

20. What principal will give \$30 yearly interest at 6%?

21. At $\frac{1}{8}$ of 1%, how much will I pay for insurance on \$10,000?

22. If insurance costs $\frac{3}{4}$%, how much can I get for \$60?

23. Bought sugar at 4¢ per lb.; sold it for 5¢. Profit %?

NOTE. — Per cent of profit is based upon the cost.

24. $\frac{3}{4}$ is what % of $\frac{5}{6}$?

(Reduce both to a common denominator.)

25. $1\frac{1}{2}$ is what % of $6\frac{2}{3}$?

PROFIT AND LOSS.

870. Slate Problems.

Find the profit or loss, and the selling price:

1. Cost \$1,876; gain 15%.
2. Cost \$36.75; loss 20%.
3. Cost \$1,012.50; gain $16\frac{2}{3}$%.
4. Cost \$875; loss 5%.
5. Cost \$934.56; gain $12\frac{1}{2}$%.

Find the profit or loss per cent.

N. B. — Profit or loss per cent is based upon the *cost*.

6. Cost \$600; selling price \$618.

Gain or loss = x% of cost. $18 = x$% of 600.

7. Cost \$1,203; selling price \$802.

8. Cost \$86.20; selling price \$73.27.

9. Cost \$908.40; selling price \$1,090.08.

10. Cost \$84; selling price \$78.75.

11. Selling price \$78.75; loss \$5.25.

Gain or loss = x% of *cost*.

12. Selling price \$150; gain \$25.

13. Selling price \$831.25; loss \$43.75.

14. Selling price \$1,051.38; gain \$116.82.

15. Selling price \$843.75; loss \$168.75.

Find the cost, and the profit or loss:

16. Selling price \$468.75; gain 25%.

Let x = cost.

17. Selling price \$73.84; loss 20%.

18. Selling price \$1,646.08; gain $33\frac{1}{3}$%.

19. Selling price \$204; loss 15%.

20. Selling price \$66.30; gain 4%.

21. A man buys a horse for \$275, and sells it at a profit of 20 per cent. How much does he gain?

22. A cow is sold for \$75, on which the profit is \$15. What is the gain per cent?

23. A lot is sold for \$960, which is 20 per cent more than it cost. Find the cost of the lot.

24. Tea that costs 32¢ per pound is sold for 48¢. What is the gain per cent?

25. A man buys a horse for $175 and sells it for $200. What per cent does he gain?

26. What per cent is lost on a horse costing $200, and sold at a loss of $25?

27. What is the selling price of dress goods costing $33\frac{1}{3}$¢ per yard, on which a profit of $12\frac{1}{2}$ per cent is made?

28. Sold a coat for $33.60, thereby losing 16 per cent. What was its cost?

29. How much did I gain on a house for which I paid $8,760, my profit being $2\frac{1}{2}$ per cent?

30. A man after spending 10 per cent of his money for clothing, 25 per cent for board, and 30 per cent for incidentals, has $70 left. What did he have at first?

31. The population of a city was 16,000 in 1880. In 1890 it was 22,000. What was the gain per cent in ten years?

(Which is the base?)

32. The population of a city was 30,000 in 1890, a gain of 6,000 over the previous census. What was the increase per cent?

(Have the correct base.)

33. 16 shares of bank stock, face (par) value $100 each, were sold at $2\frac{7}{8}$ per cent above the face value. How much was received for the stock?

34. What will be the cost of 84 shares of stock, face (par) value $50 each, at $3\frac{1}{2}$ per cent below par?

35. Find the interest on $784.50, at 4 per cent, for 3 years 7 months 15 days.

MEASUREMENTS.

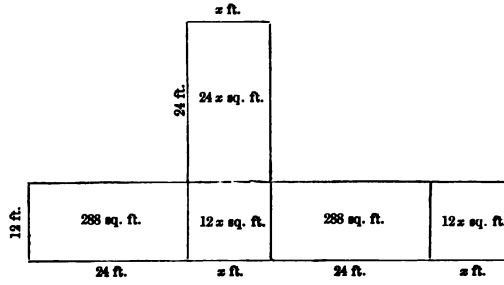
871. Slate Problems.

1. A tank 18 feet long, 15 feet wide, contains 30 cubic yards. What is its depth in feet?

$$\left(18 \times 15 \times x = 30 \times 27;\ x = \frac{30 \times 27}{18 \times 15}\right).$$

2. The surface of the walls and the ceiling of a room 24 feet long and 12 feet high is 1,440 square feet. What is the width of the room?

$$288 + 12x + 288 + 12x + 24x = 1{,}440.$$



3. A man owns a rectangular plot of ground 132 feet long, 110 feet wide. He divides it into two equal parts by a fence running from the north-east to the south-west corner. What part of an acre does each piece contain?

Indicate operations. Cancel.

4. How many yards of fence will be needed to enclose a rectangular field, 80 rods long, containing 25 acres?

5. Make a diagram of a floor 6 yards long, 4 yards wide. Show how many strips of carpet, 27 inches wide, running cross-wise, will be needed to carpet the room, no allowance being made for waste?

6. How many sheets of tin 6 inches by 4 inches will be required to cover a roof 20 feet by 30 feet, making no allowance for overlapping?

7. The owner of a lot 25 feet by 100 wishes to build around it a tight board fence 6 feet high. How many running feet of fence will there be? How many square feet?

(Draw the "development" of the fence, marking on it the dimensions.)

8. How many boards 12 feet long, 6 inches wide, will it take to build such a fence?

9. A plot of ground 300 feet by 200 feet is bought at $181.50 per acre. It is sold in lots 100 feet square at $160 each. What is the profit?

10. How many cakes of ice 4 feet long, 2 feet wide, can be cut from a rectangular pond 320 feet long, 160 feet wide, no allowance being made for waste? (Cancel.)

11. If the ice is $1\frac{1}{2}$ feet thick, how many cubic feet of ice will there be? Find its weight in tons, at 57.5 pounds per cubic foot.

12. Give the length of a shed 15 feet high, 32 feet wide, that will exactly hold the ice cut from the above pond. (Cancel.)

13. The owner of a plot of ground 640 feet long, 440 feet wide, cuts two streets, each 40 feet wide, through the middle of the plot, one running north and south, the other east and west. How many square feet of land had he originally? How many square feet has he left for building purposes? (Make diagram.)

14. What will it cost him, at 60¢ per square yard, to make the above streets?

15. If the remaining ground is divided into building lots 25 by 100, how many lots will there be?

16. How many square yards of plastering will be required for the walls and the ceiling of a room 21 feet wide, 30 feet long, 9 feet high, deducting for 2 windows, each 6 feet by 3 feet, and a door 8 feet by $4\frac{1}{2}$ feet?

PROFIT AND LOSS.

872. Slate Exercises.

1. Bought for $36; sold for $40. Gain per cent?
2. Bought for $40; sold for $36. Loss per cent?
3. Cost 36¢; selling price 40¢. Gain per cent?
4. Cost $24; gain 10%. Selling price?
5. Selling price 70¢; loss 75%. Cost?
6. Buying price 70¢; gain 75%. Selling price?
7. Cost $20; selling price $29. Gain %?
8. Cost $20; selling price $290. Gain %?
9. Cost $20; selling price $20.90. Gain %?
10. Cost $20; selling price $20.09. Gain %?
11. Selling price $300; loss $100. Loss %?
12. Cost $300; gain $100. Gain %?
13. Selling price $175; cost $150. Gain %?
14. Selling price $375; gain 25%. Profit?
15. Cost $36.50; selling price $28.50. Loss %?
16. Selling price $33.95; loss 3%. Cost?
17. Cost $75.50; loss 5½%. Selling price?
18. Selling price $20.16; gain 5%. Cost?
19. Selling price $64; profit $16. Gain %?
20. Cost $37.50; selling price $42. Gain %?
21. Selling price $26.88; loss 6⅜%. Loss?
22. Cost $24; gain 1%. Selling price?
23. Selling price $41.16; gain 5%. Cost?
24. Selling price $29.83; loss 5%. Loss?

25. Cost $19.50; loss 6%. Selling price?
26. Cost $84; selling price $184. Gain %?
27. Selling price $700; gain 250%. Cost?
28. Cost $324.80; gain 175%. Selling price?
29. Selling price $6.50; loss $13\frac{1}{3}$%. Loss?
30. Cost $346.50; selling price $339.57. Loss %?
31. Selling price $17.64; loss 2%. Cost?
32. Cost $4,613; gain $13\frac{1}{7}$%. Profit?
33. Selling price $26.69; gain $6\frac{1}{4}$%. Profit?
34. Cost $8,766; loss $\frac{7}{9}$%. Loss?
35. Selling price $50.00; profit $4.20. Gain %?
36. Cost $37.50; gain $5.40. Gain %?
37. Selling price $205.20; loss $45.90. Loss %?
38. Cost $25.60; gain $6\frac{1}{4}$%. Selling price?
39. Selling price $17.35; loss $\frac{6}{7}$%. Loss?
40. Profit $36; gain $\frac{4}{7}$%. Cost?
41. Loss $28.17; loss $3\frac{13}{100}$%. Selling price?
42. Cost $3,864.25; loss 8%. Loss?
43. Selling price $89.37; profit $6.62. Gain %?
44. Cost 8\frac{3}{4}$; loss $\$\frac{3}{4}$. Loss %?
45. Selling price $22.35; gain $6\frac{3}{7}$%. Cost?
46. Profit $47.25; gain $7\frac{1}{2}$%. Selling price?
47. Loss $38.46; loss $\frac{1}{2}$%. Cost?
48. Cost $75.52; gain $3\frac{1}{8}$%. Profit?
49. Selling price $49.95; loss $4.05. Loss %?
50. Cost $3,879; loss $\frac{5}{9}$%. Loss?

INTEREST.

873. In calculating interest, take 30 days to a month, 12 months to a year.

874. $Principal \times \frac{Rate}{100} \times Time\ (in\ years) = Interest.$

NOTE. — Change given time to years.

2 yr. 6 mo. $= 2\frac{1}{2}$ yr. $= \frac{5}{2}$ yr.

1 yr. 7 mo. $= 19$ mo. $= \frac{19}{12}$ yr.

4 mo. 10 da. $= 4\frac{1}{3}$ mo. $= \frac{4\frac{1}{3}}{12}$ yr. $= \frac{13}{36}$ yr.

1 yr. 5 mo. 15 da. $= 17\frac{1}{2}$ mo. $= \frac{17\frac{1}{2}}{12}$ yr. $= \frac{35}{24}$ yr.

5 mo. 17 da. $= 167$ da. $= \frac{167}{360}$ yr.

875. Slate Exercises.

Find the interest on:

1. $750, for $2\frac{1}{2}$ years, at 6%.
2. $84.75, $3\frac{1}{5}$ months, at 4%.
3. $308.25, from Oct. 1 to Oct. 21, at 5%.
4. $464.75, 8 mo. 12 da., at 6%.
5. $360, 33 da., at 5%.
6. $94.43, 2 mo. 3 da., at 7%.
7. $400, 1 yr. 1 mo. 1 da., at $4\frac{1}{2}$%.
8. $720, 21 da., 7%.
9. $1,000, 8 da., 5%.
10. $630, from April 1, 1890, to Jan. 16, 1892, at 6%.

$$\begin{array}{r} 1892-1-16 \\ 1890-4-\ 1 \\ \hline \end{array}$$

11. $394.50, 2 yr. 1 mo. 7 da., at 6%.
12. $1,560, 3 yr. 4 mo. 9 da., at $4\frac{1}{2}$%.

13. $960, 11 mo. 24 da., 2%.

14. $86.40, 1 yr. 9 mo. 20 da., 5%.

15. $108.36, 4 yr. 7 mo. 10 da., $3\frac{3}{4}$%.

876. *Amount = Principal + Interest.*

Find the amount:

16. $813, from April 19, 1889, to March 4, 1894, at 6%.

17. $960, from Jan 1, 1893, to Dec. 21, 1894, at 4%.

18. $27.84, for 3 yr. 6 mo. 9 da., at 6%.

19. $48.90, for 17 da., at 6%.

20. $144, for 2 yr. 5 da., at $3\frac{3}{4}$%.

21. $834.76, for 15 mo. 27 da., at $4\frac{1}{2}$%.

22. $5,760, for 1 yr. 5 mo. 29 da., at 5%.

23. $2,346.50, for 7 yr. 13 da., at 3%.

24. $1,892, for 3 yr. 5 mo., at 7%.

25. $150.40, for 1 yr. 2 mo. 3 da., at 6%.

877. Interest-bearing Demand Notes.

26. SAN FRANCISCO, Jan. 7, 1893.

On demand, I promise to pay William Britt, or order, Seven Hundred Sixty-five $\frac{40}{100}$ Dollars, value received, with interest at 6 per cent.

$765\frac{40}{100}$. ARTHUR TOWNSEND.

How much money will be required to pay the above note, with interest, July 15, 1894?

27. A demand note, dated Sept. 25, 1892, with interest at 8% from date, is paid Jan. 2, 1895. How much was due, the face of the note being $750?

28. Find the amount due March 4, 1894, on a note for $365.84, dated May 20, 1892, with interest from date at 7%.

29. Find the amount necessary, Oct. 16, 1896, to pay a note of $1,240, with interest at 6% from Aug. 15, 1892.

30. An interest-bearing note for $87.60 is dated April 3, 1886. How much is due on it for principal and interest Jan. 2, 1894? Rate 4½%.

878. Oral Problems.

1. Find the interest on $300, for 1 year 7 months, at 4%.

$12 per year is how much for 7 months?

2. On $60, for 33 days, at 6%.

$3.60 for 360 days is how much for 33 days?

3. On $120, from Jan. 1, 1893, to July 1, 1894, at 5%.

4. How long will it take $100 to produce $15 interest at 6%?

5. At what rate per cent will $50 produce $6 in 2 years?

6. What is the interest on $300, at 6%, from Feb. 1 to Feb. 21?

7. What part of a year is 72 days?

8. Find the interest at 4%, for 90 days, on $150.

9. On $240, for 36 days, at 5%.

10. What is the amount of $200, for 3 years 1 month, at 6%?

11. How long will it take $1 to make $1 interest at 5%?

12. How long will it take any sum to double itself at 6%?

13. How long will it take $14.90 to double itself at 4%?

14. At ½ per cent per month, find the interest on $90 for 16 months.

15. 5% per year is 1% for how many days?

16. 4½% per year is 1% for how many days?

17. Find the interest on $75, at 5%, for 72 days.

18. Find the interest on \$63, at $4\frac{1}{2}\%$, for 80 days.

19. Find the interest on \$570, at 6%, for 60 days.

20. Find the interest on \$840, at 6%, for 30 days.

21. Find the interest on \$150, at 3%, for 120 days.

22. Find the interest on \$275, at 4%, for 90 days.

23. Find the interest on \$360, at 5%, for 144 days.

879. Questions from Examinations for N. Y. State Certificate.

1. What fractional part of 30 rd. 5 yd. 1 ft. is 8 rd. 4 yd. 2 ft.?

2. A coal bin is 6 feet long and 4 feet wide. How deep must it be to contain 5 tons of stove coal, if one ton occupies 36 cubic feet of space?

3. A man walking at the rate of 3 mi. 96 rd. per hour will walk how far in 3 hr. 16 min.?

4. $(\frac{3}{4}+\frac{7}{8})\times\frac{4\frac{1}{2}\times\frac{8}{27}}{\frac{3}{7}\text{ of }16\frac{1}{8}}=?$

5. If a merchant pays $6\frac{1}{4}¢$ per yard for muslin, and sells the same for $7\frac{1}{4}¢$ per yard, what is his gain per cent?

6. Find the cost of 96 feet of pine lumber at \$25 per M., and 1,650 laths at \$3 per M.

7. Divide 22.5 by 51.75, and express the result in the form of a fraction.

8. In the written number 185.4, the number expressed by the first two (left-hand) figures is how many times the value expressed by the second two figures?

9. An agent buying wheat is offered a commission of 4¢ per bushel, or one of $4\frac{1}{2}$ per cent, and he chooses the former. The average price paid per bushel is $91\frac{1}{2}¢$. Does he gain or lose by his choice, and how much per bushel?

10. Express decimally, and also as a common fraction, the value of each of the following: 115 per cent; $\frac{1}{16}$ of 1 per cent; $\frac{13}{32}$ of 1 per cent.

11. Three quarts dry measure is what per cent of a bushel?

12. Make and solve a problem illustrating the application of percentage to the finding of an agent's commission.

13. How many times can a jar holding $\frac{1}{4}$ of $\frac{2}{3}$ of a gallon be filled from another jar containing $\frac{3}{4}$ of $3\frac{1}{3}$ gallons?

14. My agent collects the yearly rent of my house, and retains $13.25, the amount of his commission at $2\frac{1}{2}$ per cent. For how much does the house rent per year?

15. What decimal part of a mile is 39.27 yd.?

16. Multiply eight hundred (units) and forty-six ten-thousandths by three thousand forty millionths.

17. 3 oz. is what per cent of 5 lb.?

18. Find the interest of $1,000, for 19 days, at 5%.

19. M bought $\frac{5}{27}$ of a manufacturing business for $3,517.85, and N bought $\frac{4}{19}$ of the same business at the same rate. How much did N's interest cost him?

20. Find the number of days from March 2, 1893, to Aug. 11, 1893.

21. 3 bu. 1 pk. 5 qt. is what per cent of 20 bu. 1 pk. 6 qt.?

22. The tax on an assessment of $8,500 is $48.37. Required the rate on $1,000 of assessment.

23. Find the amount of an agent's sales, when his commission at 5 per cent amounts to $37.65.

24. What is the interest on $128.40, for 1 yr. 5 mo. 17 da., at 6 per cent?

25. Reduce to simplest form: (*a*) $\frac{3}{4}$ of $\frac{1}{2}$ of $2\frac{2}{5}$; (*b*) $\frac{\frac{5}{18}}{\frac{8}{17}}$.

880. Questions from U. S. Civil Service Examinations.

1. The Navy Department had 10,336½ square yards of bunting, which it ordered to be made into flags as follows:

10 measuring $6 \times 9\frac{1}{4}$ feet,
20 measuring $7 \times 14\frac{1}{2}$ feet,
5 measuring $14 \times 33\frac{3}{4}$ feet,

and the remainder measuring $3\frac{3}{4} \times 6\frac{1}{2}$ feet. How many of the latter were made?

2. A ship has provisions to last her crew of 450 men 5 months. How many men must be discharged to make the supply hold out 9 months?

3. The government purchased 96 acres of land at \$41.34½ per acre, 185 acres at \$53.42¼ per acre, and 36 acres at \$13.14¾ per acre. What was the area of the land, what its cost, and what the average price per acre?

4. A clerk has an income of \$1,100 per annum. He pays 20 per cent of it for board, 1½ per cent for washing, 2 per cent for incidentals, 15 per cent for clothing, 9 per cent for other expenses, and loses in various ways 50 per cent of the amount then remaining. What sum does he have left?

5. Add the following:

Two thousand (units) and three thousandths.
Two hundred-thousandths.
Nine millionths.
Seven millions seventy-nine.
Ten hundred-thousandths.

6. Multiply 9.846358 by 49.05.

7. Divide $\frac{7\frac{5}{8}}{4\frac{6}{9}}$ by $\frac{2}{3}$ of $\frac{2}{5}$ of $\frac{5\frac{3}{4}}{8\frac{5}{7}}$, and give the result in decimals.

8. What is the interest on \$765.34 for 93 days at 5¼ per cent?

9. A regiment of 940 men, during the war lost 532 of their number by death and 125 by desertion. What was the percentage of loss in each case, and what per cent remained for service?

10. Find the cost of each of the following items of a soldier's ration:

12 oz. of pork at \$17.87½ per bbl. (200 lb.).

1 lb. 6 oz. of flour at \$6.62½ per bbl. (196 lb.).

2.4 oz. of beans at \$ 2.20 per bushel (60 lb.).

1.6 oz. of rice at \$6.75 per 100 lb.

4.08 oz. of potatoes at 95¢ per bushel (60 lb.).

1.6 oz. of coffee at 22½¢ per pound.

1.4 oz. of sugar at 6⅛¢ per pound.

11. Express in words the following amounts: 2,009,001; 6.00107; 9,807,300.026; $\frac{28\frac{1}{4}}{37}$; $725\frac{45}{84}$.

12. Subtract 754.75478 from 9,026.8399.

13. Divide 12.82561 by 1.505.

14. What is the product of $\frac{1}{8}$ of $\frac{6}{9}$ of 3 of $15\frac{1}{7}$. State the result in decimals.

15. An army fought two battles. In the first it lost 15 per cent, and in the second 20 per cent of the original number, after which it mustered 19,500 men. What was the original strength of the army?

16. A owned $\frac{7}{8}$ of a ship and sold $\frac{4}{5}$ of his share to B, who sold $\frac{5}{9}$ of what he bought to C, who sold $\frac{6}{7}$ of what he bought to D. What part of the whole vessel did D buy?

17. If a letter-carrier in delivering letters takes 47,520 steps in a day, each step averaging 20 inches, how many miles does he walk?

18. A farmer exchanged 340 bushels of corn worth 75¢ per bushel, for barley worth $1 per bushel, and oats worth 50¢ per bushel. How many bushels of each did he receive, the quantity of barley and oats being equal?

19. Thirty-two clerks are to distribute 36,000 letters on a certain day. Half of the clerks are experienced men and half of them new men. If each experienced man does twice as much as a new man, how many letters will be distributed by one of each kind?

20. A merchant sold a lot of damaged sugar at a loss of 25 per cent, receiving $1,972.65. How much did the sugar cost him?

881. U. S. Civil Service. Clerkships in Department of Interior.

1. Add the following:

Two thousand (units) and two thousandths.

Two hundred-thousandths.

Nine millionths.

Seven million seventy-nine.

Four hundred thousand seven hundred ninety-seven.

One hundred sixty-seven million, eight hundred seventy-one thousand, two hundred forty-nine.

Nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred ninety-one.

Ten hundred-thousandths.

One hundred eighty-nine thousand, one hundred sixty-seven.

2. Multiply 874.0691 by 79.0473.

3. Divide 1,979 by 11.225, and divide 127.555 by 63.

4. What is a pile of wood 15 feet long, $10\frac{1}{2}$ feet high, and 12 feet wide worth, at $\$4\frac{1}{2}$ per cord?

(1 cord = 128 cu. ft.)

5. Find the interest on $419.10, for 1 year 8 months and 15 days, at 6%.

6. Find a common divisor of 72 and 90.

7. $2\frac{1}{2}¢$ is what per cent of $10¢$?

8. $10¢$ is what per cent of $2\frac{1}{2}¢$?

9. If seven men can mow a field in 19 days, in how many days will 20 men mow it?

10. When a gold dollar is worth $1.13 in paper money, what will be the value in gold of a dollar in currency?

11. Reduce $7\frac{5}{8}$ to a decimal.

12. Add $\frac{24}{48}$ to $\frac{12}{28}$.

13. Subtract $\frac{2}{5}$ from .97.

14. Divide $\frac{3}{4}$ of $\frac{7}{12}$ by $\frac{1}{3}$ of $\frac{2}{12}$.

882. Worcester Public Schools. Examination Questions. Oral.

1. How many hours in $\frac{5}{6}$ of a day?

2. What is the interest of $300 for 3 years 4 months, at 6%?

3. A man is 60 years old; $16\frac{2}{3}\%$ of his age is $\frac{2}{5}$ of his son's age. How old is his son?

4. Buy cloth at $45¢$ a yard, and sell it at $60¢$ a yard. Gain per cent what?

5. What is the difference between $\frac{2}{7}$ of $6\frac{1}{2}$ and $\frac{3}{7}$ of $4\frac{1}{3}$?

6. Five bushels is 5% of what number of bushels?

7. There is a pole standing so that $\frac{2}{5}$ of it is in water and $\frac{1}{4}$ as much is in the sand. What part of the pole is in the sand?

8. What is $\frac{5}{8}$ of $2\frac{1}{2}$?

9. A man bought a watch for $64, and sold it at a gain of $12\frac{1}{2}$ per cent. What was the selling price?

10. How many feet in a quarter of a mile?

11. How many oranges in $\frac{1}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{7}{10}$ of an orange?

12. 15 is $\frac{5}{6}$ of what number?

13. What is the cost of $7\frac{7}{8}$ pounds of sugar at 11¢ a pound?

14. In $2\frac{3}{4}$ pecks how many quarts?

883. Brooklyn Public Schools. Examination Questions.

1. Add the greatest and the least of the three fractions $\frac{17}{19}$, $\frac{6}{7}$, $\frac{19}{28}$; and divide the sum by the remaining fraction.

2. Multiply 82 ten-thousandths by 7 and 5 hundredths, and divide the product by 705 millionths.

3. To make 15 per cent profit, what must goods be marked that cost $1.10 per yard?

4. The dividend is 9,876, the quotient is 87, the remainder is 45. Find the divisor.

5. What per cent of a school is boys, and what per cent girls, there being 640 of the former and 560 of the latter?

6. 43 gallons 3 quarts 1 pint alcohol are sold for $70.20. What is the price per gallon?

7. How many planks 16 feet long and 6 inches wide will be needed for a floor 48 feet long and 32 feet wide?

8. A horse costing $160 is sold for $180. What is the gain per cent? What is the loss per cent when a horse costing $180 is sold for $160?

9. Find the interest on $375.50 for 1 year 5 months 14 days, at $3\frac{1}{2}$ per cent?

10. A merchant sold 600 barrels of flour for $3,450, at a loss of $4\frac{1}{6}$ per cent. What did the flour cost him per barrel?

11. What per cent of 9.075 is 24.2?

SPECIAL DRILLS.

884. Give sums:

425 + 99	999 + 425	\$2.63 + \$6.37	\$5.45 + \$9.99
99 + 576	576 + 999	\$4.56 + \$2.84	\$9.99 + \$6.73
685 + 99	999 + 685	\$6.49 + \$3.12	\$12.68 + \$0.99
99 + 599	599 + 999	\$3.58 + \$5.67	\$0.99 + \$13.33

885. Give differences:

565 − 99	1,424 − 999	\$7.00 − \$2.63	\$15.44 − \$9.99
488 − 99	1,575 − 999	\$6.40 − \$3.56	\$9.44 − \$6.45
794 − 99	1,684 − 999	\$9.61 − \$4.49	\$7.88 − \$4.89
898 − 99	1,598 − 999	\$8.15 − \$5.58	\$9.53 − \$2.99

886. Give products:

24×21	21×31	41×41	$19 \times 37\frac{1}{2}$
33×21	32×31	32×41	$18 \times 62\frac{1}{2}$
42×21	43×31	21×41	$22 \times 66\frac{2}{3}$
51×21	31×31	42×41	33×75

887. Give quotients:

$16 \div .25$	$228 \div 19$	$24 \div .33\frac{1}{3}$	$656 \div 16$
$18 \div .25$	$234 \div 18$	$36 \div .33\frac{1}{3}$	$544 \div 17$
$24 \div .75$	$238 \div 17$	$16 \div .66\frac{2}{3}$	$558 \div 18$
$36 \div .75$	$336 \div 16$	$66 \div .66\frac{2}{3}$	$418 \div 19$

888. Give answers:

$14\frac{7}{8} \times 7$	$13\frac{1}{2} \times 8$	$13 \times 13\frac{1}{2}$	$20 \times 8\frac{2}{3}$
$15\frac{2}{3} \times 6$	$21\frac{5}{6} \times 4$	$14 \times 12\frac{4}{9}$	$21 \times 7\frac{3}{4}$
$5\frac{5}{9} \div \frac{5}{9}$	$7\frac{1}{2} \div \frac{3}{4}$	$15 \div 3\frac{1}{8}$	$63 \div 3\frac{1}{2}$
$14\frac{2}{3} \div 1\frac{1}{3}$	$6\frac{2}{3} \div \frac{5}{6}$	$20 \div 3\frac{3}{4}$	$64 \div 5\frac{1}{8}$

889. Oral Problems.

1. What will be the cost of 48 yards of cloth at $87\frac{1}{2}¢$ per yard?

2. How many square yards in a piece of carpet 48 yards long, 27 inches wide?

3. How many yards of carpet 27 inches wide will be needed to cover a floor containing 48 square yards?

4. Paid $3.45 for groceries, $1.50 for dry goods, and 99¢ for sundries. What is the total?

5. From a chest containing $25\frac{1}{4}$ pounds of tea, $8\frac{1}{2}$ pounds were sold. How many pounds remain?

6. At $37\frac{1}{2}¢$ per peck, what will I receive for 4 bushels of potatoes?

7. $83\frac{1}{4}$ yards of cloth are divided into 9 pieces. How many yards are there in each piece?

8. I buy hardware to the amount of $6.37. I give the storekeeper two $5-bills. How much change should I receive?

9. What will be the cost of 24 yards of calico at $4\frac{3}{4}¢$ per yd.?

10. What will I have to pay for 19 base-balls at $1.25 each?

11. At $1.87\frac{1}{2}$ per yard, what will be the cost of 120 yards of silk?

12. For $120, how many yards of silk can I buy at $1.87\frac{1}{2}$ per yard?

13. What will be the cost of a ton of hay at $97\frac{1}{2}¢$ per cwt.?

14. A square field requires 320 rods of fence. How many square rods are there in the field?

15. How many acres are 6,400 sq. rd.?

16. At $4\frac{3}{4}¢$ per yd., how many yards of calico can I buy for 95¢?

17. If slate pencils cost 2 mills each, how many will I receive for $4?

18. At $5.00 per ton, how many pounds of coal can be bought for 1¢?

19. Find the cost of 3 T. 480 lb. coal at $5 per ton.

20. At $5 per ton, how many tons and pounds of coal can I buy for $10.80?

21. How many square yards are there in a field 41 yards long, 42 yards wide?

22. If I pay 15¢ for $3\frac{1}{8}$ yards of muslin, what is the price per yard?

23. How many acres of land are there in two farms containing, respectively, 347 and 495 acres?

24. At $87\frac{1}{2}$¢ each, how many base-balls can be bought for $56?

25. If one man can do a piece of work in 24 days, and another man can do it in 48 days, how long will it take both, working together?

APPROXIMATIONS.

890. Give approximate answers at sight (Art. 521):

1. Find the interest of $150, at 4%, from Jan. 1, 1893, to Dec. 30, 1895. (Nearly 3 years.)

2. What is the weight, at $57\frac{1}{2}$ lb. per cu. ft., of a cake of ice 4 ft. by 2 ft. by $1\frac{1}{2}$ ft.? (Nearly 60 lb. per cu. ft.)

3. Find the amount of goods sold, the commission at $2\frac{7}{8}$% being $11.75. (About 3%.)

4. What % of 497 is 249?

5. What % of $3\frac{15}{16}$ is $11\frac{31}{32}$?

6. Cost of 19,987 ft. boards at $30.05 per M.?

7. How much will be paid for 4 bbl. sugar, each containing 299 lb., @ $5\frac{1}{16}$¢ per lb.?

8. $18.0327 \div 4.5026$.

9. $83\frac{19}{20} \div 3\frac{29}{30}$.

10. 74 A. 155 sq. rd. land at $79 per A.?

SHORT METHODS.

891. **Slate Exercises.**

$7{,}854 \times \frac{3}{4}$	$9{,}365 \times \frac{7}{8}$
$1{,}963\frac{1}{2}$ Deduct $\frac{1}{4}$.	$1{,}170\frac{5}{8}$ Deduct $\frac{1}{8}$.
$5{,}890\frac{1}{2}$ *Ans.*	$8{,}194\frac{3}{8}$ *Ans.*

Multiply 6,578 by $9\frac{2}{8}$.

$65{,}780$ = 10 times number.
$2{,}192\frac{2}{8} = \frac{1}{8}$ number (Deduct).
$63{,}587\frac{1}{8}$ *Ans.*

892. Find products:

1. $176 \times \frac{15}{16}$	11. $4{,}844 \times 9\frac{1}{2}$
2. $273 \times \frac{12}{13}$	12. $8{,}960 \times 8\frac{7}{8}$
3. $4{,}554 \times \frac{8}{9}$	13. $3{,}245 \times 7\frac{7}{8}$
4. $1{,}001 \times \frac{10}{11}$	14. $9{,}060 \times 11\frac{4}{5}$
5. $3{,}243 \times \frac{4}{5}$	15. $658 \times 99\frac{1}{2}$
6. $6{,}776 \times \frac{5}{6}$	16. $658 \times 99\frac{3}{4}$
7. $2{,}307 \times \frac{6}{7}$	17. $725 \times 119\frac{4}{5}$
8. $7{,}284 \times \frac{7}{8}$	18. $347 \times 79\frac{5}{6}$
9. $5{,}631 \times \frac{9}{10}$	19. $418 \times 89\frac{1}{2}$
10. $9{,}657 \times \frac{11}{12}$	20. $543 \times 49\frac{3}{8}$

893. Multiply:

21. 418×99	26. $724 \times 36 \times 45$
22. $674 \times 87\frac{1}{2}$	27. $484 \times \frac{10}{11} \times 9\frac{3}{4}$
23. $36 \times 999 \times 25$	28. $576 \times 9\frac{11}{12} \times 12\frac{1}{2}$
24. $48 \times 125 \times 71$	29. $95 \times 36 \times 19\frac{4}{5}$
25. $64 \times 7\frac{7}{8} \times 33\frac{1}{3}$	30. $74 \times 31 \times 13 \times 9\frac{5}{6}$

TABLE.

894. Domestic Exports.

	1890.	1891.	Increase.	Decrease.
Agricultural implements	$3,859,184	$3,219,130		
Animals	33,638,128	32,935,086		
Breadstuffs	154,925,927	128,121,656		
Carriages and cars	4,746,678	4,901,120		
Coal	6,856,088	8,391,026		
Copper and manufactures	8,402,628	11,875,490		
Cotton and manufactures	260,968,069	304,317,755		
Fertilizers	1,618,681	2,182,274		
Fruit and nuts	4,059,547	2,434,793		
Furs and skins	4,661,934	3,236,705		
Hops	1,110,571	2,327,474		
Iron and steel, and manufactures	25,542,208	28,909,614		
Leather and manufactures	12,438,847	13,278,847		
Musical instruments	1,105,134	1,326,389		
Naval stores	2,853,515	3,523,473		
Oils	52,743,957	51,682,372		
Provisions	135,287,578	137,974,947		
Spirits	1,633,110	1,887,431		
Sugar, refined	1,901,386	6,138,746		
Tobacco and manufactures	25,355,601	25,220,472		
Wood and manufactures	28,257,783	26,263,014		
All other articles	?	?		
Total	$845,293,828	$872,270,283		

Supply the value of "All other articles," and give "Increase" or "Decrease" of the value of each item, and the total increase.

895. Divide. Do not write products (Arts. 385, 616):

1.	611,463,874 ÷ 87,659	6.	703,205,104 ÷ 71,685
2.	279,864,597 ÷ 45,678	7.	923,812,701 ÷ 18,789
3.	387,250,005 ÷ 34,567	8.	575,646,828 ÷ 59,764
4.	800,700,900 ÷ 68,439	9.	1,234,567,890 ÷ 169,375
5.	453,211,687 ÷ 576,258	10.	3,126,045,000 ÷ 483,729

896. Write answers (Art. 385):

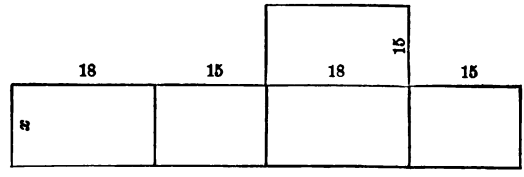
11.	$\frac{876,459}{94,317}$	14.	$\frac{512,000}{69,999}$	17.	$\frac{208,040}{17,613}$
12.	$\frac{763,154}{82,915}$	15.	$\frac{453,124}{123,456}$	18.	$\frac{963,018}{126,748}$
13.	$\frac{654,817}{73,295}$	16.	$\frac{375,005}{59,687}$	19.	$\frac{862,304}{87,925}$

MEASUREMENTS.

897. Slate Problems.

1. How many boards 16 ft. long, 8 in. wide, will be required for a tight fence 8 ft. high, around a piece of ground 240 ft. long, 180 ft. wide? How many posts, 6 ft. apart, will be needed?

2. A room is 18 ft. long, 15 ft. wide. The walls and the ceiling contain 930 sq. ft. What is the height of the room?



3. What will it cost to cover a table 6 ft. long, $2\frac{1}{4}$ ft. wide, with baize $\frac{3}{4}$ yd. wide, at 75¢ per yd.?

4. 160 square rods make 1 acre. How many square yards are there in an acre? About how many yards long is a square field containing 1 acre?

5. A 40-acre field is 160 rods long. How many rods of fence are needed to enclose the field?

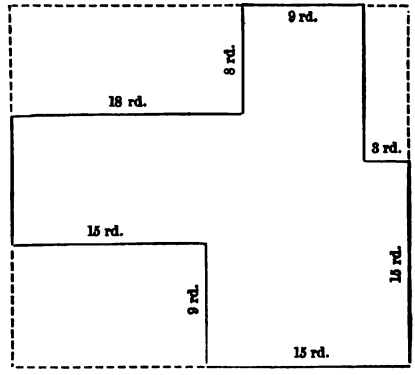
6. A room 30 ft. long, 24 ft. wide, 15 ft. high, contains 40 persons. How many square feet of floor space are there for each occupant? How many cubic feet of air space are there for each?

7. How many yards of carpet, 27 inches wide, would be needed for the floor of such a room?

8. How many bundles of laths, each bundle covering 3 square yards, would be needed for the walls and ceiling of the above room, no allowance being made for doors and windows?

9. A farmer owned a rectangular piece of ground 30 rods long, 27 rods wide. He sold three lots 18×8 rods, 12×3 rods, 15×9 rods.

Find the number of square rods in the original piece. Mark in the diagram the area of each lot sold, and the area of the part remaining.



10. How many rods of fence will be needed to enclose the part remaining?

11. In a box containing 50 square feet of glass, how many panes will there be 6 inches long, 4 inches wide?

898. 1 gallon = 231 cubic inches.
1 bushel = 2,150.4 cubic inches.
1 cord = 128 cubic feet.

12. How many gallons will fill a tank 22 feet long, 14 feet wide, 9 feet deep?

Indicate operations. Cancel where possible.

13. How many cords of wood are there in a pile 48 feet long, 16 feet wide, 12 feet high?

14. A bin measures 49 feet long, 32 feet wide, 10 feet 8 inches deep. How many bushels of grain will it hold?

15. In building a sewer one thousand yards long, an excavation 6 feet wide was made to an average depth of $10\frac{1}{2}$ feet. How many cubic yards were dug out?

16. If the sewer pipe occupies a space equal to a box $4\frac{1}{2}$ feet wide, $4\frac{1}{2}$ feet deep, and 1,000 yards long, how many cubic yards of earth will be left after the excavation is filled in?

899. St. Paul Public Schools. Examination Questions.

1. I borrowed \$241.50 at 8%, and kept it 3 yr. 8 mo. 18 da. What was due the lender at the expiration of that time?

2. Made a loan of \$1,040 for 1 yr. 9 mo. at $7\frac{1}{2}$%. What interest did I receive?

3. Mr. B's ground is 14 rods by 10 rods, which is 40% larger than A's. Find the value of A's ground at \$5 a sq. rd.

4. What per cent of a sum of money can be earned in 1 day if 72% of it can be earned in 9 days?

5. Write the analysis of the second.

6. At $\$3\frac{1}{2}$ a cord, what is the value of the wood that can be piled under a shed 50 ft. long, 25 ft. wide, and 12 ft. high?

7. In building a house, 350 joists 12 in. by 4 in. were used, which together amounted to 3,456 cu. ft. Find length of each.

8. If I buy $9\frac{1}{2}$ bu. of chestnuts at $\$4\frac{3}{4}$ a bu. and retail them at $\$.12\frac{1}{2}$ a pint, what is my whole gain?

9. If 14 acres of meadow yield $32\frac{2}{3}$ tons of hay, how much do $5\frac{1}{4}$ acres produce? Analyze.

10. What is the value of

$$\left(\frac{31\frac{1}{2}}{8} \times \frac{6\frac{3}{4}}{2\frac{1}{12}}\right) \div 4.23?$$

BANK DISCOUNT.

900. Wm. Brown and Sons receive the following note in settlement of their account with Thomas Tierney:

BOSTON, May 30, 1893.

Thirty days after date, I promise to pay to the order of Wm. Brown and Sons, Three Hundred Fifty-Four $\frac{75}{100}$ Dollars, value received, at the Park National Bank.

$\$354\frac{75}{100}$. THOMAS TIERNEY.

901. This money is payable 33 days after May 30, 3 days of grace being allowed. If Wm. Brown and Sons desire to use the money at once, they may have the note discounted at a bank. In this case, the bank deducts from the face of the note ($354.75), the interest thereon for 33 days, and pays over the difference (*proceeds*).

Face of note $354.75
Discount (Int. for 33 da.) 1.95 (at 6%)
Proceeds $352.80

902. Slate Exercises.

Find the discount at 6% on the following, allowing 3 days of grace in each case. (See Appendix.)

1. A 30-days note for $75.
2. 15-days note for $183.60.
3. 60-days note for $275.40.
4. 20-days note for $96.
5. 4-months note for $336.

Find the proceeds, at 7%, on

6. A 6-months note for $180.
7. A 3-months note for $36.90.
8. A 24-days note for $795.60.
9. A 90-days note for $180.
10. A 72-days note for $1,000.

903. In computing the discount on a note, the banks ascertain the exact number of days.

A 3-months note, dated February 1, is payable three days after May 1, which is May 4. The discount is taken for $27 + 31 + 30 + 4 = 92$ days. A 90-days note of the same date is payable 93 days after February 1, which is May 5.

The year, however, is considered to contain 360 days; the interest in the first case being taken for $\frac{92}{360}$ of a year, and in the latter for $\frac{93}{360}$ of a year.

Find the discount, at 6%, on

11. A 1-month note for $600, dated Feb. 6, 1894. ($\frac{31}{360}$.)
12. A 2-months note for $240, dated July 17, 1894. ($\frac{65}{360}$.)
13. A 3-months note for $360, dated April 8, 1894.
14. A 4-months note for $84, dated Dec. 24, 1894.
15. A 6-months note for $172.60, dated March 4, 1895.
16. A 60-days note for $240, dated July 17, 1894.
17. A 90-days note for $360, dated April 8, 1894.

904. In each of the preceding examples, it has been assumed that the note has been presented for discount the day on which it was made.

In some of the following examples, the notes are discounted at a later date, and the *term of discount* is to be ascertained; that is, the time between the date of discount and that of maturity (including the days of grace).

The term of discount of a 30-days note dated May 1, and discounted May 19, is 15 days.

905. In the following examples, find (*a*) date of maturity; (*b*) term of discount; (*c*) discount; (*d*) proceeds:

	Dated.	Face.	Time.	Discounted.	Rate.
18.	June 16, 1894;	$87.60;	30 days;	July 1, 1894;	6%.
19.	Sept. 9, 1894;	$124.18;	4 months;	Nov. 18, 1894;	8%.
20.	Dec. 5, 1894;	$504.60;	30 days;	Dec. 12, 1894;	7%.
21.	Nov. 14, 1894;	$72.36;	3 months;	Dec. 20, 1894;	6%.
22.	Oct. 30, 1894;	$234;	90 days;	Jan. 5, 1895;	6%.
23.	Jan. 2, 1895;	$95.90;	2 months;	Feb. 13, 1895;	6%.
24.	Aug. 5, 1895;	$164;	60 days;	Aug. 31, 1895;	8%.
25.	Feb. 27, 1895;	$83.20;	100 days;	March 9, 1895;	6%.

INTEREST.

906. Find interest:

When the time is less than one year, ascertain the exact number of days. When greater than a year, find the time by compound subtraction, taking the month of 30 days.

1. $160; Jan. 2, 1893, to May 16, 1896; $4\frac{1}{2}\%$.

   3 yr. 4 mo. 14 da. $= 3\frac{134}{360}$ yr. $= 3\frac{67}{180}$ yr. $= \frac{607}{180}$ yr.

2. $342.18; April 5, 1895, to Sept. 30, 1895; 6%.

   Time, 178 days $= \frac{178}{360}$ year.

3. $59.80; Feb. 24, 1896, to Dec. 24, 1896; 5%.

   (Leap year.)

4. $1,234.56; Aug. 3, 1890, to Jan. 1, 1896; $5\frac{1}{2}\%$.

5. $387.90; March 15, 1894, to Sept. 1, 1894; 6%.

6. $96; July 6, 1894, to Feb. 4, 1895; $3\frac{3}{4}\%$.

7. $240.72; May 20, 1893, to Jan. 15, 1894; 7%.

8. $983.25; Dec. 15, 1899, to March 3, 1900; 6%.

   How many days in February, 1900? Be *sure* you are right.

907. An Invoice (German).

ELBERFELD, Dec. 16, 1892.

INVOICE.

For MESSRS. BROWN & JONES, New York,

From HAUSELMANN & SCHMIDT.

We send you, through MESSRS. VON DER OHE & MEYER, at Antwerp,

1 case, marked H. & S. 1,276, containing

39^{90}	m.			
42^{20}	"			
33^{00}	"			
30^{70}	"			
40^{00}	"			
44^{25}	"	12 pieces Rhaetia		
43^{45}	"			
49^{90}	"	black		
38^{30}	"			
42^{30}	"			
40^{95}	"			
40^{25}	"			
	m.	@	M. 3.78	M.
35^{05}	m.			
29^{20}	"	4 pieces Caledonia		
23^{00}	"	brown		
44^{50}	"			
	m.	@	M. 5.66	
				M.

(39^{90} m. means 39.90 meters. M. stands for marks.)

Find the total number of yards of dress goods in the above case (1 meter = 39.37 inches), and the total cost in U. S. money (1 mark = 23.8 cents).

SHORT METHODS.

908. When multiplying mixed numbers, many accountants prefer not to reduce them to improper fractions.

909. Multiply without reducing:

1. $43\frac{3}{4} \times 8\frac{1}{2}$

   350

   $21\frac{7}{8}$

2. $57\frac{1}{8} \times 9\frac{1}{4}$

   516

   $14\frac{1}{8}$

3. $17\frac{3}{5} \times 7\frac{1}{8}$

4. $24\frac{7}{8} \times 5\frac{1}{6}$

5. $42\frac{2}{3} \times 12\frac{1}{8}$

6. $35\frac{5}{16} \times 8\frac{1}{5}$

7. $56\frac{3}{7} \times 4\frac{1}{7}$

8. $83\frac{3}{8} \times 3\frac{1}{8}$

9. $72\frac{1}{4} \times 10\frac{1}{6}$

10. $95\frac{5}{6} \times 11\frac{1}{4}$

910. $76\frac{1}{2} \times 9\frac{5}{6}$

765 product by 10

$12\frac{3}{4}$ less product by $\frac{1}{6}$

$752\frac{1}{4}$ *Ans.* $9\frac{5}{6}$

$64\frac{1}{2} \times 11\frac{5}{7}$

$709\frac{1}{2}$ product by 11

$9\frac{3}{14}$ product by $\frac{1}{7}$

$36\frac{12}{14}$ product by $\frac{4}{7}$

$755\frac{4}{7}$ *Ans.* $11\frac{5}{7}$

$84\frac{3}{5} \times 5\frac{2}{5}$

$421\frac{7}{5}$ product by 5

$16\frac{7}{5}$ product by $\frac{1}{5}$

$16\frac{7}{5}$ product by $\frac{1}{5}$

$455\frac{5}{5}$ *Ans.* $5\frac{2}{5}$

911. Multiply:

11. $19\frac{3}{8} \times 4\frac{2}{5}$ $(4 + \frac{1}{5} + \frac{1}{5})$

12. $24\frac{1}{2} \times 3\frac{2}{3}$ $(4 - \frac{1}{3})$

13. $35\frac{3}{5} \times 7\frac{3}{7}$ $(7 + \frac{1}{7} + \frac{2}{7})$

14. $40\frac{2}{3} \times 8\frac{3}{9}$ $(9 - \frac{1}{9})$

15. $51\frac{1}{8} \times 6\frac{3}{5}$

16. $63\frac{1}{2} \times 10\frac{5}{8}$

17. $29\frac{2}{3} \times 2\frac{3}{4}$

18. $31\frac{1}{4} \times 12\frac{5}{6}$

19. $42\frac{3}{5} \times 9\frac{9}{10}$

20. $56\frac{5}{8} \times 7\frac{2}{3}$

912. Boston Examination Questions. Mental.

1. If $5\frac{1}{2}$ yd. of cloth cost 66¢, what cost $12\frac{1}{2}$ yd.?

2. What part of a cu. yd. is 3 cu. ft.?

3. What is the difference between a floor 40 ft. square and two others each 20 ft. square?

4. What is the interest of $12, for 1 yr. 4 mo., at 6%?

5. Sold an article for $60, thereby losing 70%. Cost?

6. If I buy a dozen pencils at 2¢ each, and sell at 3¢ each, what is the gain per cent?

7. How many sq. yd. in the surface of a cube that contains one cu. yd.?

8. If a yard of ribbon contains 144 sq. in., how wide is it?

9. At 75¢ a yard, what will be the cost of carpeting a floor 15 ft. long, 4 yd. wide?

10. How long will it take a boy to pay for a suit worth $6.50 and a pair of boots worth $2.50, if he earns $66\frac{2}{3}$¢ a day?

11. 180 is 10% less than what number?

12. In $1\frac{2}{3}$ yd., how many inches?

13. Sold a saddle for $18, which was $12\frac{1}{2}$% more than the cost. Cost?

14. What % is gained on goods sold at double the cost?

15. What is the interest of $1,500, for 60 days, at 6%?

16. Sold flour at a profit of $2.00, and gained 25%. What was the cost per barrel?

17. Bought $\frac{1}{2}$ a ton of sugar at $0.0625 per lb. How much did I pay for it?

18. If $2\frac{1}{2}$ pk. berries cost one dollar, what would 3 qt. cost at the same rate?

19. Bought 5 bu. nuts at a dollar a pk., and got 5% off for cash. How much did I pay for the nuts?

20. In a certain school 40 pupils are present and 10 are absent. What % are absent?

21. In a certain class $\frac{1}{2}$ of the pupils are under 10 years, $\frac{1}{3}$ of them are between 10 and 12, and the rest are over 12. What % are over 12 years?

22. Mr. Eastman collects bills for me, and I pay him $12\frac{1}{2}\%$. He pays over to me $56. How much did he collect?

23. With $33\frac{1}{3}\%$ of his money, Robert bought 12 oranges. How many oranges could he have bought with all his money?

24. Mrs. Allen bought 7 chairs at $4 apiece, 2 tables at $9 apiece, and a carpet for $30. She paid two $50 bills. How much change was due her?

25. If a boy earns $4.00 a week and saves 2% of it, how much will he save in 10 weeks?

26. What is the % of gain, when boots which cost $2.00 a pair are sold for $2.50?

27. 30% of Mr. Brown's money is in the bank, and 50% in real estate; the remainder, $2,000, is in railroad stocks. How much has he in all?

28. How much difference between $\frac{1}{2}$ of 20 and 20 divided by $\frac{1}{2}$?

29. What is the premium for insuring $3,600 on my house at $\frac{3}{4}\%$?

30. 20% of 115 is $33\frac{1}{3}\%$ of what number?

913. Boston Examination Questions. Slate.

1. Change .03125 to a common fraction in smallest terms.

What is the product of one hundred one thousandths by ten thousand one hundred one hundred-thousandths?

Divide .01001 by .001.

2. If I am compelled to lose $12\frac{1}{2}\%$ on damaged goods, how must I sell those that cost me $\$5.60$?

3. What is the interest of $\$750.25$, for 2 yr. 6 mo. 24 da., at 6%?

What is interest? The principal? The amount? The rate?

4. Find the interest on a note for $\$250$, dated Jan. 21, 1890, and paid May 30, 1890, at 6%.

Find the amount of $\$198.76$, from May 15, 1887, to July 21, 1890, at 4%.

5. What is $87\frac{1}{2}\%$ of $\$832$?

$\$832$ is $87\frac{1}{2}\%$ of what sum?

6. How large a sale must a merchant make, at a profit of 15%, that his gain may be $\$3,750$?

7. By the census of 1880, the population of a certain city was 26,275. By the census of 1890, its population was 31,530. Find the per cent of increase.

8. Each of two boys bought 100 apples for a dollar. The first boy sold his, 4 apples for $5\not{¢}$; the second sold his, 5 apples for $6\not{¢}$. Which boy gains the more per cent? How much more?

9. From 4 A. of land I sold one piece 20 rods square, and another piece containing 16 square rods How much did I have remaining?

10. Multiply one hundred eleven millionths by five and six-tenths.

11. A quantity of coal was bought for $\$900$. For what must it be sold to gain $33\frac{1}{3}\%$?

By selling a house for $\$5,760$, a man gained on the cost 25%. What was the cost?

Bought a horse for $\$800$, and sold it for $\$600$. What $\%$ did I lose?

12. If a man paid $\$18\frac{4}{7}$ for a load of hay weighing $1\frac{4}{5}$ tons, what would he pay at the same rate for $\frac{3}{5}$ of a ton?

13. Sold my house and farm of $94\frac{4}{5}$ acres for $12,300. Allowing $7,000 for the house, what did I receive per acre for the land?

14. What number is that which, diminished by $2\frac{1}{5}$, will leave $2\frac{1}{30}$?

How long will 200 lb. flour last 18 persons if each person is allowed $1\frac{3}{4}$ lb. per day?

If $\frac{3}{4}$ of $\frac{7}{8}$ of a ship cost $84,000, what is $\frac{2}{3}$ of it worth?

15. The dividend was $4,689.036, the quotient .027. What was the divisor?

$19.406 \times 10.403 = ?$

Write your rule in pointing off in division of decimals.

16. Change to other methods of expression, $\frac{1}{4}$, $\frac{1}{8}$, $.37\frac{1}{2}$, $\frac{5}{4}$, $.16\frac{2}{3}$.

17. A note of $1,260, dated July 5, 1888, was paid June 7, 1890, with interest at 8%. What was the amount paid?

18. Maine had, by census of 1880, 29,895 sq. mi. and 648,936 inhabitants; New Hampshire had 9,005 sq. mi. and 346,991 inhabitants; Vermont had 9,135 sq. mi. and 332,286 inhabitants; Massachusetts had 8,040 sq. mi. and 1,783,085 inhabitants; Connecticut had 4,845 sq. mi. and 622,700 inhabitants; Rhode Island had 1,085 sq. mi. and 276,531 inhabitants; Texas had 262,290 sq. mi. and 1,591,749 inhabitants. Texas is how many times as large in area as New England? Carry the answer to two decimal places. What was the average population per square mile in New England? In Texas? If the land in Rhode Island is worth on the average $\frac{1}{2}$ of a mill per square foot, what is the value of the entire land in the state?

19. A flock of sheep has been increased by 250% of its number, and now numbers 1,050. What was the original number?

20. Bought a house for $6,240, and sold it so as to gain $37\frac{1}{2}$%. What did I sell it for?

21. If this room is 35 ft. long, 23 ft. wide, and 13 ft. high, how many square yards of plastering will it require, making no allowance for doors, etc.?

22. A commission of \$121.29 was charged for selling \$1,866 worth of goods. What was the rate of commission?

Sold goods at a loss of 20%, an actual loss of \$57.50. What was the prime cost?

23. Harry Hedge earns \$12 a week. He pays \$4.25 for board, \$0.625 for car fare, \$0.375 for library fees, and \$4.875 for other expenses. In how many weeks would he save \$97.50?

24. The milk from a herd of 15 Jersey cows, sold at 6¢ a qt., amounted in one summer to \$2,025. How many quarts were sold, and what was the average quantity from each cow?

25. A woman has three children. She pays for each \$15 a year for having his clothes made, \$1.50 a month for his mending, and \$0.35 a week for his washing. How much could she save in a year if she knew how to wash, make clothes, and mend?

26. A coal dealer bought 25,784 tons of coal at \$5 a ton. He sold 40% of it at \$7, 20 % of it at \$8.50, and the remainder at \$4.50. How much did he gain?

27. A man insured his house for \$6,500, his store for \$3,500, and his goods for \$7,000 at ½%. What did his insurance come to?

28. A pole stands $\frac{1}{7}$ in the mud, $\frac{2}{5}$ in the water, and 32 ft. in the air. How long is the pole?

29. What is the amount of the following: 23 yd. at \$1.25; 17 yd. at \$2.75; 16 yd. at \$1.12½; 10 yd. at \$1.37½; 14 yd. at \$1.62½; 20 yd. at \$2.34; 12 yd. at \$0.88?

30. Bought flour for \$8.25, and sold it for \$9. What is the per cent of gain?

Bought for \$9 and sold it for \$8.25. What is the per cent of loss?

What per cent is gained by selling goods for 2 times their cost?

914. Kansas City Examination Questions. Mental.

1. $\frac{2}{3}$ of my pens, diminished by 10, equals $\frac{1}{4}$ of my pens. How many have I?

2. A and B together have $24. A has $\frac{5}{7}$ as much as B. How many dollars has each?

3. The sum of A's money and B's money is $31. A has $5 more than B. What has each?

4. A and B have $165. $\frac{3}{5}$ of A's money equals $\frac{4}{5}$ of B's. What has each one?

5. A and B can build $\frac{1}{6}$ of a boat in a day. Twice what A builds equals what B builds. How much can each do in a day?

6. If 5 men in 8 days can build 20 rd. of wall, how many men in 2 days can build $\frac{1}{2}$ as much wall?

7. If $\frac{2}{3}$ of a yard of cloth costs $\frac{4}{5}$ of a dollar, what will $\frac{3}{4}$ of a yard cost?

8. If a certain quantity of grain will last 4 horses 6 days, how long will it last if one more horse is fed from the same quantity after the four have been eating two days?

REVIEW.

915. Slate Problems.

1 pound troy = 5,760 grains.
1 pound apothecaries' = 5,760 grains.
1 pound avoirdupois = 7,000 grains.

How many grains in a troy ounce? In an avoirdupois ounce?

1. Find the value of a dozen silver spoons, each weighing 3 oz. 5 pwt., at $1.20 per oz. (See table, Art. 828.)

2. A gold chain weighs 384 grains. What is its cost at $1.15 per pwt.?

3. Add 4 lb. 6 oz. 18 gr., 5 oz. 9 pwt., 3 lb. 20 gr., and 9 lb. 11 oz. 15 pwt. 5 gr.

4. Gold coin contains 90 per cent gold, 9 per cent silver, 1 per cent copper. Find the quantity of each metal in 50 double-eagles ($20), each containing 516 grains.

5. How many spoons, each weighing 2 oz. 18 pwt., can be made from 5 lb. 9 oz. 12 pwt. silver?

6. How much money in silver dollars, 412½ grains each, will weigh 165 lb. avoirdupois (7,000 grains)?

7. What fraction of a pound avoirdupois is a pound troy? What per cent of an ounce avoirdupois is a troy ounce?

8. What is the value, at $1.60 per oz. troy, of a silver pitcher weighing 4 lb. 8 oz. avoirdupois?

9. At 75¢ per ounce, what is the value of the silver contained in a half-dollar, which weighs 192.9 grains, $\frac{9}{10}$ being pure silver?

10. What is the capacity, in gallons (231 cu. in.), of a barrel that will contain 2½ bu. (2,150.4 cu. in.)?

11. A tank 16 ft. long, 14 ft. wide, 8 ft. deep, is lined with lead on the bottom and sides. How many square feet of sheet lead will be required? (Draw the "development" of the tank.)

12. How many cu. ft. of lead will be needed for the above, if the sheet lead is $\frac{1}{32}$ in. thick?

13. A cubic foot of water weighs 1,000 oz. Lead is 11½ times as heavy. How many pounds of lead will be needed to line the tank? (Cancel.)

14. How many gallons will the above tank hold? How many bushels?

15. 8 lb. 3 oz. troy is what per cent of 4 lb. 7 oz.?

16. What per cent of a lb. avoirdupois is a troy pound?

17. If 3 T. 740 lb. hay cost $60.66, how much will 17 T. 380 lb. cost?

18. A man sold 18 bbl. sugar, each containing 306 lb.; 21 bbl., each containing 297 lb.; 5 bbl., each containing 291 lb. What is the average weight per barrel?

19. A, B, and C buy a farm. A pays $8,700, B pays $7,200, C pays $4,100. What per cent of the purchase money does each furnish?

20. If 11 weavers in 9 days weave 1,584 yards, what will 1 man do in 1 day? 6 men in 7 days?

21. The tax rate of a certain city is $1\frac{3}{4}\%$ upon the assessed value of property. If this value is 75% of the actual value, how much taxes does Mr. Smith pay upon a house and lot, the actual value of which is $24,000?

916. Philadelphia Public Schools. Oral Work.

1. $96 \div (12 + 24 - 17) =$

2. What decimal is equal to $\frac{2}{5}$?

3. What part of 100 is $62\frac{1}{2}$?

4. $\frac{3}{4} - \frac{3}{8} + .25 =$

5. 80 is $\frac{4}{5}$ of what number?

6. $\frac{5}{8}$ of 40 yards + $\frac{3}{5}$ of 15 yards =

7. At $12 per ounce, what is $\frac{5}{6}$ of a pound of gold worth?

8. 1.25 pecks are how many quarts?

9. How many square feet in a board 10 feet long and $1\frac{1}{2}$ feet wide?

10. If 9 pounds of sugar cost 36¢, what will 12 pounds cost?

11. What fractional part of a minute is $12\frac{1}{2}\%$ of it?

12. 25 is 20% of what number?

13. What per cent is lost in buying potatoes at 80¢ a bushel, and selling them at 60¢ a bushel?

14. If I buy butter at 30¢ a pound, how much per cent do I gain by selling it at 36¢ a pound?

917. Philadelphia Public Schools. Slate Work.

1. Multiply eight hundred nine and one hundred twenty-five thousandths by six and seventy-eight hundredths. Divide the product by fifty-four.

2. Find the quotient of $5205\frac{1}{5} \div \frac{4}{5}$ of 91.

3. Find the greatest common divisor of 216 and 360.

4. Find the least common multiple of 13, 15, 26, and 39.

5. If two-thirds of a yard of silk can be bought for $\$\frac{3}{4}$, how many yards can be bought for $\$3\frac{3}{8}$?

6. A drover sold 250 sheep for $\$1,150$, which was 15% more than they cost. What was the cost of each sheep?

7. A man shipped 600 barrels of flour, and lost $16\frac{2}{3}\%$ of it by storm; he sold 75% of the remainder. What per cent of the whole remained?

8. A owns 65 shares (par value, $\$50$ each) of an insurance company, which declares a dividend of $2\frac{1}{2}$ per cent. Required the amount of A's dividend.

918. St. Paul Public Schools. Examination Questions.

1. How many rolls of paper, each 8 yd. long and 18 in. wide, will paper the sides of a room 16 ft. by 14 ft., and 10 ft. high, deducting 174 sq. ft. for doors and windows? Illustrate.

2. Find the sum of $\frac{8}{80}$, $\frac{2}{7}$, $\frac{43}{56}$, $\frac{7}{24}$, $\frac{75}{486}$, in decimals, correct to fourth place.

3. $66\frac{2}{3}\%$ of 200 bu. is $2\frac{1}{2}\%$ of how many bushels?

4. If a grocer's gallon measure is too small by 1 gi., what does he make dishonestly in selling 2 hhd. of molasses, averaging 58 gal. 2 qt. 1 pt. each, worth $\$.80$ a gallon? (Art. 525.)

5. What will it cost to cement a cellar bottom 48 feet 6 inches long and 25 feet wide, at $.45 a square yard?

6. If corn selling for 21¢ a bushel more than cost gives a profit of 30%, what did it cost?

7. Bought a hhd. of sugar containing 8 cwt. 48 lb. for $75.86, and paid $3.85 freight and cartage. At what price per pound must it be sold to gain 20%?

8. How many tablespoons, each weighing 2 ounces, can be made from 2 pounds 10 ounces of silver? Analyze.

9. After taking out 15% of the grain in a bin, there remained 40 bu. 3½ pk. How many bu. were there at first?

10. Write the analysis of the fifth.

DISCOUNT OF INTEREST-BEARING NOTES.

919. Slate Problems.

BROOKLYN, N. Y., Oct. 15, 1894.

Sixty days after date I promise to pay to the order of Harman P. Payne, Forty-eight $\frac{50}{100}$ Dollars, value received, with interest at 6%.

$48\frac{50}{100}$. GEORGE P. POST.

1. Find the amount due on the above note at maturity.

If the above note is discounted at a bank, the discount is taken on the amount due at maturity.

Find the proceeds of the above note if discounted Dec. 1, 1894, at 6%.

2. Find the proceeds of a 90-days note for $175, bearing interest at 6%, discounted 33 days after date, at 6%.

3. Find the proceeds of a 60-days note for $350, bearing interest at 6%, discounted at 6%, 10 days after date.

4. Find the proceeds of a three-months note for $840, bearing interest at 7%, discounted at bank 47 days before maturity, at 8%.

5. A four-months note for $720, dated March 17, 1894, bearing interest at 6%, is discounted at 7%, May 10. What are the proceeds?

REVIEW.

920. Slate Problems.

1. Change 7,643 inches to rods, etc.

2. Change 1,875 feet to rods, etc.

3. Change 964 yards to rods, etc.

921. English Money.

12 pence (*d.*)	1 shilling (*s.*)
20 shillings	1 pound (£)

A farthing is one-fourth of a penny, and is generally written as a fraction.

4. Reduce 4,000*d.* to £, *s.*, *d.*

5. How many pence in £87 17*s.* 6*d.*?

6. What will be the cost of 150 yd. silk at 3/6 per yd.?

3/6 = 3*s.* 6*d.*, read three and sixpence.

7. If £1 = $4.8665, what will be the cost in U. S. money of 75 books at 18 pence each?

8. A merchant sells 37 coats at £3 5*s.* each, less 10%. What is the amount of his bill in English money?

9. Find 25% of £183 14*s.* 8*d.*

10. A silver dollar weighs 412½ grains. How many ounces of pure silver are there in 1,000 silver dollars if the coin is $\frac{9}{10}$ pure silver?

11. The one-cent pieces weigh 48 grains. How many dollars would weigh 120 pounds avoirdupois (7,000 grains to pound)?

12. A coal dealer buys 150 tons of coal, 2,240 pounds each, at $4.50 per ton. He sells it at $4.75 per ton, giving 2,000 lb. to the ton. What is his profit?

13. Three workmen receive $283.50 for doing a piece of work. One worked 32 days, the second worked 53 days, the third worked 41 days. What is the share of each?

14. Three men engage in a business venture. One furnishes $3,000, another furnishes $5,000, a third furnishes $4,000. They gain $1,800. What is each one's share of the profit?

15. A garrison of 1,200 men has rations for 40 days. Ten days later it receives a reinforcement of 300 men. How many more days will the rations last?

16. Two trains start at 9 o'clock from towns A and B, 120 miles apart. The train leaving A travels 20 miles per hour, the other 30 miles per hour. What time do they meet, and at what distance from A?

17. Divide $900 among four persons, so that the second will have three times as much as the first; the third, twice as much as the second; the fourth, as much as the three others.

18. Divide $540 among three persons, so that the first will have $48 more than the second, and the second $75 more than the third.

19. Four men working 7 hours a day need 15 days (105 hours) for a piece of work. How many days would it take 6 men, working 10 hours per day, to do the same work?

20. A grocer buys some eggs at 15¢ per dozen. He breaks 15, and then finds that by selling the rest at 16¢ per dozen, he will neither gain nor lose. How many eggs did he buy?

21. If a person lends me $250 for 8 months, for how long ought I to lend him $400 as an equivalent?

22. A fort had a garrison of 4,000 men, and provisions for 18 weeks. If 1,000 men were sent away, how long would the provisions last?

23. At $20.60 per ounce, what is the value of 12 bars of pure gold, each weighing 12 lb. 5 oz. 16 pwt. 5 gr.?

24. A plank that is 20 feet long and 12 inches wide has the same area as another plank 24 feet long. What is the width of the latter plank?

25. Mr. Jones owes $3,240, and has only $2,025 to pay his debts. How much should he pay Mr. Smith, to whom he owes $576?

922. U. S. Civil Service. Assistant Weighers, Custom House.

1. Add the following:

5,673,911,987.87
44,376,013,705.90
32,673,231,695.25
7,736,910,286.16
642,155.14
26,105,321,266.57
132,873.63
8,856,764,397.49

Express in figures:

2. Twenty-five thousand one hundred one.

3. Forty-four million, one hundred one thousand one, and one ten-thousandth.

Express in words:

4. 24,763.

5. 20,101,001.0015.

6. A merchant bought 51 tons 17 cwt. 3 qr. 25 lb. of wool, and sold 27 tons 4 cwt. 2 qr. 27 lb. Of the remainder, one-half was lost by fire. How much had he left?

28 lb. = 1 quarter; 4 quarters = 1 cwt.

7. To $\frac{3}{5}$ of $\frac{3}{4}$ add $\frac{1}{2}$ of $\frac{7}{10}$, and reduce to lowest terms; multiply the sum so obtained by $1\frac{2}{3}$, and reduce to a mixed number; from the product subtract $\frac{5}{6}$, and reduce to lowest terms; divide the remainder by 5, and convert the quotient into a decimal fraction; add 1.1; multiply by 2.5; subtract .9; and divide the remainder by .007.

8. A can weigh a certain quantity of goods in 15 days, by working 7 hours a day. How long will it take him to do the same work by working 9 hours a day?

9. What is the total weight of 4 hogsheads of sugar, weighing respectively $936\frac{1}{2}$, $1{,}025\frac{9}{10}$, $846\frac{2}{5}$, and $987\frac{7}{15}$ pounds, deducting tare at 10 per cent?

10. An invoice of wool weighs 32 tons 17 cwt. 2 qr. 11 lb. State the value in £ *s.* *d.*, at 10*d.* sterling per pound.

1 ton = 2,240 lb.

923. Questions from the Regents' Examination, University of the State of New York.

1. Express in Arabic notation ten billion ninety million five thousand three.

2. Express in Roman characters 1893.

3. Name and define three terms used in multiplication.

4. In an example in division the remainder is 14, the divisor is 16, and the quotient is 18. What is the dividend?

5. Define numerator and denominator.

6. $6\frac{4}{5} - 3\frac{5}{12} + 7\frac{3}{8} =$

7. $(9\frac{3}{4} \times \frac{5}{7}) \div (8\frac{2}{5} \div 11\frac{2}{3}) =$

8. State two differences between a common and a decimal fraction.

9. $(.025 \times 6.25) \div .000125 =$

10. The product of three factors is 2,749; two of the factors are 23 and 11. What is the third factor?

11. Solve by cancellation:

How many pieces of cotton cloth, each piece containing 42 yd., at $9\frac{1}{2}¢$ per yd., can be bought for 14 firkins of butter, each containing 56 lb., at $19¢$ per lb.?

12. How many minutes are in November?

13. What must be the depth of a bin which is 4 ft. wide and 6 ft. long, to contain 40 bu. oats?

14. If a locomotive runs 25 mi. 48 rd. in 50 min., how far will it run in 12 hours?

(Give answer in miles and decimal of a mile.)

15. A farmer sold 9,875 lb. hay at $\$12\frac{1}{2}$ per ton, and took in part payment 5,000 ft. of boards at \$11 per thousand. How much remained due him?

16. Name and define five terms used in percentage.

17. A store rents for \$500, which is $10\frac{1}{2}$ per cent of its value. What is its value?

18. Bought 80 barrels of flour at \$6 per barrel, paying for freight \$30. At what price must I sell it per bbl. to gain 30% on the total cost?

19. What is the amount of \$720.50, for 3 yr. 5 mo. 19 da., at 6 per cent?

20. Three men buy a house for \$2,500. A pays \$500, B pays \$900, C pays \$1,100. They rent it for \$250. What is each one's share of the rent?

21. If 12.875 acres of land cost \$1,030, what will 4.75 acres cost?

22. What will be the duty, at 55 cents per square yard, on 6 pieces of cloth, each containing 54 yards, 32 inches wide?

COMMERCIAL DISCOUNT.

924. Slate Problems.

1. On a bill of goods amounting to $583.40, a discount of 5% is given for cash. What is the amount paid?

2. What will be the cost of 16 gross of Roman candles at $26.75 per gross, less 60%?

3. Sept. 1, 1894, I bought tea amounting to $1,876.50. If 5% is deducted for payment within ten days, how much would I have to pay if I paid the bill Sept. 9?

4. What will be the cost of 15 cases cocoa @ $13.20 each, less 20%?

5. Bought 5 gross of essence of lemon at 50¢ per doz., less 5%. What is the amount of my bill?

6. Find the cost of 15 cases of chloride of lime, 50 lb. per case, at 9½¢ per lb., less 15%.

7. What will be the net cost of a bill of plated ware amounting to $84.75, on which a discount of 33⅓ and 10% is allowed?

$84.75
less ⅓ 28.25
56.50
less $\frac{1}{10}$ 5.65
Ans. $ net.

This means 33⅓% discount on the gross amount, and 10% discount on the remainder.

8. Find the difference between $390 less 43⅓% discount, and $390 less 33⅓ and 10% discount.

9. Thurber, Whyland Co. sold the following goods. Make out the bill, less 50 and 10 and 10 and 10% discount.

500	¼-pound	bags	at	$1.00	per M.
1,500	½-pound	"	"	1.20	" "
3,000	1-pound	"	"	1.60	" "
5,500	1½-pound	"	"	1.70	" "
2,000	2-pound	"	"	2.00	" "

10. Find the net cost of 18,500 bags at $4.40 per M. less 60 and 10 and 5%.

11. Which is the better discount for the buyer, 40 and 10% or 30 and 20%? What will be the difference on a bill of $100?

12. $100 less 33⅓ and 10% discount is equal to what? What per cent discount is 33⅓ and 10% equal to? To what per cent net is it equivalent?

13. What per cent discount is equal to 40 and 20% discount?

14. In price-lists the "gross" price is generally given. What is the "gross," or "list," price of plated ware costing $60 net, on which a discount of 33⅓ and 10% was allowed?

15. What per cent of the list price is the net price of goods on which 20 and 10% discount is allowed?

16. What per cent of the gross price is the net price of goods on which 10 and 20% discount is allowed?

925. Wilmington, Del., Public Schools. Examination Questions.

1. Find 12½ per cent of $648.

2. What per cent of 16 is 12?

3. What per cent of 50 dollars is 5 cents?

4. 62.75 is 10 per cent of what?

5. Write three-fourths of one per cent, first as a pure decimal, and again as a common fraction.

6. ⅛ + ⅘ of a number is what per cent of it?

7. 30 and 6 per cent off the price of an article marked $4.00 reduces the price to what?

8. The selling price was $30, and the gain 25 per cent. What was the cost?

9. What number diminished by 15 per cent of itself is 510?

10. If a piece of property is taxed $28.60, at a tax rate of $\frac{4}{5}$ of one per cent, what is the assessed value of the property?

11. Mr. Wilson bought 96 shares of stock, par $50, for $4,476. What was the rate of discount?

12. A boy deposited $15 in bank. This was 30 per cent of what he had in bank before making this deposit. What had he there after this deposit?

13. What is the interest of $500, for 3 years 4 months and 18 days, at 6 per cent?

14. What is the interest of $3,000, for 2 years and 6 months, at 5 per cent?

15. What is the exact interest of $500, for 100 days, at 8 per cent? (Take 365 days to the year.)

16. What is the interest of $700, from March 10 to November 3, at 5 per cent? (360 days.)

COMMERCIAL DISCOUNT.

926. Oral Problems.

1. A piano, marked $800, is sold at a discount of 25 and 10%. What is the selling price?

2. Bought goods amounting to $600, less 5% for cash. What is the net cost of the goods?

3. What single discount is 50 and 10% equal to?

4. What single discount is 30 and 30% equal to?

5. Paid $729 for goods, on which 10% was allowed. What was the "gross" price?

6. How much will I have to pay for 12 doz. bottles flavoring extract, at 60¢ per doz., less 10%?

7. What is the "list" price of an article for which I paid $48, after a discount of 25% was deducted?

8. Which is the larger discount, 40 per cent or 20 and 20%? Why?

9. What is the difference between a discount of 50 and 10% and one of 10 and 50%?

10. Find the net cost of a $700 bill of fire works, on which a discount of 60% is allowed.

927. Brooklyn Public Schools. Examination Questions.

1. A man received $2.75 per day, exclusive of Sundays, during 1891. He paid $73 for clothing for himself and family, $15 per month rent, $1.10 per day for provisions, $8 per month for fuel and light, and 25¢ per day for other expenses. How much had he left at the end of the year?

2. Divide the product of $8\frac{3}{4}$ and $11\frac{2}{3}$ by their difference.

3. If a meter measures 39.37 inches, how many yards will be equal to 2,400 meters?

4. A merchant bought 340 bushels of potatoes at 80¢ per bushel; 20 per cent of them proved worthless, and were thrown away. He sold the remainder at $1.10 per bushel. How much did he gain or lose?

5. What would it cost, at 12¢ per square yard, to paint the outside of a fence, 6 feet high, that encloses a lot 100 feet long and 25 feet wide?

6. Divide eighty-four and eighty-four hundredths by forty-eight thousandths.

7. What is the interest on $250.75, at 4 per cent, for 197 days, taking 360 days to the year?

8. A man can do a certain work in $18\frac{3}{4}$ days. What per cent of it can he do in $6\frac{2}{3}$ days?

9. What is the amount of $1,395, at 4 per cent, for 7 months 24 days?

10. A man spent 30 per cent of his money for clothes, 20 per cent for rent, and had $75 left. What rent did he pay?

11. What is the value of $(\frac{3}{8}$ of $\frac{5}{6}$ of $3\frac{3}{4} + 8\frac{2}{3}) \div (10\frac{1}{2} - 7\frac{11}{12})$?

12. From 16 ten-thousandths take 27 millionths, multiply the difference by 20.5, and divide the product by 410.

13. How many bushels of grain will fill a bin 8.5 feet long, 4.25 feet wide, $3\frac{3}{4}$ feet deep?

14. I wish to put 111 bu. 2 pk. 4 qt. of grain into 47 bags. What quantity must each contain?

15. What is the difference between $\frac{1}{2}$ per cent of $15,000, and 50 per cent of $15,000?

16. A man bought silverware for $120, and sold it for $250 less $33\frac{1}{3}$ and 10 per cent. What was his profit per cent?

17. What is the interest on $356.75, at 4 per cent, for 3 yr. 5 mo. 14 da.?

18. A note for $600, drawn January 16, payable 4 months after date, is discounted March 25 at a bank, at 6 per cent. What are the proceeds?

19. There are 350 pupils on the third floor of a school, 450 on the second floor, and 600 on the first floor. What per cent of the pupils are on each floor?

20. A dry-goods merchant sells goods $12\frac{1}{2}$¢ per yard more than their cost, and realizes a profit of 8 per cent. What is the cost per yard?

21. A man bought 396 acres of land for $40,293. He sold 150 acres at $120 per acre, 134 acres at $80 per acre, and the remainder at cost. Did he gain or lose, and how much?

22. If $44\frac{4}{9}$ yards of calico cost $1.99, how much must be paid for 80 yards?

23. Divide the sum of 75 thousandths and 75 ten-thousandths by the difference between 75 hundredths and 75 tenths.

24. What number divided by 320 gives 47 for quotient and 163 for remainder?

25. What is the cost of a pane of glass 30 inches wide and 50 inches long, at 40¢ per square foot?

26. If a river current carries a raft of lumber at the rate of 4 miles 4 furlongs 20 rods (4 miles 180 rods) per hour, how long will it take the raft to float 365 miles?

27. In a school room there are 35 pupils and a teacher. The room is 30 feet long, 20 feet wide, and 15 feet high. How many cubic feet of air space has each person?

28. What is the interest of $247.50, for 8 months 19 days, at 4 per cent?

29. A merchant sold a quantity of flour for $282, losing 6 per cent. How much money did he lose?

30. I bought 2,500 bushels of wheat at 80¢ per bushel, and sold it for 84¢ per bushel, on a note for 60 days, which I had discounted immediately at a bank, at 6%. How much did I gain?

INTEREST AND BANK DISCOUNT.

928. Slate Problems.

1. Find the interest on x dollars, for 2 years 3 months, at 5%.

$$x \times \tfrac{5}{100} \times 2\tfrac{1}{4}.$$

2. What principal will produce $180 interest in 2 years 3 months, at 5%?

Let x = the principal.

3. Find the interest on $4,250, for x years, at 6%.

4. A man loaned $4,250 at 6%. He received $765 interest. Find the time.

5. What sum, at 4%, will produce $240 in 3 years?

6. How long will it take \$2,020 to produce \$606 interest, at 6%?

7. Find the interest on \$6,000, for 2 years 6 months, at x%.

8. At what rate per cent will \$6,000 produce \$900 interest in 2 years 6 months?

9. Find the amount of \$1,875, for 7 mo. 15 da., at x%.

10. What is the amount of \$682.90, for x months, at $3\frac{3}{4}$%?

11. Find the discount on a 60-days note, for x dollars, at 6%.

$$x \times \tfrac{6}{100} \times \tfrac{6}{360} =$$

12. A 30-days note for \$400 is discounted at x%. What is the discount?

13. What is the discount on an x-days note for \$300, at 6%?

(Do not forget days of grace.)

14. Find the proceeds of a 4-months note for \$200, discounted at x per cent.

15. Find the proceeds of an x-days note for \$600, discounted at 6%.

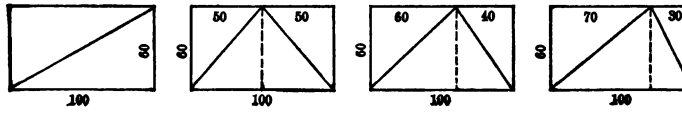
16. What is the amount of x dollars, for 4 years 3 months 10 days, at $4\frac{1}{2}$%?

17. Find the proceeds of a 3-months note for x dollars, discounted at 6%.

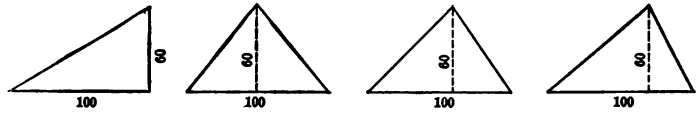
MEASUREMENTS.

929. Slate Problems.

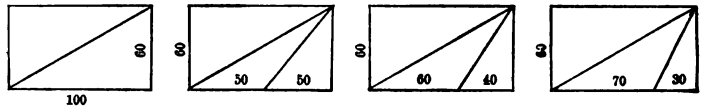
1. Mark in each division of each of the following rectangles its area in square feet. Dimensions of each rectangle 100 ft. by 60 ft.



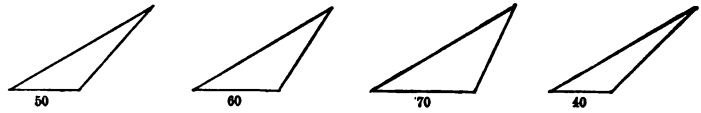
2. Find the area of each of the following triangles in square feet; base of each 100 feet, altitude 60 feet.



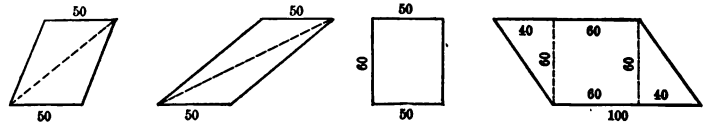
3. Mark in each of the following eleven triangles its area in square feet; altitude of each, 60 ft.



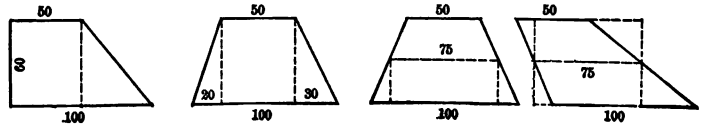
4. Find the area of each of the following four triangles in square feet; altitude of each, 60 ft.



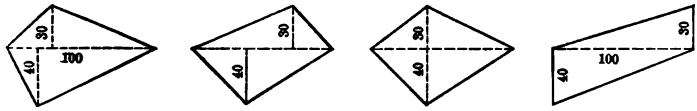
5. Find the area of each of the following four quadrilaterals; altitude of each, 60 ft.



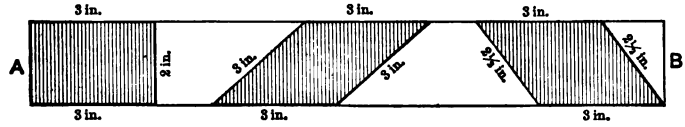
6. Find the area of each of the following trapezoids; altitude of each, 60 ft. The number of square feet in each is equal to 60 multiplied by what?



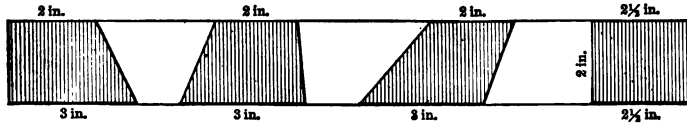
7. One diagonal of each of the following quadrilaterals measures 100 feet. The perpendiculars let fall on this diagonal from the opposite corners measure 30 ft. and 40 ft., respectively. Find the area of each in square feet.



8. Cut from a strip of paper, AB, two inches wide, a rectangle, a rhombus, and a rhomboid, as given in the accompanying diagram. Show that the three parallelograms are equal in surface.



9. Cut from a strip of paper, two inches wide, three trapezoids. Make one parallel side of each 2 inches long, and the other parallel side 3 inches long. Divide up each trapezoid in such a way as to show that its surface is equal to that of a rectangle $2\frac{1}{2}$ in. by 2 in.



SURFACES AND VOLUMES.

930. Slate Problems.

1. How many cubic feet are there in a block of granite whose base is $4\frac{1}{2}$ feet square, and whose height is 6 feet?

2. Find the value of the above block at 60¢ per cubic foot, and the cost of polishing its six faces at 60¢ per square foot.

3. A contractor charged 30¢ per cubic yard for digging a [illegible] wide, 81 feet long, 9 feet deep. What was the [illegible]

4. What will be the cost of cementing the floor of the above [illegible] per square yard? Deduct from the given dimensions of the inside of the four walls, eighteen inches each. Make [illegible]

5. A cubic foot of water weighs 1,000 ounces. Marble is 2.8 [illegible] times as heavy. Find the weight in tons, of a marble shaft [illegible] feet high. [illegible]

6. A [illegible] a cubical box whose inside measurements [illegible] The sides consist of a single piece of wood 1 [illegible] the dimensions in inches of each of the six [illegible]

7. How many [illegible] would there be in a brick [illegible] nearly 2½ times as heavy as [illegible]

8. [illegible] cubic inches of a kind of pine wood, which [illegible]

9. [illegible] 12 inches on a side, is enclosed for [illegible] a wooden box made of material [illegible] the inside dimensions of the box? [illegible] the wood and the marble together? [illegible]

10. [illegible] long weighs 2 tons. How many times [illegible] Calculate the weight of an iron cube [illegible] long.

CHAPTER XII.

SIMPLE AND COMPOUND INTEREST. — DISCOUNT. — CAUSE AND EFFECT. — PARTNERSHIP. — BONDS AND STOCKS. — EXCHANGE. — LONGITUDE AND TIME. — SURFACES AND VOLUMES.

TO FIND PRINCIPAL, RATE, OR TIME.

931. At what rate per cent will \$723.60 amount to \$759.78 in 1 yr. 1 mo. 10 da.?

Let x = rate.

Then $723.60 \times \frac{x}{100} \times \frac{402}{360} = 8.04x$ = interest.

$723.60 + 8.04x$ = amount = 759.78

Transposing, $8.04x = 759.78 - 723.60 = 36.18$

Clearing of decimals, $804x = 3{,}618$

$x = \frac{3618}{804} = 4\frac{1}{2}$ *Ans.* $4\frac{1}{2}$ per cent.

932. In what time will \$85.50 produce \$8.17 interest, at 4 per cent?

Let x = time in years.

Then $85.50 \times \frac{4}{100} \times x = 3.42x$ = interest.

$3.42x = 8.17$

Clearing of decimals, $342x = 817$

$x = \frac{817}{342}$ = time in years.

2 yr. 4 mo. 20 da.

342)817 yr.

133 yr. remainder.

12

1596 mo. new dividend.

228 mo. remainder.

30

6840 da. new dividend.

0 *Ans.* 2 yr. 4 mo. 20 da.

933. Slate Exercises.

Find rate, time, etc.

1. Principal, \$2,000; time, 3 yr.; interest, \$300. Rate?

2. Principal, \$1,800; rate, 4%; interest, \$144. Time?

3. Time, 8 mo.; rate, $4\frac{1}{2}\%$; interest, \$2.88. Principal?

4. Principal, \$38; time, 2 yr.; amount, \$40.28. Rate?

5. Principal, \$140; rate, $3\frac{1}{2}\%$; time, 3 mo. 15 da. Interest?

6. Amount, \$39.60; rate, 4%; time, 2 yr. 6 mo. Principal?

7. Amount, \$484.15; rate, $3\frac{1}{2}\%$; principal, \$460. Time?

8. Principal, \$39.60; rate, 4%; time, 1 yr. 7 mo. 15 da. Amount?

9. Time, 8 yr.; rate, 3%; amount, \$6,200. Principal?

10. Principal, \$7,548; time, 3 mo. 5 da.; interest, \$119.51. Rate?

11. Principal, \$9,000; rate, 4%; interest, \$632. Time?

12. Time, 2 yr. 3 mo. 20 da.; rate, 5%; amount, \$160.60. Principal?

13. Principal, \$756; rate, $3\frac{1}{4}\%$; time, 3 yr. 4 mo. 20 da. Interest?

14. Principal, \$120; time, 1 yr. 2 mo. 15 da.; interest, \$4.35. Rate?

15. Amount, \$97.57; rate, 4%; interest, \$7.57. Time?

16. Time, 3 yr. 8 mo. 19 da; rate, $4\frac{1}{2}\%$; amount, \$93.39. Principal?

17. Principal, \$1,848; rate, $3\frac{3}{4}\%$; time, 4 yr. 9 mo. 25 da. Amount?

18. Rate, 5%; time, 4 yr. 6 mo. 23 da.; interest, \$16.43. Principal?

19. Rate, 3%; amount, \$3,651.90; interest, \$51.90. Time?

20. Principal, \$60; amount, \$73.91; time, 3 yr. 10 mo. 11 da. Rate?

21. Amount, \$196.92; rate, 4%; time, 2 yr. 4 mo. 6 da. Interest?

22. Interest, \$27.82; rate, 5%; time, 1 yr. 5 mo. 25 da. Amount?

23. Principal, \$9,000; rate, 4%; interest, \$37. Time?

24. Principal, \$14.40; time, 3 yr. 9 mo. 20 da.; interest, \$2.74. Rate?

25. Amount, \$147.59; rate, $2\frac{1}{2}$%; interest, \$3.59. Time?

934. Oral Exercises.

1. In what time will \$100 amount to \$109, at 6% interest?

2. At what rate will \$200 produce \$16 interest in 2 years?

3. What principal will produce \$12 interest in 3 years, at 4%?

4. In what time will \$300, at 4%, produce \$29 interest?

5. In what time will \$170 produce \$1.70 interest, at 5%?

6. In what time will \$360 produce \$3.60 interest, at 4%?

7. In what time will \$725 produce \$7.25 interest, at 6%?

8. In what time will \$45 produce 45¢ interest, at $4\frac{1}{2}$%?

9. In what time will \$72 produce \$1.44 interest, at 6%?

10. Find the interest on \$84 for 144 days, at 5%.

11. Find the interest on \$125, at 5%, for 2 months 12 days.

12. At what rate will \$64 produce 64¢ interest in 80 days?

13. At what rate will \$40 produce \$1.20 interest in 6 months?

14. A certain principal produces \$120 interest, at 6%. What would be the interest if the rate were 4%?

15. A man borrows $1,000, at 4%, and loans it at 6%. What is his profit in $2\frac{1}{2}$ years?

16. What is the interest on $240, for 3 years, at 5%?

17. Find the interest on $1,420, at 4%, for 6 months?

18. What is the interest on $60, at 6%, for 11 months 29 days?

19. Find the amount of $250, at 5%, for 2 years.

20. What is the interest on $90, at 4%, for 99 days?

935. U. S. Civil Service. Copyist Examination.

1. Add the following, placing the sum at the bottom:

79,654,321,908.25
47,776,013,705.30
92,773,331,693.25
7,774,910,236.15
44,297,793,329.37
6,105,723,266.57
132,173.63
8,857,367,397.45
42,233,001,764.86
63,437,476,074.03
3,335,602,047.90
173,827,764,501.77

2. Express the following in figures: One billion ten million eleven thousand twenty, and (decimal) one thousand twenty-five ten-millionths.

3. Express the following in sign and figures: Ten billion one hundred ten million one hundred thousand five dollars six cents and three and three-tenths mills.

4. Express in words the following figures, signs, and abbreviations: $\frac{1}{2}$ of (6 hhd. 1 bbl. 16 gal. 2 qt. − 1 bbl. 2 qt.) = 6 hhd. 16 gal. ÷ 2 = 3 hhd. 8 gal.

5. Express in words the following:

CXCII. XLIX. $11,111,111,111.115. £25 6*s.* 4*d.*

6. Add .5, .05, .125, .0625, and divide the sum by 2. (Express answer in decimal fraction.)

7. A merchant purchased 540 bushels of potatoes at 90¢ per bushel, lost 30% of them by fire, and sold the remainder at $1.10 per bushel. Did he gain or lose, and how much?

8. A miller grinds for customers 3,600 bushels of wheat, taking from each bushel 4 quarts as toll. How much does he receive for the work, wheat being worth $62\frac{1}{2}$¢ per bushel?

9. A man purchased a house, paying for it in four payments as follows: On the first payment $\frac{1}{3}$ of the purchase price; on the second payment $\frac{1}{2}$ of the remainder; on the third payment $\frac{3}{5}$ of what then remained due; and on the last payment $2,000. What was the full amount paid for the house?

10. A field 250 feet long and 125 feet wide is enclosed by a tight board fence $6\frac{1}{2}$ feet high. What will it cost to paint the outside of the fence at $12\frac{1}{2}$¢ per square yard?

INTEREST BY ALIQUOT PARTS.

936. Slate Exercises.

1. Find the interest on $387.45, for 2 yr. 8 mo. 18 da., at 7%.

$387.45 × .07.

	$27.1215	interest for 1 year.
	27.1215	interest for 1 year.
6 mo. = ½ yr.	13.5607	interest for 6 months.
2 mo. = ⅓ (of 6 mo.)	4.5202	interest for 2 months.
15 da. = ¼ (of 2 mo.)	1.1301	interest for 15 days.
3 da. = ⅕ (of 15 da.)	.2260	interest for 3 days.
Ans.	$73.68	interest for 2 yr. 8 mo. 18 da.

2. Find the interest on $432.90, at 6%, for 1 yr. 7 mo. 12 da.

	$432.90 × .06.	
		interest for 1 yr.
6 mo. = ½ yr.		interest for 6 mo.
1 mo. = ⅙ (of 6 mo.)		interest for 1 mo.
10 da. = ⅓ (of 1 mo.)		interest for 10 da.
2 da. = ⅕ (of 10 da.)		interest for 2 da.
		interest for 1 yr. 7 mo. 12 da.

3. Find the amount of $874.16, at 5%, for 1 yr. 9 mo. 4 da.

	$874.16	principal.
5% = 1/20	43.708	interest for 1 yr.
6 mo. = ½ yr.		interest for 6 mo.
3 mo. = ½ (of 6 mo.)		interest for 3 mo.
3 da. = 1/30 (of 3 mo.)		interest for 3 da.
1 da. = ⅓ (of 3 da.)		interest for 1 da.
		amount for 1 yr. 9 mo. 4 da.

4. What is the amount of $95.72, for 3 yr. 6 mo. 20 da., at 5%?

	$95.72	principal.
(10% =) 1/10	9.572	interest for 2 yr.
1 yr. = ½ (of 2 yr.)	4.786	interest for 1 yr.
6 mo. = ½ yr.		interest for 6 mo.
20 da. = ? of 6 mo.		interest for 20 da.
		amount for 3 yr. 6 mo. 20 da.

5. Interest of $1,806.45, at 4%, for 1 yr. 7 mo. 25 da.

1 yr., 6 mo., 1 mo., 15 da., 5 da., 5 da.

6. Interest for 10 mo. 29 da., at 4%, on $380.40.

	$380.40 × .04.	
	$15.2160	interest for 1 year.
1 mo. = 1/12 yr.		interest for 1 month } deduct from interest
1 da. = 1/30 mo.		interest for 1 day } for 1 year.
		interest for 10 mo. 29 da.

7. Amount, at 6%, of $125.73, for 2 yr. 10 mo. 4 da.

8. Interest on $84.66, at 7%, for 1 yr. 4 mo. 12 da.

9. Interest, at 5%, for 4 yr. 2 mo. 7 da., on $250.

10. Amount of $1,000, at 6%, for 33 days.

937. When the time is less than a year, the following facts should be remembered:

6% for a year is 1 per cent for 60 days.
5% for a year is 1 per cent for 72 days.
4½% for a year is 1 per cent for ? days.
4% for a year is 1 per cent for ? days.

11. Find the interest for 81 days, at 5%, on $876.40.

72 days = 1% = $8.764
9 days = ⅛ (of 72 da.) = 1.095
$9.86 interest for 81 days.

12. Amount of $954, at 4%, for 4 mo. 10 da.

Principal $954.
3 months' interest = 1% 9.54
1 mo. = ⅓ (of 3 mo.) 3.18
10 da. = ⅓ (of 1 mo.)
amount for 4 mo. 10 da.

13. Interest of $1,874, at 4½%, for 93 days.

80 days = 1%.
10 days
2 days
1 day

14. Interest of $753.20, at 5%, for 158 days.

72 da., 72 da., 12 da., 2 da.

15. Amount of $1,234.50, for 193 days, at 6%.

60 da., 120 da., 12 da., 1 da.

16. Find the proceeds of a 90-days note, for $873.60, at 6%.

Face	$873.60	
60 da.	8.736	} Deduct.
30 da.	4.368	
3 da.	.437	
	$860.06	proceeds.

17. Find the discount on a 3-months note, for $1,596, at 6%.

18. What are the proceeds of a 6-months note, for $785, discounted at 6%.

19. Find the interest on $484.40, for 1 yr. 3 mo. 17 da., at 7%.

20. Find the amount of $683, for 3 yr. 4 mo. 11 da., at 4½%.

938. N.B. — Do not use unnecessary figures.

21. Principal, $360; 5%; 3 yr. 7 mo. 18 da. Interest?

22. Principal, $613; 4½%; 157 da. Amount?

23. Principal, $1,774; 3¾%; 17 mo. 23 da. Interest?

24. Principal, $875; 6%; 2 yr. 3 mo. 1 da. Amount?

25. Principal, $976; 7%; 325 da. Interest?

939. By the *time* of a note is meant the number of days, etc., for which it is drawn. Find the discount for three additional days.

26. Face of note, $254; time, 30 days; 7%. Proceeds?

27. Face of note, $515; time, 6 months; 5%. Discount?

28. Face of note, $493; time, 60 days; 8%. Proceeds?

29. Face of note, $717; time, 15 days; 6½%. Discount?

30. Face of note, $1,000; time, 90 days; 4%. Proceeds?

940. Find the exact number of days. Take 360 days to year.

31. Principal, $1,836.50; 6%; Jan. 2 to Dec. 1. Amount?

32. Principal, $1,295.70; 7%; March 8 to April 9. Interest?

33. Principal, $1,433.11; 5%; Feb. 13 to Sept. 4. Amount?

34. Principal, $765.90; 4%; Oct. 1 to Dec. 17. Interest?

35. Principal, $275.84; 5½%; May 9 to July 3. Amount?

941. By the *term* of a note is meant the number of days it has to run after it has been discounted, including days of grace.

36. Face of note, $100; term, 63 days; 7%. Discount?

37. Face of note, $200; term, 93 days; 6½%. Proceeds?

38. Face of note, $300; term, 24 days; 5½%. Discount?

39. Face of note, $400; term, 117 days; 8%. Proceeds?

40. Face of note, $500; term, 88 days; 5%. Discount?

942. In examples 41–45, inclusive, find the time by compound subtraction.

41. Principal, $25.83; 6%; Jan. 14, 1892, to Sept. 5, 1894. Interest?

42. Principal, $47.96; 5%; Feb. 6, 1893, to Aug. 1, 1896. Amount?

43. Principal, $85.30; 7%; March 25, 1894, to Jan. 13, 1897. Interest?

44. Principal, $75.00; 4%; April 15, 1888, to Feb. 6, 1895. Amount?

45. Principal, $92.75; 3%; May 6, 1893, to Dec. 29, 1898. Interest?

46. Face of note, $600; time, 60 days; dated June 13, discounted June 19. Proceeds?

47. Face of note, $700; time, 90 days; dated July 2, discounted July 3. Discount?

48. Face of note, $800; time, 3 months; dated Aug. 14, discounted Oct. 1. Proceeds?

49. Face of note, $900; time, 4 months; dated January 5, discounted May 5. Discount?

50. Face of note, $1,000; time, 6 months; dated March 17, discounted March 17. Proceeds?

943. U. S. Civil Service. Day Inspectors, Custom House.

1. Add these across, placing the totals in the spaces provided, and find grand total.

			Totals.
9,277	3,185	2,054	.....
1,326	8,520	5,291	.....
2,657	5,873	5,212	.....
5,281	3,535	8,841	.....
23,493	23,285	2,479	.....
15,725	2,165	2,957	.....
3,013	4,425	4,869	.....
8,890	31,715	1,789	.....
504	2,071	4,131	.....
6,284	290	3,152	.....
770	7,291	87,247	.....
13,323	7,400	329	.....
491	3	33,001	.....
978	1,940	535	.....
19	486	1,405	.....
856	2,718	6	.....
769	210	7,457	.....
28,923	71,614	892	.....
		Grand total....	.....

2. Express the following in words: 10,111,303.0125.

3. Express the following in words: $1,405,090.625.

4. Express in sign and figures one million one thousand ten dollars and twenty cents.

5. What is the sum of

$$7{,}752.5 + 775.75 + 77.625 + 7.6625 + .66625?$$

6. Divide $\frac{1}{3}$ of $4\frac{1}{2}$ by $\frac{2}{5}$ of $\frac{7}{8}$.

Reduce fraction to its lowest terms.

7. A merchant purchased 3 hhd. wine at 50¢ per gal. He lost by leakage, from one hhd. 10%, from another 5%, and from the other 4% of the quantity. He sold the remainder at 29¢ per qt. Did he gain or lose by the transaction, and how much? (1 hhd. = 63 gal.)

8. If a laborer received for his services $2.75 per day, exclusive of (53) Sundays during the year 1887, and paid $175 per year for house rent, $1.10 per day for provisions, $8 per month for fuel and lights, $6.50 per month for clothing for self and family, and 25¢ per day for incidentals, how much had he left at the close of the year?

9. Five lots of coal, containing, respectively, $43\frac{1}{2}$ tons, $38\frac{1}{4}$ tons, $45\frac{1}{8}$ tons, $33\frac{1}{2}$ tons, and $66\frac{1}{8}$ tons, were divided equally among five dealers. How much did each receive? (Ton = 2,240 lb.)

Express answer in tons, hundredweights, and quarters.

10. What will it cost to carpet a room, 13 feet long and 18 feet wide, with carpeting $\frac{3}{4}$ yd. wide, at $1.25 per lineal yard?

COMMERCIAL DISCOUNT.

944. Slate Problems.

1. On a bill of goods amounting to $874.40, a discount of 5% is allowed. How much must be paid?

$$\begin{array}{rl} & \$874.40 \\ 5\% = \frac{1}{20} & \underline{\quad 43.72} \\ & \$830.68 \ \textit{Ans.} \end{array}$$

Divide by 2, placing the quotient figure one place to the right of the corresponding figure of the dividend.

2. Find the cost of a wagon, the catalogue price of which is $750, the discount being 30%.

$750 × .70 = $525. *Ans.*

The net cost is .70 of the catalogue price.

3. What will be the cost of goods amounting to $1,837.60, on which there is allowed a discount of 17½%?

	$1,837.60	
10% = ⅒	183.76	} Deduct.
5% = ½ (of 10%)	91.88	
2½% = ½ (of 5%)	45.94	
	$1,516.02 *Ans.*	

For 10% rewrite the original amount, placing first figure one place to the right. 5% is ½ of 10%. 2½% is ½ of 5%.

4. $784.68 less 75%.

¼ of $784.68 = *Ans.*

5. $937.52 less 36%.

		$937.52
Deduct	25% = ¼	234.38
	10% = ⅒ of $937.52	
	1% =	

6. Find the net cost of 1,630 yd. silk, invoiced at $1.10 per yard, less 16% discount.

7. What is the cost, in francs, of 843.72 meters silk, at 5.75 francs per meter, less 12%?

8. What is the net cost of a lot of musical instruments amounting to $1,875.60, on which a discount of 10, 5, and 2½% is allowed? (Art. 924, Ex. 7.)

9. What would be the net cost of the same articles, if the discount were 2½, 5, and 10%?

10. Find the net cost of the same, at 17½% discount.

11. Goods catalogued at x dollars are sold at a discount of 20 and 10%. What is the net price?

12. A piano marked \$600 is sold at a discount of $33\frac{1}{3}$ and $x\%$. Find the selling price.

13. What is the catalogue price of an article on which 20 and 10% discount is allowed, the net price being \$360?

14. A wagon catalogued at \$750 is sold at a discount of $33\frac{1}{3}$ and $x\%$. Find value of x, when the selling price is \$450.

15. Which is the better discount, 40 and 30% or 30 and 40%?

945. U. S. Civil Service. Departmental Examination. Copyist.

1. Add the following, placing the sum at the bottom:

79,654,321,908.35
47,776,013,703.30
92,773,331,673.25
7,774,910,336.15
44,297,794,329.37
6,105,733,266.59
232,173.63
8,859,367,397.45
42,223,001,764.86
63,337,476,074.03
2,335,602,047.90
293,827,764,501.77

2. Express the following in figures:

Two billion three million one hundred thousand eight, and (decimal) four hundred six millionths.

3. Express the following in sign and figures:

Three hundred sixteen million two hundred sixty-four dollars, five cents and six and seven-tenths mills.

4. Express in words the following figures, signs, and abbreviations:

(7 mi. 3 fur. 24 rd. 4 yd. 2 ft. 9 in. + 2 mi. 2 yd. 1 ft.) ÷ 3 = 3 mi. 1 fur. 8 rd. 2 yd. 1 ft. 3 in.

(fur. = furlong.)

5. Express in words the following:

CCCLVI.

$105,003,200.153.

£13 15*s.* 8*d.*

$(\frac{1}{7} + \frac{16}{17} + \frac{84}{48}) \times \frac{1}{8} = \frac{5119}{8568}$.

6. Add .026, .0137, and .4. From the sum subtract .3998, and divide the difference by 21. (Express answer in decimal fraction.)

7. A merchant bought 84 yd. of linen at 55¢ per yd., and 105 yd. of muslin at 20¢ per yd. He sold all the linen at 40¢ per yd. What must he charge per yd. for the muslin in order to make up exactly his loss on the linen?

8. A fruit dealer bought a lot of oranges for $240. He sold $\frac{1}{4}$ of them for $\frac{1}{2}$ of the entire cost; $\frac{1}{8}$ of the remainder for $\frac{2}{5}$ of the entire cost; $\frac{1}{2}$ of what then remained for $\frac{1}{8}$ of the entire cost; and the final remainder for $\frac{1}{8}$ of the entire cost. What was his gain or loss?

9. The owner of 165 shares of gas stock sold them at $25 per share, and with the proceeds purchased two lots, 32 ft. by 115 ft., and 30 ft. by 105 ft., respectively, and had just $27 left. What was the price per square foot of the lots?

10. At 18¢ a square yard, what will it cost to plaster the walls and ceiling of a room 16 ft. long, 12 ft. wide, 14 ft. high, deducting for two doors, each 8 ft. by 4 ft., and for three windows, each 7 ft. by 3 ft.?

946. Oral Problems.

1. A can do a piece of work in 5 hours, B in 7 hours. How long will it take both working together?

2. An agent collected a bill, and sent to his employer the amount, less $2\frac{1}{2}\%$ commission. If his commission was \$1.60, how much did he remit to his employer?

3. My house, worth \$12,000, is insured for $\frac{3}{4}$ of its value, at $\frac{1}{4}\%$. What premium do I pay?

4. A floor 6 yards long, 4 yards wide, needs 32 yards of carpet to cover it. What is the width of the carpet?

5. What will be the interest on \$87, at 5 per cent, for 144 days?

6. Find the discount, at 8 per cent, on a note for \$176, which has 90 days to run.

7. An agent receives \$8,200 to invest after deducting his commission of $\frac{1}{40}$ of the amount invested. What is the agent's commission?

8. By selling a house for \$3,500, I lose \$500. What is my loss per cent?

9. A lot is sold for \$1,200, at a loss of 20 per cent. What part of \$1,200 is the loss?

10. A merchant's receipts are \$1,200; his gain is 20 per cent. What part of his receipts is profit?

11. If 3 men earn \$72 in 8 days, how many dollars will 5 men earn in 11 days?

12. If a dealer loses 25% by selling a horse for \$225, what per cent would he gain or lose by selling the horse for \$325?

13. Find the cost of 4 yd. 1 ft. of ribbon, when 2 yd. 2 ft. cost 40 cents.

TO FIND FACE OF NOTE, RATE OF DISCOUNT, AND TIME.

947. I wish to obtain \$1,000 from a bank. What must be the face of a 30-days note, which will give the above proceeds, if it is discounted at 6%?

Let x = face of the note.

$$x \times \frac{6}{100} \times \frac{33}{360} = \frac{11x}{2000} = \text{discount.}$$

$$x - \frac{11x}{2000} = \text{proceeds} = 1{,}000$$

Clearing of fractions, $2{,}000x - 11x = 2{,}000{,}000$

$$1{,}989x = 2{,}000{,}000$$

$$x = \frac{2000000}{1989} = 1{,}005.53$$

Ans. \$1,005.53, face of note.

Proof. Face of note, \$1,005.53

Deduct { 30 days' discount = 5.0276 } ½%.
Deduct { 3 days' discount = .5027 } $\frac{1}{10}$ of 30 days.

Proceeds \$1,000.00

948. A note for \$1,980 was discounted at 6%. The proceeds were \$1,959.21. How many days had the note to run?

Let x = *term* in days.

$$1{,}980 \times \frac{6}{100} \times \frac{x}{360} = \frac{33x}{100} = \text{discount.}$$

$$1{,}980 - \frac{33x}{100} = \text{proceeds} = 1{,}959.21$$

Clearing of fractions, $198{,}000 - 33x = 195{,}921$

$$-33x = -2{,}079$$

$$x = 63$$

Ans. 63 days.

949. Slate Exercises.

1. Three-months note; face, $108; rate 6%. Find proceeds.

(Term of discount is 93 days.)

2. 90-days note; face, $360; discount, $6.51. Find rate.

3. Proceeds, $717.60; rate, 5%; face, $720. Find term.

4. Discount, $11.20; rate, 7%; term, 48 days. Find face.

5. 15-days note; face, $1,560; rate, 6%. Find discount.

6. Term, 20 days; face, $158.40; proceeds, $157.96. Find rate.

7. Rate, 7%; discount, $2.10; face, $150. Find term.

8. Two-months note; discount, $14.70; rate, 7%. Find face.

9. For what amount must a 60-days note be drawn, so that the proceeds will be $300 when the rate of discount is 8 per cent?

10. A note for $120 was discounted at a bank March 15, 1894. What is the date of the maturity of the note, the proceeds being $119.52 and the rate of discount 6 per cent?

11. Find the proceeds of a 6-months note for $875 drawn Jan. 2, 1894, and discounted at 6 per cent 35 days after that date.

12. A merchant bought 300 barrels of flour at $4.75 per bbl., cash, and sold it for $5 per bbl., taking in payment a 60-days note for the amount. If he has the note discounted immediately at a bank, at 7 per cent, what does he gain by the transaction?

950. U. S. Civil Service. Bureau of Pensions. Special Examiner Examination.

1. Write in figures as decimal fractions: (*a*) Five thousand six hundred forty-three ten-millionths. (*b*) Two thousand twenty-two hundred-thousandths.

2. Express in words: (*a*) \$1,500,675,001.375. (*b*) £17 8*s*. 3*d*.

3. Find the value of $2\frac{1}{2} + \frac{3}{5} - \frac{5}{6}$.

4. Divide 4.5 by .012.

5. The regular salary of a special examiner was \$350 per quarter, with \$3 additional for each day actually employed. During the first quarter of 1888 he was employed an average of 20 days per month. His expenses when employed were \$3 a day, and when unemployed \$2 a day. What was his net income for that quarter? (Leap year.)

6. A pension is granted to a widow from April 16, 1879, at \$8 per month, and \$2 per month for each of two children up to the age of 16; widow's pension increased to \$12 per month from March 19, 1886. One child attains the age of 16 on September 1, 1880, and the other on November 22, 1882. She receives her first payment on March 4, 1887. How much should this be?

7. Calculate the interest on \$5,650 for 1 year 5 months 12 days, at 7% per annum.

8. An examiner purchased a house for \$5,500. At the end of five years he found that after paying interest on his money and all taxes, he had received a net income from rent amounting to \$10 a month. He sold the house for one-fifth more than its original cost. How much did he make by the transaction?

9. Special examiner A reports 16 cases per month, B 14 cases per month, and C 12 cases per month. A receives \$1,600 per year. If paid in proportion to work done, what should be the salaries of B and C?

10. Owing to a deficiency in the appropriation bill, the salaries of the clerks in a bureau were reduced 18 per cent for the last quarter of the fiscal year. How much did a clerk who was paid \$287 for the last quarter receive during the whole fiscal year?

SPECIAL DRILLS.

951. Find sums:

$23 + 37 + 48$	$\$7.56 + \$5.38$	$52 + 41 + 34$	$325 + 865$
$44 + 66 + 19$	$\$2.74 + \$8.54$	$28 + 38 + 43$	$472 + 935$
$75 + 42 + 37$	$\$3.49 + \$9.89$	$81 + 49 + 24$	$567 + 629$
$16 + 71 + 62$	$\$4.83 + \$6.52$	$63 + 47 + 33$	$784 + 796$

952. Find remainders:

$1{,}300 - 654$	$90 - 34 - 39$	$\$63.20 - \$48.50$	$\$98.63 - \$75.21$
$1{,}295 - 986$	$94 - 27 - 66$	$\$27.80 - \$19.90$	$\$63.44 - \$50.20$
$1{,}111 - 777$	$85 - 42 - 37$	$\$34.10 - \$17.30$	$\$86.75 - \$42.50$
$1{,}463 - 684$	$79 - 16 - 12$	$\$56.70 - \$20.70$	$\$73.24 - \$31.10$

953. Find products:

91×19	51×29	27×99	67×101
82×19	42×29	38×99	78×101
73×19	32×29	49×99	89×101
64×19	23×29	56×99	98×101

954. Find quotients:

$378 \div 18$	$256 \div 16$	$468 \div 18$	$175 \div 12\frac{1}{2}$
$462 \div 14$	$289 \div 17$	$900 \div 75$	$75 \div 6\frac{1}{4}$
$475 \div 19$	$493 \div 17$	$675 \div 75$	$675 \div 37\frac{1}{2}$
$448 \div 16$	$465 \div 15$	$975 \div 75$	$375 \div 62\frac{1}{2}$

955. Find answers:

$136 \times \frac{7}{8}$	$64\frac{1}{2} \div 5$	$22 \times 19\frac{1}{2}$	$65\frac{1}{2} \div 13$
$290 \times \frac{9}{10}$	$73\frac{2}{3} \div 8$	$24 \times 19\frac{3}{4}$	$109\frac{4}{5} \div 12$
$315 \times \frac{14}{15}$	$33\frac{2}{5} \div 7$	$32 \times 29\frac{7}{8}$	$160 \div 1\frac{1}{2}$
$378 \times \frac{17}{18}$	$47\frac{1}{6} \div 9$	$45 \times 9\frac{14}{15}$	$18\frac{4}{5} \div 1\frac{4}{5}$

956. Oral Problems.

1. Paid 23¢ for calico, 27¢ for ribbon, and 48¢ for collars. What was the amount of my bill?

2. A farmer had 95 sheep. He sold 39, and 17 died. How many had he left?

3. What will be the cost of 16 base balls, at 49¢ each?

4. How much paint will there be in 27 casks, each containing 75 lb.?

5. A man divided a 429 acre farm into plots of 13 acres each. How many such plots were there?

6. There are 900 men in a certain regiment. How many companies of 75 men each are in the regiment?

7. Find the cost of 136 lb. sal-soda, at $\frac{7}{8}$¢ per lb.

8. At $19\frac{1}{2}$¢ per yd., what will I have to pay for 64 yd. gingham?

9. How many square inches in a sheet of paper $10\frac{1}{2}$ inches long by $4\frac{1}{2}$ inches wide?

10. If $2\frac{2}{3}$ yards of cloth are needed for a jacket, how many jackets can be made from $18\frac{2}{3}$ yd.?

11. How many yards around a field 96 yards long, 75 yards wide?

12. What will be the area, in square rods, of a triangle 33 rods base, altitude 42 rods?

13. How many acres in 4,960 square rods?

14. How many feet in a mile?

15. I paid $16.25 for cloth at $1.25 per yard. How many yards did I buy?

16. What will be the cost of 3 lb. 7 oz. of tea, at 64¢ per lb.?

17. Half a number $+ \frac{1}{3}$ of the same number $= 85$. What is the number?

18. I mix 4 lb. of coffee costing 20¢, with 6 lb. costing 25¢. What is the mixture worth per lb.?

19. A tailor makes up 99 yd. cloth into trousers, using $2\frac{3}{4}$ yd. per pair. How many pairs of trousers does he make?

20. How many feet in $2\frac{1}{4}$ rods?

21. At 60¢ per pound, what will be the cost of a chest of tea weighing 45 lb.?

22. A man owns a strip of land with a frontage of 576 feet. How many lots 18 feet front can he make?

23. How much will be paid for 21 lb. butter, at 28¢ per lb.?

24. The circumference of a circle is about $3\frac{1}{7}$ times its diameter. Find the diameter of a circle whose circumference is 132 ft.

25. A square field contains 169 sq. rods. How many rods of fence will be needed to enclose it?

957. Questions from U. S. Civil Service Examinations.

1. The duty on brown and bleached linens, valued at 30¢ or less per square yard, is 35 per cent ad valorem, or upon the value. What is the amount chargeable on a bale containing 56 webs, each being $37\frac{1}{2}$ yards long, 30 inches wide, and valued at 27¢ per square yard?

2. What would be the total cost of 17,350 pounds of flour at \$4.875 per bbl. of 196 lb., 12,275 lb. of sugar at \$4.9375 per 100 lb., and $2\frac{1}{4}$ tons of rice at \$.0825 per pound? (2,240 lb. = 1 ton.)

(Give operation and result in decimal fractions.)

3. An officer of the United States deposits in the Treasury \$49,739.55$\frac{3}{8}$, having retained $1\frac{3}{4}$ per cent of the whole amount collected. What amount did he collect?

4. The circulation in Great Britain at the close of 1882 was estimated to be, of gold £120,761,000, of silver £19,144,000, and of paper £43,721,000. The value of the pound sterling being $4.8665, what would be the equivalent of this circulation in the currency of the United States?

5. Express in figures:

Five hundred two million one thousand one and one ten-thousandth.

6. Express in words:

163,003,601.0044.

7. Add the following:

5,673,911,987.87
44,376,013,705.90
32,673,231,695.25
7,736,910,286.16
6,444,642,155.14
44,297,763,429.39
26,105,321,266.57
9,708,132,873.63
42,231,001,161.86
63,497,476,084.03
1,362,004,706.22

8. The consular fees collected by the U. S. Consul at Liverpool during the year ending June 30, 1882, amounted to $16,561.67. The salary was $7,500. What percentage of the fees was required to pay the salary?

9. The silver coinage in France, in 1882, amounted to 1,159,859.50 francs. The value of the franc is 19.3¢. What was the value of this coinage expressed in the money of the United States?

10. If $\frac{6}{95}$ of a pound of zinc costs $\frac{8}{96}$ of a dollar, how many cents will $\frac{17}{20}$ of a pound cost?

11. What must be the annual sales of a merchant that he may realize $4,500 after paying $2,500 expenses, when his rate of profit is 35 per cent?

12. The duty on woolen bunting being 20¢ per square yard, and 35 per cent ad valorem, give the total duty on 45 pieces, each containing 38 lineal yards, the width being 42 inches, and costing 55¢ per lineal yard.

13. Express in figures the following numbers:

Ten million seventeen hundred seven dollars thirty-seven cents five mills.

14. Express in words the following sign and figures:

$170,001,310,525.625.

15. Add the following columns of figures so as to show (1) total expenses, and (2) total receipts; and subtract the expenses from the receipts so as to show (3) the profits.

Expenses.		Receipts.	
Office rent	$360.00	Day visits	$1,550.00
Feed and care of horse	240.00	Night visits	735.00
Wages of servants	276.00	Surgical operations	975.00
Purchase and repair of instruments	75.75	Office prescriptions	923.50
Repairs of carriage and harness	25.50	Mileage	842.85
R. R. fare	40.60	Medicines	427.20
Medicines	12.25		
Stationery and printing	9.35		
Total		Total	
Less expenses			
Profits			

16. Reduce the following decimal fractions to common fractions, and reduce to lowest terms:

.75, .125, .1875, .4.

17. Give the value of two and one-half pounds of sulphate of eserine, at 33¢ per grain. (5,760 gr. per pound.)

18. There were 984 patients in a certain hospital, classified as follows: 369, pulmonary diseases; 246, nervous diseases; 123, diseases of heart; and 246, various other diseases. Give the per cent of each class.

19. What will it cost to carpet an office room measuring 21 feet in length, and 19½ feet in width, the carpeting being ¾ yard wide, and costing $1.35 per lineal yard?

20. A physician accepts, in payment of a bill, a note for $275.75, due in one year and three months, without grace, at 7 per cent. What amount will be due at maturity?

SHORT METHODS.

Slate Exercises.

6,748
× 427
47 236 Multiply by 7. Multiply this product by 6. Why?
2 834 16
2,881,396

958. Find products:

1. 3,925 × 328
2. 12,345 × 273
3. 2,087 × 287
4. 20,308 × 142
5. 4,321 × 189
6. 31,265 × 164
7. 5,763 × 426
8. 87,093 × 486
9. 6,905 × 364
10. 64,271 × 357

3,289
832
2 631 2 Multiply by 8. Multiply this product by 4. Where is the second product placed? Why?
105 248
2,736,448

959. Find products:

11. $4{,}008 \times 214$	**21.** $7{,}214 \times 99$	**31.** $876 \times 9\frac{7}{8}$
12. $8{,}736 \times 742$	**22.** $3{,}281 \times 999$	**32.** $547 \times 19\frac{3}{8}$
13. $3{,}764 \times 327$	**23.** $6{,}085 \times 75$	**33.** $734 \times 29\frac{4}{5}$
14. $1{,}087 \times 848$	**24.** $6{,}984 \times 25$	**34.** $615 \times 39\frac{1}{2}$
15. $8{,}319 \times 416$	**25.** $5{,}796 \times 62\frac{1}{2}$	**35.** $427 \times 71 \times 16$
16. $6{,}352 \times 927$	**26.** $8{,}383 \times 12\frac{1}{2}$	**36.** $284 \times 31 \times 19$
17. $2{,}781 \times 525$	**27.** $3{,}428 \times 37\frac{1}{2}$	**37.** $876 \times 27\frac{7}{8}$
18. $9{,}060 \times 1{,}166$	**28.** $7{,}154 \times 87\frac{1}{2}$	**38.** $973 \times 24\frac{4}{5}$
19. $6{,}329 \times 618$	**29.** $6{,}419 \times 33\frac{1}{3}$	**39.** $5{,}147 \times 126\frac{6}{11}$
20. $2{,}345 \times 1{,}272$	**30.** $6{,}208 \times 66\frac{2}{3}$	**40.** $4{,}284 \times 451\frac{1}{9}$

960. Supply missing amounts:

41.	**42.**	**43.**
\$834,682.50	\$16,945.84	\$380,086.77
95.00	123,456.78	64,593.25
734.18	9,876.54	8,737.84
.69	385.89	695.27
3.75	57.40	47.16
28.14	.98	
1,059.23	7.23	96.86
22,965.89	16.84	408.08
387.42	275.30	2,766.59
1,369.78	8,888.88	32,059.87
777,777.77		165,384.26
83,008.08	64,935.27	32,564.37
699.69	148,376.95	6,999.88
.88	834.11	840.30
3.86	2,070.08	27.63
50.05	12,316.99	5.98
	7,456.83	.86
\$2,000,000.02	\$456,789.01	\$743,869.05

44. £7 16*s.* 7*d.*
4 18*s.* 11*d.*
£20 1*s.* 3*d.*

45. 8 yd. 2 ft. 6 in.
9 yd. 1 ft. 10 in.
30 yd. 0 ft. 3 in.

46. 6 bu. 3 pk. 5 qt.
5 bu. 2 pk. 7 qt.
25 bu. 1 pk. 1 qt.

961. Divide. 2 decimal places (Arts. 385, 616):

47. 46,893,647 ÷ 3,986,048

48. 26,053,862 ÷ 1,898,637

49. 38,627,000 ÷ 2,679,835

50. 76,538,061 ÷ 5,736,804

51. 92,647,318 ÷ 4,863,978

52. 57,913,246 ÷ 2,597,384

962. Write answers (Art. 385):

53. $\frac{46,893,647}{9,728,759} =$

54. $\frac{26,053,862}{2,198,684} =$

55. $\frac{38,627,000}{3,568,879} =$

56. $\frac{76,538,061}{8,736,804} =$

57. $\frac{92,647,318}{9,863,978} =$

58. $\frac{57,913,246}{7,384,597} =$

TABLE OF EXPORTS.

963. The following table contains the values of the goods exported in 1891 by the United States to the various countries of the world.

Find the total value of the goods exported, and the per cent of this value for each section. Carry out to two places of decimals.

Countries to which Exported.	1891.	Per cent.
1. Europe	$697,614,106	?
2. Asia and Oceanica	43,813,519	?
3. British North American Possessions	37,345,515	?
4. West Indies	33,416,178	?
5. South America	33,226,401	?
6. Mexico, Central America, and British Honduras	21,236,545	?
7. Africa	4,738,847	?
8. All other	879,172	?
	?	100.00

REVIEW.

964. Oral Problems.

1. $\frac{3}{7}$ of a number is 48. What is the number?

2. A base ball club won 17 games, and lost 13 games. What per cent of its games did it win?

3. What per cent of 4 is 64?

4. $2\frac{2}{3}$ is what per cent of $3\frac{1}{3}$?

5. How many acres in a rectangular farm 1 mile long, $\frac{7}{8}$ mile wide?

6. What per cent of the "list" price is paid by a buyer who receives a discount of 20 and 10 per cent?

7. A tank is filled by two pipes, one of which can fill it in 6 hours, and the other in 8. How long will it take both together to fill the tank?

8. Find the interest on \$80, for 72 days, at 6%.

9. A man sold a wagon for \$420, which was 16% less than it cost. How much did he lose?

10. A kilo is 2.2046 lb. How many pounds in 1,000 kilos?

965. Slate Problems.

In the first four examples, carry out to two places of decimals, as: 135.67%, 25.83%, 6.03%, 0.52%, 0.09%.

1. The population of Montana was 39,159 in 1880, 132,159 in 1890. Find the per cent of increase.

2. The population of South Dakota in 1890 was 328,808, a gain of 230,540 over the population in 1880. What was the gain per cent?

3. The enrollment in the South Dakota schools was 9,972 in 1880, 66,150 in 1890. Find the gain per cent.

4. What was the gain per cent in the population of a state that had 416,396 inhabitants in 1880, and 416,552 in 1890?

5. A man marks an article $1.50, and sells it at a discount of 25% from the marked price. If the article cost him 90¢, what is his gain per cent?

6. Goods costing $8 are sold at an advance of 20 per cent. The marked price is $12. What per cent reduction is made on the marked price?

7. A rug costs $20. It is sold at a profit of 20%. The selling price is 20% below the marked price. How much is received for the rug? What is the marked price?

8. What price must cloth, which costs $2 per yard, be marked so that a profit of 20% will be made when the cloth is sold at 20% less than the marked price?

9. Find the net cost in English money of a bill of goods amounting to £83 2*s*. 6*d*. less 12%.

$$10\% = \tfrac{1}{10} \text{ of } £83\ 2s.\ 6d.$$
$$2\% = \tfrac{1}{5} \text{ of } 10\%.$$

10. A can do a piece of work in 12 days, B in 15 days. How long will it take both working together?

966. St. Paul Public Schools. Examination Questions.

1. What is a promissory note? An indorser? Purpose of an indorser? Write a promissory note.

2. Find the interest, by any of the ordinary methods, of $721.56, for 1 yr. 4 mo. 10 da., at 6%.

3. At what rate will $1,500 amount to $1,684.50, in 2 yr. 18 da.?

4. The net profits of a mill in two years were $6,970, and the profits the second year were 5% greater than the profits the first year. What are the profits each year?

5. How shall I mark goods that cost me $.96 a yard, in order to abate 15% and still make 15%?

6. A man receives semi-annually $350 interest on a mortgage, at 7%. What is the amount of the mortgage?

7. Explain the terms: interest, usury, principal, tax, insurance, duty.

8. What will it cost to insure a factory valued at $21,000, at $\frac{4}{5}$%, and the machinery valued at $15,400, at $\frac{5}{8}$%?

9. John Ray imported from Havana 25 hhd. of W. I. molasses, which was invoiced at 36¢ per gallon. Allowing $\frac{1}{2}$% for leakage, what was the duty at 24 per cent? (1 hhd. = 63 gal.)

10. Analyze the ninth, and explain what a port of entry is, why it is necessary to impose a duty, and give some uses to which the money so obtained is put.

967. Worcester Public Schools. Examination Questions. Oral.

1. If A can do a piece of work in 2 days, B in 3 days, and C in 4 days, in what time can they do it, working together?

2. What is the interest of $400, for 2 yr. 6 mo., at 4%?

3. At what rate will $300 gain $24 in 2 yr.?

4. In what time will $150 double itself, at 5%?

5. What sum of money will gain $30, in 2 yr. 6 mo., at 6%?

6. If a staff 12 ft. long casts a shadow of 3 ft., what is the length of a pole that casts a shadow of 27 ft. at the same time?

7. If 20 men can perform a piece of work in 8 days, how many men will it take to do the same work in 5 days?

8. What number is that which, being increased by its half and its third, will equal 22?

9. What will be the cost of fencing a lot 20 ft. long, and 10 ft. wide, at 10¢ a foot?

10. How many days from July 1 to Sept. 6?

11. At 1¢ a sq. ft., what will a piece of wire netting 7 ft. long and 6 ft. wide cost?

12. What will it cost to insure a house worth $5,000, at ½% premium?

13. How many eggs, at the rate of 2 for 5 cents, can be bought for 60¢?

14. 75 is what per cent of 300?

15. What is 33⅓% of 600?

968. Worcester Public Schools. Examination Questions. Written.

1. What is the interest of $458.64 from June 6, 1886, to Feb. 16, 1891, at 5%?

2. In what time will $750 gain $195 interest, at 4%?

3. What is the rate per cent when the amount of $500 is $593.75, for 2 yr. 6 mo.?

4. What principal will gain $360 in 5 yr. 4 mo., at 9%?

5. What is the duty, at 6¢ a gallon, on 250 hogsheads of molasses, 60 gallons in a hogshead, leakage 2%? (Deduct leakage.)

6. A house valued at $24,000 was insured for two-thirds of its value, at ⅘%. What is the insurance, including $1.00 for the policy?

7. An agent collected 20% of an account of $750, charging 4% commission. What was his commission, and what sum should he have paid over?

8. Bought 480 barrels of flour, at $4.50 a barrel, and sold it for $2,880. Find the gain per cent.

9. By selling a house for $10,304, a man gained 15% on the cost. What was the cost?

10. What sum of money put on interest June 1, 1885, will amount to $522 Dec. 1, 1892, at 6%?

11. The tax assessed in a town is \$4,500. The property is valued at \$1,750,000. There are 500 polls, each taxed at \$2. Find the rate of taxation, and a man's tax whose property is valued at \$24,000, and who pays one poll tax. (\$3,500 is raised on property, \$1,000 being raised by poll tax.)

12. Find the difference between the greatest common divisor of 480 and 520, and the least common multiple of 5, 6, 15, and 20.

13. Find the value of a pile of wood 40 ft. long, 8 ft. wide, and 4 ft. 6 in. high, at \$5.50 a cord.

14. A cargo of flour was bought for \$690. For what must it be sold to gain $66\frac{2}{3}\%$?

15. Reduce to a simple fraction: $(\frac{2}{7} \div \frac{4}{5}) \times \left(\frac{\frac{5}{6}}{\frac{7}{8}} - \frac{2}{3}\right)$.

REVIEW FRACTIONS.

969. Slate Exercises.

NOTE. — Do not use too many figures.

1. Add $\frac{4}{5}$, $2\frac{1}{4}$, $\frac{3}{7}$, $\frac{5}{8}$.

2. Divide each of the following fractions by 6:

$$\frac{5}{7}, \frac{12}{13}, \frac{25}{46}, \frac{84}{108}.$$

3. Reduce $\frac{7}{8}$ of $\frac{6}{11}$ of $\frac{5}{12}$ of $2\frac{2}{7}$ to a simple fraction.

4. $38\frac{5}{9} - 21\frac{11}{12}$.

5. What fraction of £1 18*s.* 9*d.* is 5*s.* 6*d.*?

6. Multiply $24\frac{1}{3}$ by $\frac{4}{5}$ of $\frac{3}{7}$.

7. What is the greatest common divisor of 657 and 1,168? The least common multiple of 12, 16, 20, 30?

8. What must be taken from $8\frac{5}{16}$ to leave $3\frac{7}{12}$?

9. Reduce $\frac{432}{528}$ and $\frac{408}{663}$ to their lowest terms.

10. Which is the greatest and which is the least, of $\frac{4}{5}$, $\frac{2}{3}$ of $\frac{5}{8}$, and $2\frac{1}{3}$ of $\frac{2}{11}$?

11. What must be added to $3\frac{4}{11}$ to make $5\frac{4}{9}$?

12. Add $\frac{3}{8}$ of a week, $\frac{5}{6}$ of an hour, $\frac{7}{12}$ of a minute.

13. How much is 9 times each of the following fractions?

$$\frac{5}{8},\ \frac{6}{11},\ \frac{7}{27},\ \frac{19}{68}.$$

14. $30\frac{2}{7} \div \frac{3}{5}$ of 7.

15. $\frac{7}{12} + \frac{5}{6}$ of $\frac{1}{10} + \frac{5}{8}$ of $\frac{2}{3}$.

16. What part of a ten-acre field is 4 A. 100 sq. rods?

17. What is the least number that will contain each of the numbers 6, 15, 18, and 20?

18. What must be multiplied by $4\frac{1}{3}$ to produce $16\frac{1}{4}$?

19. What is the value of $\frac{\frac{1}{2} + \frac{3}{7}}{4\frac{1}{4}}$?

20. What quantity must be divided by $4\frac{2}{7}$ to produce $8\frac{5}{9}$?

21. Find the value of $\frac{2\frac{1}{4} \div \frac{3}{5}}{4\frac{1}{12} - \frac{5}{6}}$.

22. How much is $\frac{\frac{3}{4} - \frac{2}{8}}{2\frac{1}{6}}$ of 3 da. 15 hr. 32 min.?

23. Reduce $\frac{4}{15}$ mile to rods.

24. Add $\frac{2}{3}$, $\frac{4}{9}$, $5\frac{1}{6}$. Subtract $4\frac{5}{12}$ from the sum.

25. Multiply $\frac{3}{4}$ of $5\frac{1}{8}$ by $7\frac{1}{5}$. Divide the result by $1\frac{4}{5}$.

MEASUREMENTS.

970. Slate Problems.

1. A tank 18 ft. long, 15 ft. wide, requires 63 sq. yd. of lead to line its sides and bottom. How many feet deep is it?

(Make diagram.)

2. A farmer has a 3-acre field in the form of a right-angled triangle. If one perpendicular side measures 242 yards, what is the length of the other?

3. One parallel side of a field in the shape of a trapezoid measures 150 yd., the other measures 200 yd. How many square yards in the field, the perpendicular distance between the sides being 50 yards? (Make diagram.)

4. The shorter parallel side of a trapezoid is x yards, the other is 100 yd., the perpendicular is 60 yd. Find the area in square yards.

How long is the shorter parallel side when the area is 5,400 sq. yd.?

5. One parallel side of a trapezoid is 80 yd., the other is 120 yd., the perpendicular is x yd. Find the area in square yards.

How long is the perpendicular when the area is 4,000 sq. yd.?

6. One parallel side of a trapezoid is x yd., the other is $x + 40$ yd., the perpendicular is 60 yd. Find the area.

Find the length of the parallel sides when the area is 6,000 sq. yd.

7. The sum of the parallel sides of a trapezoid is 200 yards, the perpendicular is 100 yards. How many square yards in the area?

8. How many flagstones $5\frac{1}{2}$ ft. long, 3 ft. wide, will be needed to lay a sidewalk 1 mile long, 6 ft. wide?

9. What would it cost, at 10¢ a square yard, to paint the walls of a room 16 ft. 6 in. long, 14 ft. 9 in. wide, 13 ft. 4 in. high?

10. A room 20 ft. long and 17 ft. 6 in. wide will require how many yards of carpet 2 ft. 6 in. wide to cover it, making no allowance for waste?

11. Find the weight of a plank 15 ft. 9 in. long, 10 in. wide, and 2 in. thick, at $41\frac{1}{4}$ lb. per cu. ft.

12. Find the entire surface of a block of marble $3\frac{3}{4}$ ft. long, $2\frac{1}{2}$ ft. wide, and $1\frac{1}{3}$ ft. thick. (Draw "development.")

13. The area of a floor is $135\frac{1}{3}$ sq. ft., and its length is 12 ft. 8 in. What is its width?

14. How many bushels will a bin contain, its dimensions being 10 ft. 8 in. × 12 ft. 8 in. × 8 ft. 9 in.? (Cancel.)

15. Find the capacity in gallons of a tank 6 ft. 5 in., by 3 ft. 9 in., by 4 ft. 6 in.

16. A farmer wishes to construct a post and rail fence around a square field containing 40 acres. He pays 15¢ each for the posts, which are placed $\frac{1}{2}$ rod apart. The rails cost 10¢ each. If the fence is 5 rails high, how much will the material cost?

17. A farm 1 mile square is divided into square fields each containing 40 acres. Make a diagram of the farm, and say how many miles of fence will be needed to enclose all the fields.

(640 A. = 1 sq. mi.)

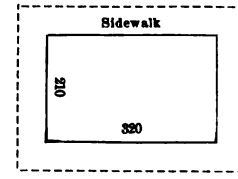
18. How many cords of wood in a pile 164 ft. long, 16 ft. wide, 30 ft. high?

19. A cubic foot of water weighs 1,000 oz. What will be the weight of a gallon of water? (Give answer in pounds and fraction.)

20. Find the weight of a quart of mercury, considering mercury 13 times as heavy as water.

21. A plot of ground 320 ft. long, 210 ft. wide, is enclosed by a tight board fence 6 ft. high. How many square yards in the surface of the fence?

22. Find the number of square yards in a sidewalk, six feet wide, on the outside of the above plot.



The inner rectangle measures 320 × 210 ft. What are the dimensions of the outer rectangle? Find the difference between the area of the outer rectangle and that of the inner one.

23. Find the area of a gravel walk, 6 ft. wide, just inside a fence surrounding a plot 320 ft. long, 210 ft. wide.

Make a diagram. Find difference of areas, as above.

CAUSE AND EFFECT.

971. Oral Problems.

1. If 4 books cost \$1.25, what will a dozen cost?

2. If 3 lb. sugar cost $13\frac{1}{2}$¢, what will be the cost of 50 lb.?

3. If 48 lb. tea cost \$20, what will 12 lb. cost?

4. Bought 17 yards of cloth for \$30. How many yards could I have bought for \$90?

5. If 36 men do a piece of work in 105 days, how long will it take 72 men to do it?

6. If 7 railway trucks weigh 14 tons, how much would 29 trucks weigh?

7. How long will it take 8 horses to plow a field, if 3 horses can do it in 8 days?

8. What is the height of a steeple that casts a shadow of 300 ft., if an 8 ft. pole casts a shadow of 12 ft.?

9. If 18 men mow 90 acres of grass in 5 days, how many acres will 36 men mow in 5 days? In 10 days?

10. If 60 yd. carpet $\frac{3}{4}$ yd. wide will cover a floor, how many yards $\frac{5}{8}$ yd. wide will be required?

972. Slate Problems.

1. A piece of cloth, measured with a yard measure that is 1 inch too short, appears to be 25 yd. long. What is its true length?

2. Exchanged 40 yd. muslin, worth $10\frac{1}{2}$¢ per yd., for 15 yd. linen. What is the value of the linen per yd.?

3. If 3 men or 6 women can do a piece of work in 56 days, in what time will 1 man and 2 women working together do it?

4. If 5 men can do as much in a day as 8 boys, how long will it take 32 boys to finish a piece of work which 15 men can do in 12 days?

5. If \$100 gain \$4 in 1 year, what will \$350 gain in $3\frac{1}{2}$ years?

6. If 48 horses in 10 days consume 180 bu. oats, how many bushels will 32 horses consume in 10 days? In 12 days? In 15 days?

7. If 5 men mow 45 acres of grass in 6 days, in how many days will 12 men mow 90 acres?

973. If 5 men mow 45 acres in 6 days,

1 man will mow 45 acres in 6×5 days.

1 man will mow 1 acre in $\frac{6 \times 5}{45}$ days.

12 men will mow 1 acre in $\frac{6 \times 5}{45 \times 12}$ days.

12 men will mow 90 acres in $\frac{6 \times 5 \times 90}{45 \times 12}$ days.

Canceling, $\frac{\overset{2}{\cancel{6}} \times 5 \times \overset{2}{\cancel{90}}}{\underset{3}{\cancel{45}} \times \underset{2}{\cancel{12}}} = 5$. *Ans.* 5 days.

974. In practice, the work is somewhat shortened. Since the number of days is required, we write the given number of days last, with a line underneath.

$$\left.\begin{array}{l} 5 \text{ men mow } 45 \text{ acres} \\ 1 \text{ man mows } 1 \text{ acre} \\ 12 \text{ men mow } 90 \text{ acres} \end{array}\right\} \overset{\text{days}}{\frac{6 \times 5 \times 90}{45 \times 12}}.$$

If 5 men do the work in a certain time, 1 man will require 5 times as many days. We place 5 in the numerator (as a multiplier). To cut 1 acre, he will take $\frac{1}{45}$ of the time required to cut 45 acres. Place 45 in the denominator (as a divisor).

12 men will take $\frac{1}{12}$ of the time 1 man requires. Place 12 in the denominator. To cut 90 acres will require 90 times as long. Place 90 in the numerator.

8. If 12 horses eat 60 bushels of oats in 6 days, how many bushels will 24 horses eat in 3 days?

Make bushels the last term.

12 horses in 6 days eat
1 horse in 1 day eats } bu.
24 horses in 3 days eat } 60

975. This example can be solved more easily. 6 days' food for 12 horses will supply how many horses for 1 day? 3 days' food for 24 horses will supply how many horses for 1 day?

9. If 24 men use 240 lb. of beef in 2 weeks, how many pounds will 18 men use in 8 weeks?

24 men in 2 weeks use 240 lb.

10. If 6 printers can print 1,656 sheets in 9 days, how many sheets will 15 printers print in 10 days?

11. How much will it cost to feed 520 sheep for 36 days, if it costs $128 to feed 160 sheep 48 days?

12. In what time will 8 masons build a wall 84 ft. long, working 10 hours a day, if 12 masons build a wall 96 ft. long in 8 days, working 8 hours a day?

13. How much money must I lend for 1 year and 3 months, when the rate of interest is 5 per cent, in return for $60 lent me for 9 months, which I borrowed at 4 per cent?

14. If 27 men build 54 rods of wall in 6 days, how many rods will 32 men build in 9 days?

15. If 50 men can do a piece of work in 90 days, working 8 hours a day, in how many days will 72 men do it, working 10 hours a day?

16. If $350 earns $42 interest in 3 years, how much will $225 earn in 5 years?

17. If a wall 34 feet high could be built by 68 men in 15 days, how many men could build a wall 32 feet high in 8 days?

18. If a ship's crew of 500 men have provisions to serve for 48 days, at the rate of 27 ounces a day for each man, how many men will the same provisions serve for 60 days, allowing each man 30 ounces a day?

19. How many hours a day must 9 men work so that they may do as much in 16 days as 12 men can do in 15 days of 8 hours each?

20. If 30¢ is paid for 6 lb. 14 oz. of bread, when wheat is $1.14 per bu., what should be paid for 23 lb. 12 oz., when wheat is $1.32 per bu.?

NOTE. — Reduce weights to ounces, or to pounds and fractions.

976. Admission to N. Y. State Normal Schools.

1. Express in words:

(*a*) 3,050,050,813; (*b*) 50,002.004; (*c*) $10,103,042.5075; (*d*) 103.75 per cent; MDCCCLXXXVIII.

2. Find the sum of all the prime numbers to 50.

3. If A and B can mow a field in seven days; and A, B, and C mow it in five days, for $25, what ought C to receive?

4. With 5 bushels of wheat, at 80¢ per bushel, and 5 bushels of rye, at 60¢ per bushel, how many bushels of oats, at 30¢ per bushel, must be added to make a mixture worth 50¢ per bushel?

x = number. Mixture contains $(x + 10)$ bu. @ 50¢.

5. To $\frac{5}{6}$ of a score add $\frac{4}{5}$ of a dozen, and from the sum subtract $\frac{3}{8}$ of a hundred. What is the remainder?

6. What must be the length of a load of wood that is four feet wide, and five and one-third feet high, to contain two cords?

x = length.

7. Bought a hogshead of molasses containing 128 gallons, at 65¢ a gallon; paid 80¢ for cartage, and lost 16 gallons by leakage. At what price per gallon must the remainder be sold to gain one-fifth of the entire cost?

8. What is the least number that will exactly contain 48, 20, 21, 24?

9. Find the least common multiple of 24, 44, 65, 100.

10. Sold 50 sofas for $2,250. 25 of them were sold at a gain of 20 per cent, and 25 at a loss of 20 per cent. What was the gain or loss on the transaction?

11. Bought a number of eggs, and sold 11 of them for what 18 cost me. What was my gain per cent?

12. A bookseller wishes to mark up the price of a book which he is now selling for $2, so that he can deduct 15 per cent, and yet receive the present price. What must be the marked price?

13. What is the difference between .75 divided by 75, and 75 divided by .75?

14. A pole extended into the mud $5\frac{5}{8}$ feet; $\frac{1}{8}$ of its length was in the river, and $\frac{1}{4}$ of it in the air. What was the length of the pole?

x = length.

15. Find the exact amount of $68, from March 13 to June 15, 1888, interest 6 per cent.

Take 366 days to the year.

16. How many acres in a field 210 ch. long and 50 ch. wide?

1 acre = 10 sq. chains.

17. It took 9 horses to move a stick of timber weighing 12,590 lb. How many pounds would a stick weigh that would take 7 horses to move?

18. Mr. A sold a horse for $275, which was 5 per cent less than he asked for it, and his asking price was 5 per cent more than the horse cost him. What was the cost of the horse?

19. Write in Roman numerals 1889.

20. A man purchased a rectangular field 18 rods wide, containing 10 acres 146 square rods of land. The land cost him $80 per acre, and he surrounded it by a fence costing 80¢ per rod. How much did the land cost, and how much the fence?

21. By selling a carriage for $117, a dealer lost 10 per cent of its cost. For how much should he have sold it to gain 10 per cent of its cost?

22. A man, dying, left $\frac{2}{3}$ of his estate to his wife, $\frac{2}{5}$ of the remainder to his son, and the remainder to his daughter, who received $5,000. What was the value of the estate, and what was the son's share?

x = value of estate.

23. What is the interest of $10, for 10 years 10 months and 10 days, at 10 per cent?

24. If it takes one man $7\frac{1}{4}$ days to do a piece of work, how long will it take 3 men to do $2\frac{2}{3}$ times as much?

PARTNERSHIP.

977. Slate Problems.

1. B and C gain by trade $182. What is the gain of each, B having put in $300, and C $400?

The gain of $700 is $182. What should $300 gain? What should $400 gain?

2. A, B, and C invest $720, $340, and $960, respectively. The profits are $101. What is each one's share?

How many dollars of capital produce $101 profits?

3. Two men hire a pasture for $45. One puts in 15 cows, the other puts in 12 cows. What should each pay?

4. A and B hire a boat for 50 days, paying $30. A uses it 27 days, B uses it 23 days. How much should each pay?

5. Our standard gold coin consists of 900 parts gold, 90 parts silver, 10 parts copper. What is the quantity of each metal in 50 pounds of coin?

6. Gunpowder is composed of 15 parts of saltpeter, 2 of sulphur, and 3 of charcoal, mixed together. How many pounds of each are there in 72 pounds of powder?

7. Three farmers hired a threshing-machine for $54. A used it to thresh his crop of 900 bu., B to thresh his crop of 828 bu.; C, 672 bu. How much should each pay?

8. A, B, and C rented a warehouse. A stored in it 2,400 bales cotton; B, 1,500; C, 1,100. A fire destroyed 1,800 bales. How much of the loss should each sustain?

9. X and Y rent a field for $32. X puts in 8 horses for 6 months, and Y 10 horses for 8 months. How many dollars should each pay?

8 horses for 6 months = how many for one month?

10 horses for 8 months = how many for one month?

10. M and N entered into partnership. M puts $200 into the business for 5 months, and N, $300 for 4 months. They gained $132. Find the share of each.

REVIEW DECIMALS.

978. Slate Exercises.

1. Express as decimals $\frac{57}{100}$, $\frac{7}{1000}$, and $\frac{27}{820}$.

2. .395 + 86.7 + 209.0043 + .81 + 3.075 + 27.

3. Divide 34,020.072 by 5.309.

570 ÷ .005 = ?

4. Multiply 80.037 by 10.

Seventy-three one hundred-thousandths by one hundred.

.2054 × 1,000 = ?

5. Subtract 48.8067 from 53.07.

.0539 × 26.08 = ?

6. The smaller of two numbers is 8.5307, and their sum is 25.07. Find the larger number.

7. Express .39, 6.175, .00036, and 74.0005 as common fractions (or mixed numbers).

8. Divide .826 by 100; 543.71 by 10,000; and fifty-nine ten-thousandths by one thousand.

9. Find the difference between 9.84 and 38.005, and the continued product of 83.09, .734, and 5.007.

10. Reduce 6 shillings 9 pence to the decimal of a pound sterling.

11. Express as decimals seven hundredths, forty-three ten-thousandths, and ninety-one millionths.

12. Change $\frac{1}{16}$, $8\frac{1}{80}$, $1\frac{1}{125}$, and $\frac{1}{256}$, into decimals. Find their sum.

13. Express .42796 as a common fraction, and the sum of $\frac{7}{10}$, $\frac{3}{100}$, and $\frac{878}{10000}$ as a decimal.

14. $3.009 \times .07 \times .0907$.

15. Divide .0075 by .15, and .00044408 by .0112.

16. Divisor, 403.6; quotient, 2.709. Dividend?

17. What is the value of $\frac{.035 \times .0056}{.00007}$?

18. Change 69 rods to the decimal of a mile.

19. Change .4285 month (30 days) to days, hours, etc.

20. How many meters, each 39.37 inches, in 3 miles 220 rods?

21. Change .1875 bu. to quarts.

22. What decimal of a pound is 13 oz.?

23. Reduce 4 ft. $1\frac{1}{2}$ in. to the decimal of a rod.

24. How many links of 7.92 in. each in a 4-rod chain?

25. A chain is 66 ft. What decimal of an acre is 1 sq. chain?

979. An English Invoice.

Invoice of Brass Wire Cloth, per steamship Furnessia, for entry at New York, shipped to Arthur Jackson by Robt. MacDonald & Bro., of Leith, the 21st day of April, 1893.

6 cases marked A. J. 1–6.

2 pieces, each 33 ft. × 6 ft. 4 in.	418 @ 1/2	£24 7 8
2 " " 41 " × 5 ft. 6 in.	451	
4 " " 42 " × 7 ft. 2 in.		
3 " " 43 " × 7 ft. 8 in.		
	@ 10	
		£
	2½% discount	
		£

Find cost in English money.

NOTE. — 1/2 = 1 shilling 2 pence, the price per square foot. The remaining pieces cost 10 pence per square foot.

980. A German Bill.

DRESDEN, Sept. 4, 1892.

MR. C. G. SCHMIDT,

Bought of MEYER & SON.

10	doz.	Scarfs	M. 6.75	
5	"	"	9.	
5¾	"	"	11.70	
9	"	"	9.85	
3	"	"	7.60	
3	"	"	11.60	
2	"	"	8.50	
2¼	"	"	11.50	

Find cost in marks. What is the duty in U. S. money at 35% on the value?

1 mark = 23.8 ¢.

APPROXIMATIONS.

981. Give approximate answers at sight (Art. 890):

1. $487\frac{3}{4}$ is what per cent of 960?

2. If 17 bu. 37 lb. of corn cost \$8.75, what will 52 bu. cost?

3. About how many cords of wood in a pile 25 ft. long, 4 ft. wide, 5 ft. high?

4. How many bushels ($1\frac{1}{4}$ cu. ft.) can be placed in a bin 6 ft. long, 5 ft. wide, 4 ft. high?

5. How many acres in a field 50 rods long, 30 rods wide?

6. About how many yards are there in the side of a square field containing 1 acre (4,840 sq. yd.)?

7. At $7\frac{1}{2}$ gal. to cu. ft., about how many gallons will a tank hold 6 ft. long, 4 ft. wide, 3 ft. high?

8. $64.3 \div .0987 = ?$

9. About how many dollars are equal to £199 17*s.* 6*d.*?

10. A mark = 23.8¢. How many marks in \$100?

BONDS AND STOCKS.

982. Slate Problems.

1. The people of a certain town wish to build a street railroad. A company is formed. Five hundred shares of stock, of the par value of \$100 each, are sold.

At the end of 6 months it is found that the profits are \$2,000. How much should the owner of 10 shares receive?

2. Profits thus distributed are termed *dividends*. What % semi-annual dividend is declared on the stock of the above railroad? To what per cent interest per year is it equal?

3. Mr. H. has \$4,500 in the savings bank, on which he receives a low rate of interest. Hearing of the success of the

new road, he gives that amount for 30 shares of the stock. What price does he pay per share? What per cent of the par value?

4. If the semi-annual dividend is again 4%, how much more income does Mr. H. receive from the railroad stock than he would obtain from the savings bank in six months, interest 4 per cent per annum?

5. What per cent, for six months, does the stock pay on his investment of $4,500? What % per year?

6. If he sells the stock (30 shares) at 164½ (per cent), how much more does he receive for it than it cost him?

7. Which investment will pay better, one in a gas company paying 6 per cent dividends annually, their stock selling at 150, the other in a bank paying 7 per cent dividends annually, stock selling at 175?

8. What annual dividend should be declared on railroad stock bought at 125, so that the buyer will receive 4% per annum on his investment? What semi-annual dividend?

9. What will be the cost of 17 shares of canal stock, par value $50, at 93⅞, and 143 shares gas stock, par value $10, at 102¾?

10. If the above stock is purchased through a broker, what commission does the latter receive at ⅛% on the par value?

11. A railroad company needing more money to extend its road, issues bonds, promising to pay the holder the face value in twenty years, with interest at 4%.

If these bonds are sold at 95, what rate of interest on the money invested does the owner of a bond receive?

12. Government 4 per cent bonds sell for 116½. What per cent interest is received on the amount invested?

How is it that these bonds bring higher prices than railroad bonds?

13. Can you state a difference between stocks and bonds as to the rate of income received from each?

Bonds are redeemed at maturity. How about stocks?

If a railroad prove unsuccessful, which claims are first met, those of the stockholders or those of the bondholders?

14. Why is it necessary sometimes to employ a broker to purchase stocks or bonds? What is his fee called?

15. Mention some other persons, not owners, through whom buyers regularly make purchases.

16. What is the base in the following?

(*a*) Insurance; (*b*) taxes; (*c*) brokerage; (*d*) commission; (*e*) interest; (*f*) discount; (*g*) stocks; (*h*) bonds.

17. At $24.50 per thousand, what will have to be paid in taxes by the owner of property assessed at $18,750?

18. Mr. Cartwright owns a house and lot worth $36,000. The tax rate is $2\frac{1}{4}\%$, and his tax bill is $540. What is the assessed value of the property? What per cent of the actual value is the assessed value?

19. If the property in the last problem were assessed at its real value, what should be the rate to make Mr. Cartwright's tax bill the same?

20. For insuring his property, Mr. Cartwright pays a yearly premium of $135. If the rate is $\frac{3}{4}\%$, for how much is his property insured?

21. Reduce 1,674 feet to rods, etc.

22. A man paid $8,575 for bank stock at 245. How many shares, par value $100, did he buy? If a quarterly dividend of $2\frac{1}{2}\%$ is declared, how much should he receive?

23. Reduce 7,481 inches to rods, etc.

24. A woman deposited $100 in a savings bank Jan. 1, 1892. On the first of July, interest at the rate of 4% per annum was

calculated, and entered on the depositor's bank book. Jan. 1, 1893, interest on the new principal was placed to the credit of the depositor. The same was done July 1, 1893. How much was there to the woman's credit at the date last mentioned?

25. Reduce 3,[illegible]8 feet to rods, etc.

COMPOUND INTEREST.

983. Find the amount of $375, for 1 year, at 6%. Considering this as a new principal, find the amount for a year, same rate. Find the amount of this last principal for 3 months.

26. What is the amount of $375, for 2 years 3 months, at 6%, compound interest?

27. What is the amount of $375, for 2 years 3 months, at 6%, the interest compounded semi-annually?

Principal, $375.

3%	11.25	6 months' interest.
	386.25	Amount 6 months.
3%	11.5875	6 months' interest.
		Amount 1 year.
3%	_____	6 months' interest.
		Amount 1½ years.
	etc., etc., etc.	

Find the "compound interest" on $375, for 2 years 3 months, at 6 per cent, compounded semi-annually.

28. What is the amount of $100, at compound interest, for 3 years, interest at 6%, compounded annually?

29. Find the compound interest of $1,800, at 4%, for 2 years, interest compounded quarterly.

	$1,800.00
1%	18.00
	1,818.00
1%	18.18
	etc., etc., etc.

30. Find the difference between the simple interest of $100, for 2 yr. 3 mo., at 5%, and the compound interest for the same time, interest compounded semi-annually.

	$100.00
$2\frac{1}{2}\% = \frac{1}{40}$	2.50
	102.50
$2\frac{1}{2}\%$	2.5625
	105.0625
$2\frac{1}{2}\%$	2.6266
	$107.6891

Divide by 4, and put first quotient figure one place to the right.

(four places of decimals are sufficient.)

984. Compound interest is allowed by savings banks. It is not collectible on notes or other debts.

REVIEW.

985. Oral Problems.

1. A capitalist wishes to realize 5% on money invested in stock. What must be the annual dividend on stock costing 300, in order to produce this rate?

2. What will be the taxes on property assessed at $25,000, the rate being $16 per $1,000?

3. Find the compound interest on $1,000, for two years, at five per cent, interest compounded annually.

4. What will be the net cost of an article marked $8, on which a discount of 50, 25, and 10% is allowed?

5. Find the "list" price of an article sold for $10 after a discount of 50 and 50 per cent had been deducted.

6. Paid 90¢ for an article. The discount is 25 and 25 per cent. What is the list price?

7. One boy can do a certain piece of work in 2 hours, a second boy requires 3 hours, a third needs 6 hours. How long will it take the three working together?

8. Sold a cow for $60, losing 25%. What was the loss?

9. Sold a cow for $60, gaining 25%. What was the gain?

10. Sold two horses at $240 apiece. On one I gained 20%, on the other I lost 20%. Did I gain or lose on both, and how much?

SUGGESTION. — $240 in the first case represents 120% of cost of horse. The gain is 20%, which is $\frac{1}{6}$ of selling price, or $40.
The loss in the other case is 20%, which is what part of the selling price? Do not find the cost.

986. U. S. Civil Service. Meteorologic Clerk Examination. War Department. Signal Office.

1. Express the following in figures, using such signs, monetary or otherwise, as may be necessary:

(*a*) Five million three thousand seven, and four thousand one ten-millionths; (*b*) three million two hundred nine dollars thirty-seven cents two mills.

2. Express the following in words:

(*a*) $\overline{\text{XCIV}}$CCXLVIII.; (*b*) $14,000,027.034; (*c*) 3,010,004-.05007; (*d*) 6 T. 3 cwt. 7 lb. 5 oz. (Art. 444.)

3. Divide 52.28178 by 3.0047, and multiply the quotient by 4.09.

4. Find the value of $\frac{2\frac{1}{2} + 4\frac{5}{12}}{1\frac{3}{4} \times 3\frac{1}{3}} \div \frac{1}{8}$ of $6\frac{5}{7}$.

5. What is the length of a fence required to inclose a rectangular piece of ground 121 feet wide, containing one acre? (4,840 sq. yd. = 1 acre.)

6. A grocer sells goods to a customer for $352, by weights averaging $15\frac{1}{4}$ ounces to the pound. He afterwards sells said customer goods for $320, by weights averaging $16\frac{1}{2}$ ounces to the pound. How much does the grocer make or lose by the false weights?

7. A merchant buys 42 gallons of alcohol, at $2.50 per gallon, and keeps it for three years. He then finds that he has lost 7 gallons by leakage and evaporation. Estimating the value of

money at 6 per cent per annum, how much per gallon must he charge in order that he may realize the full amount of the cost, including the estimated interest.

8. The owner of £4,500 in English consols (3 per cents) sells them at 96, and invests the proceeds in 6 per cent $100 bonds which he buys at 108. What is the difference in dollars and cents between his income from the consols and from the bonds? (£ = $4.85.)

9. What is the weight (in tons, cwt., etc.) of the water which fills a cistern 9 feet 8 inches long, 9 feet 4 inches wide, and 6 feet 9 inches deep, a cubic foot of water weighing 1,000 ounces? (1 ton = 2,000 lb.)

10. A grocer pays 18¢ per pound for coffee, and roasts it, losing 10% of the weight in the process. What must he charge per pound for the roasted coffee in order to make a profit of 20%?

987. Boston Public Schools. Examination Questions. Mental Problems.

1. What would be the cost of 2 bu. blueberries, at 5¢ per qt.?

2. What will 18 oranges cost, at 35¢ per doz.?

3. A man collected a bill of $300 for me, at ½% commission. How much was his commission?

4. A man put $15, which was 16⅔% of his month's salary, in the bank. What was his month's salary?

5. If I sell for $4.50 a book which cost me $3, what per cent do I gain?

6. Out of 500 pupils, 50 are absent. What is the per cent of attendance?

7. A garden is 200 ft. long and 100 ft. wide. What will it cost to build a fence around it, at 50¢ a ft.?

8. What is the interest of $200, for 90 days, at 3%?

9. What cost 4 lb. 4 oz. of butter, at 16¢ a lb.?

10. How many men could you hire with $50, for a week's job, if each man received $12\frac{1}{2}$ per week?

11. How many doz. yeast cakes, at 2¢ each, could be bought for 96¢?

12. 75 is 25% more than what number?

13. Sold a watch for $44, and so lost 12% on the cost. What was the cost?

14. One acre of corn yields 80 bu., and another acre 20% more. What does the second acre yield?

15. In $1\frac{2}{3}$ yd. how many inches?

16. What will it cost to fence a garden 10 rd. long and 6 rd. wide, at $1 a rod?

17. If each boy eats $\frac{3}{8}$ of a loaf of bread, how many boys will eat 6 loaves?

18. May bought $6\frac{1}{4}$ yards of cloth, at 12¢ a yard. How many cents did the cloth cost her?

19. How many half-pint mugs can a person fill with $\frac{1}{2}$ gallon of milk?

20. If your school-room is 40 ft. long and 30 ft. wide, what is the distance around it?

21. If a cup holds $\frac{2}{3}$ of a pint, how many cups in a gallon?

22. A rectangular field containing 1 acre is 40 rods long. How many rods around the entire lot?

23. Find the sum of $\frac{3}{4}$ and $\frac{5}{6}$.

24. What per cent of a number is $\frac{6}{20}$ of it?

25. Two men, 96 miles apart, travel toward each other, one at the rate of $5\frac{1}{2}$ mi. an hour, the other $4\frac{1}{2}$. In how many hours will they meet?

26. Change $\frac{7}{8}$ and $\frac{3}{20}$ to decimals, and add them.

27. From a school of fifty pupils, six are absent. What per cent are present?

28. What will it cost to fence a piece of land that is 6 rods square, at $3 per rod?

29. What will one bushel and one peck of chestnuts cost, at 8¢ a quart?

30. A can do a piece of work in four days; B can do it in five days. In what time can A and B do it if they work together?

988. Boston Public Schools. Examination Questions. Slate.

1. A merchant bought 48 bales of cotton, and then sold the lot for $2,008.80, losing 7%. What was the cost per bale?

2. Mrs. Stetson owned three pieces of land, containing $5\frac{4}{21}$ acres, $6\frac{7}{24}$ acres, and $10\frac{15}{28}$ acres. She gave to her son $11\frac{1}{7}$ acres. How many acres had she left?

3. If 57.6 bbl. of flour cost $266.40, how much will 97.8 bbl. cost at the same rate?

4. What is the cost of sawing a pile of wood 20 ft. long, 4 ft. wide, and 6 ft. high, at $1.20 a cord?

5. After increasing the wages of his workmen $33\frac{1}{3}\%$, a manufacturer paid them $2.00 a day. What did he pay them before?

What should a bookseller charge for a book for which he paid at the rate of $54 a dozen, that he may make 20% on the cost?

What is the per cent profit or loss when a hundred logs which cost $65.00, are sold at 78¢ each?

6. What is the duty on 100 watches, worth $100 apiece, on which there is a specific duty of $10 each, and an ad valorem duty of 50%?

7. Paid $25 for an insurance policy on my house. If the rate is $\frac{3}{4}\%$, for how much is my house insured?

8. Interest of $796.28, from Jan. 7, 1880, to July 28, 1883, at $7\frac{1}{2}\%$?

Amount of $396.80, from July 25, 1883, to Jan. 5, 1885, at $7\frac{1}{2}\%$?

9. A man spent $\frac{8}{21}$, and invested in his business $\frac{4}{15}$, of his income. He deposited the remainder, $1,850, in a bank. What was his income?

Mr. Smith's annual income is $2,500. He pays $37\frac{1}{2}$ a month for rent. What per cent of his income does he pay for rent?

10. Sold a horse for $322, and thereby lost 8%. What should I have sold it for, to gain 15%?

Bought a horse for $340; paid $60 for keeping him, and then sold him for $540. What per cent was gained?

11. My agent collected 80 per cent of a debt of $4,500, and charged $7\frac{1}{2}$ per cent commission. What amount should he pay me?

John bought $12\frac{1}{2}$ lb. of sugar at $8\frac{1}{2}¢$ a pound, spending 25% of his money. How much had he at first?

12. When 10.25 bushels of wheat cost $12.71, what will $7\frac{1}{2}$ bushels cost?

13. Mr. Jones paid $15.12 for the use of a sum of money, for 1 yr. 6 mo., at 5%. What was the sum?

Find the interest on $867.15, for 3 yr. 7 mo. 17 da., at 4%.

What were the proceeds of a note for $725.14, due July 7, discounted at a bank June 20, at 8%?

14. How many more pounds of sugar can be bought for $1.00 when sugar is $6¢$ a pound, than when it has advanced 20% on that price?

15. After Mr. Jones had spent $18\frac{1}{2}\%$ of his money, he found that he then had enough to buy 80 lb. of rice at $6\frac{1}{2}¢$ a pound. How much could he have bought with the whole of his money?

16. Bought 28,500 pounds of hay at $12\frac{1}{2}$ a ton, and sold it at $0.87\frac{1}{2}$ per hundredweight. What was the gain?

17. A farmer bought 6 cows through an agent. He sent $525.30 to pay for the cows and a commission of 3%. How much did each cow cost?

18. What are the avails of a 90-days note for $600, dated May 16, 1890, and discounted June 20, 1890, at the Rockland National Bank, rate of discount being 6%?

On the 10th day of November, 1893, you lent William Rogers $864.50. How much does he owe you to-day, the rate of interest being $4\frac{1}{2}$%?

19. I sold a carriage for $300, which was six-fifths of the cost. What per cent did I make? Write out the analysis.

20. A man bought wheat for $10,867, and sold it at a gain of $4\frac{1}{2}$%. What did he receive for it?

21. What number diminished by 5% of itself equals 665?

22. A lawyer collected 65% of a note of $950, and charged $6\frac{1}{4}$% commission. What was his commission?

23. Divide three million by six thousand, and multiply the quotient by .024.

24. What will be a broker's commission, at $2\frac{7}{8}$%, for selling a farm of 673 acres @ $52 per acre?

25. How many square feet in a mat $8\frac{3}{4}$ ft. long and $7\frac{1}{3}$ ft. wide?

A field contains $199\frac{1}{2}$ sq. rd. If it is $18\frac{3}{4}$ rd. long, how wide is it?

26. How much must I have invested @ 5% that my income may be $2,880 per year?

27. A merchant sold 87 cases of shoes, 12 pr. to the case, for $2,479.50. What was that a pair?

28. Find the cost of the following: 36 rolls of paper at $33\frac{1}{3}$¢; 64 yd. matting at $37\frac{1}{2}$¢; $97\frac{3}{4}$ yd. carpeting at $1.68; 7 window shades at $1.17; 7 curtains at $16.50; 10 chairs at $4.75; and $5\frac{1}{3}$ days' work at $2.75 per day?

29. $1,150 is 15% more than what?

A cord of wood, costing $4.50, sold for $9.00. What was the gain per cent?

30. A man paid for a house $4,500, and for repairs $150, and then sold it for 18% above the entire cost. What did he receive for it?

989. Massachusetts Civil Service. Clerks. Limited Examination.

1. Write in figures the following number:

Fifty-seven million six thousand thirty-one.

Write in words the number expressed by the following figures:

9,805,010.68

2. Add these *across*, placing the totals in the space indicated; then add the totals:

						Totals.
14,305	10,702	18,346	37,946	43,865	17,387	
22,324	17,437	18,438	3,741	22,972	25,960	
13,849	67,431	34,965	12,674	32,905	1,468	
15,607	27,865	32,476	18,430	33,301	18,695	
19,898	13,460	27,686	23,492	13,852	26,973	

3. What is the sum of $\frac{13}{7}$, $\frac{2}{3}$, and $\frac{27}{21}$, using the lowest common denominator? Give the answer in mixed numbers.

Divide $\frac{64}{9}$ by $\frac{8}{8}$. Give the answer in mixed numbers.

4. Subtract $\frac{11}{16}$ from $\frac{53}{64}$, and change the result to a decimal.

To eighty-one and three hundredths add seven and fifty-eight thousandths.

5. Multiply 5.0173 by 1.16.

Divide twenty-five by twenty-five thousandths.

6. How many yards of paper will it require to cover the walls and ceiling of a room 20 feet long, 15 feet wide, 10 feet high, the width of the paper being ten inches?

7. If $\frac{7}{8}$ of a pound of butter costs $\frac{3}{5}$ of a dollar, what will $5\frac{1}{4}$ lb. cost?

8. If the rent of a house worth \$4,000 is $\$33\frac{1}{3}$ a month, what is the value, at the same rate, of a house that rents for \$87.50 a month?

9. If 1,998, or 27 per cent of the inhabitants of a town, are voters, how many inhabitants has the town?

10. If the tax rate is \$13.80 on \$1,000, what is the assessed value of property that pays a tax of \$144.90?

990. Philadelphia Public Schools. Oral Work.

1. $[(96 \div 12) \times 9] \div 12 =$

2. $\frac{3}{4}$ of 48 is 4 times what number?

3. A horse was sold for \$80, which was $\frac{1}{8}$ of the cost. How much was lost on the horse?

4. What part of 5 gal. is 3 qt.?

5. 24 is 60 per cent of what number?

6. If a bushel of English walnuts costs \$1.60, what will 6 quarts cost?

7. A man had \$600 in the bank. He drew out $16\frac{2}{3}$ per cent of it. How many dollars remained in the bank?

8. A man put 5 gal. 2 qt. of syrup into bottles holding two quarts each. How many bottles did it require?

9. (*a*) What is the interest of \$300, for 30 days, at 6 per cent?

(*b*) A lost 40 per cent of his money, and had \$750 left. How much had he at first?

991. Written Work.

1. (*a*) Add 61.027; 64.3006; 7.986071; 832.026; 610.016.
   (*b*) Multiply .71298 by .0398.

2. Divide five thousand fifty and five tenths by ten thousand one hundred one hundred-thousandths. From the quotient take twenty-five thousand one hundred-thousandths.

3. (*a*) Find the greatest common divisor of 949 and 1,387.
   (*b*) Find the least common multiple of 28, 32, 56, 72, 96.

4. Ten cows were sold for $690, at a gain of 15 per cent. For how much per head on the average should they have been sold to gain 20 per cent?

5. Find the interest of $575.50, for 1 year, 10 months, 15 days, at 5%.

6. A, B, and C formed a partnership. A put in $400; B, $350; C, $600. In one year they gained $540. What was each one's share of the gain?

7. A man invested $9,562.50 in the stock of a city bank at 127½, the par value being $100 per share. If a dividend of 3½ per cent is declared, what amount of dividend would he get?

EXCHANGE.

992. If I wish to pay a bill in a distant city, ought I to enclose the money in a letter? Why?

Can money be sent by express? Can the telegraph be used in paying money at a distance?

What is a money-order?

Can I buy from the postal authorities a money-order payable in Europe?

What will be the cost of a money-order for $85, payable in San Francisco?

What is the largest money-order that can be purchased?

What is a check? Can you tell why a draft rather than a check is used in paying a bill at a distance?

Pupils should be encouraged to look up answers to the foregoing.

Bills of exchange are either foreign or domestic. A domestic bill of exchange is called a *draft*, the term *bill of exchange* being generally applied only to foreign bills.

DOMESTIC EXCHANGE.

993. Slate Problems.

William F. Smith, of Memphis, Tenn., owes John M. Thomson, of New York, $3,475.86. He purchases from a Memphis banker, Joseph E. Washington, a sight draft for the above amount on the Chemical Bank of New York. The following is the form of the draft:

$3,475$\frac{86}{100}$. MEMPHIS, TENN., Aug. 9, 1893.

At sight, pay to the order of John M. Thomson Three Thousand Four Hundred Seventy-five and $\frac{86}{100}$ Dollars, value received, and charge to the account of

To CHEMICAL BANK, New York. JOSEPH E. WASHINGTON.

1. What must William F. Smith pay for the above draft, the rate being $1.50 premium per $1,000?

(A draft for $1,000 costs $1,001.50.)

2. Find the cost of a Boston draft on New York for $1,875, at 12¢ discount per $1,000.

(A draft for $1,000 costs $999.88.)

3. What will a St. Louis merchant have to pay for a draft on New York for $2,460.53, at 50¢ premium per $1,000?

4. If the rate of exchange is 50¢ discount per $1,000, what is the face of the sight draft on Boston, that can be bought in New York for $1,000?

5. When the premium is $1.25 per $1,000, Mr. Brown pays $1,634.04 for a draft on Louisville. What is the face of the draft?

6. At $\frac{1}{8}\%$ premium, find the cost of a sight draft for $\$1,843.60$.

$$\frac{1}{8}\% = \begin{array}{r} \$1,843.60 \\ 2.30 \\ \hline \$ \end{array} \text{ Add.}$$

7. At 75¢ discount per $\$1,000$, how much will cost a sight draft on Milwaukee for $\$946.75$?

$$\text{less} \begin{cases} \text{50 per M.} \\ \text{25 per M.} \end{cases} \begin{array}{r} \$946.75 \\ .473 \\ .237 \end{array}$$

8. Paid $\$632.18$ for a sight draft on Milwaukee. What was the face of the draft, the discount being $\frac{3}{40}\%$?

9. I sent a commission merchant $\$1,000$ to buy grain. How much will he spend for grain, if his commission at $1\frac{1}{2}\%$ is included in the amount sent?

(Let x = amount spent for grain. ? = commission.)

10. A farmer ships produce to a commission merchant, which the latter sells for $\$339.66$, charging 2 per cent commission. For the remainder of the money he buys groceries and dry-goods, charging 2 per cent commission on the amount spent. What is the cost of the goods purchased?

REVIEW.

994. Slate Problems.

1. A joiner worked on Monday 9 hr. 45 min., on Tuesday and Wednesday 10 hr. 45 min. each day, on Thursday and Friday 10 hr. 15 min. each day, and on Saturday 6 hr. 45 min. What was the average length of his day's work?

2. A watch that loses 35 seconds in an hour was set right at noon on Monday. What time did it show at 6 P.M. the following Thursday?

3. There are 5 boys whose heights are 4 ft. 9 in., 5 ft. 1 in., 4 ft. 5 in., 3 ft. 11 in., and 4 ft. 4 in., respectively. What is their average height?

4. A man had a plot of ground 20 yards long and 12 yards wide, which he planted in cabbage. How many plants did he require, if the rows, which ran lengthwise, were 2 feet apart and 2 feet from the fence surrounding the plot, and the plants in the rows 16 inches from each other and from the fence?

Get the correct number of rows, and the correct number of plants in a row. How many plants would have been needed if the rows ran crosswise?

5. How long would it take a person to count a million silver dollars, at the rate of 100 a minute, and working 8 hours a day?

6. The front wheel of a wagon is 13 ft. 4 in. in circumference. How many revolutions will it make in a journey of 14 miles? How many more revolutions will it make than the hind wheel, the circumference of the latter being 17 ft. 6 in.?

7. The wheels of an engine being 16 ft. 8 in. in circumference, and the number of revolutions 150 per minute, how far does it go in an hour? Give answer in miles and rods.

995. Circular Measure.

60 seconds (″)	1 minute (′)
60 minutes	1 degree (°)
360 degrees	1 circle.

8. If the equatorial circumference of the earth is 25,000 miles, how many miles apart are two places on the equator, the distance between them being 20°?

9. What is the length of a degree on a circle whose diameter is 18 feet?

The circumference = diameter × 3.1416.

10. The 60th parallel of latitude is a circle about one-half as long as the equator. How far due east of Christiania is St. Petersburg, both situated on this parallel, the former being 10° east of Greenwich, and the latter 30° east?

11. How many miles north of the equator is a place in latitude 46° 22′ 30″? Take 69¼ miles to a degree.

12. Two places in latitude 45° are 22° 30′ apart, measured on that parallel. Find the distance in miles, assuming the 45th parallel to be a circle .7071 times the length of the equator, and considering the length of the latter to be 25,000 miles.

996. Time Drafts.

987\frac{65}{100}$. NEW ORLEANS, June 15, 1893.

At three days' sight, pay to the order of John D. Hallock, Nine Hundred Eighty-seven $\frac{65}{100}$ Dollars, value received, and charge to account of

FRANK PHILLIPS.

To NATIONAL PARK BANK,
New York.

When Mr. Hallock receives the above, he presents it to the National Park Bank for *acceptance*. The proper bank official writes across the face of the draft in red ink "Accepted," with the date, say "June 18, 1893," and signs his name. Three days thereafter, plus three days of grace, or June 24, the draft will be payable.

997. Sight drafts are usually not allowed days of grace. Time drafts are generally allowed three days of grace. (See Appendix.)

998. The premium on the above draft at $1.50 per $1,000 is calculated on the face of the draft, and amounts to $1.48.

999. Since it is not payable until six days after acceptance, the interest (or bank discount) for that time is deducted.

Interest on 987\frac{65}{100}$ for 6 days at 6% = $.99.
Cost of draft = $987.65 + $1.48 − $.99 = $988.14.

N.B. Take 6% as the interest rate, unless a different rate be expressed.

1000. Slate Exercises.

1. What will I have to pay for a 90-days draft on San Francisco for $840, at $1.75 premium per $1,000?

2. Face $400; 30 days' sight; discount $\frac{1}{8}$%. Cost?

3. Face $560; 60 days' sight; premium 50¢ per $1,000. Find cost.

4. What will be the cost of a sight draft for $625.38 at $7\frac{1}{2}$¢ discount per $1,000?

5. Find the cost of a 60-days draft for x dollars, premium 25¢ per $1,000.

6. Find the cost of an x-day draft for $1,200, discount $\frac{1}{8}\%$.

7. Find the cost of a 30-days draft for $1,600, premium x dollars per $1,000.

8. Paid $1,188.90 for a 60-days draft, at $\frac{1}{8}\%$ premium. What was the face of the draft?

9. A time draft for $1,800 at $1 premium per $1,000, cost $1791.90. At how many days' sight was it drawn?

10. At what rate did I purchase a 90-days draft for $900, its cost being $884.70?

LONGITUDE AND TIME.

NOTE. — This topic should be taught in connection with the study of Mathematical Geography.' The globe should be used to show the pupils that all places on the same meridian have the same time, that a difference in longitude of 15 degrees produces a difference in time of 1 hour, and that the more easterly of two places has the later time.

1001. Oral Problems.

1. The difference in time being 1 hour for each 15 degrees, find the difference in longitude between two cities differing in time $3\frac{1}{3}$ hours.

2. Two places differ in longitude 61 degrees. What is their difference in time?

3. London is 75° east of Philadelphia. When it is 1 o'clock at Philadelphia, what is the time at London?

4. When it is 2 P.M. at London, what is the time at Philadelphia?

5. When it is noon at a city 25 degrees west of Vienna, what is the time at the latter place?

6. How many degrees of longitude correspond to a time difference of 3 hours 40 minutes?

7. What is the difference in longitude between Philadelphia, 75° west longitude, and St. Petersburg, 30° east longitude?

8. When it is 3 P.M. at St. Petersburg, what is the time at Philadelphia?

9. Washington is in 77° west longitude, and uses "standard time," that is, the time of 75° west longitude. What is the difference between the correct time at Washington and its clock time?

10. A town in 84° west longitude uses standard time (of 90°). What is the correct time when the clocks are striking 12, noon?

1002. Slate Problems.

1. Find the difference in longitude between two places differing in time 3 hr. 44 min.

2. Two places differ in longitude 37° 18′. What is their difference in time?

3. Chicago is 87° 35′ west of Greenwich. What is the difference in time between the two places?

Is it earlier or later than noon at Chicago when it is noon at Greenwich? Why?

What is the standard time at Chicago when it is 1 P.M. at Greenwich?

4. When a captain's observation of the sun shows that it is exactly noon, the ship's chronometer, keeping Greenwich time, reads 30 minutes past 2 P.M. How many degrees west of Greenwich is the vessel?

5. Find the difference in time between two places in longitude 74° 31′ and 93° 14′ west of Greenwich, respectively.

6. When it is noon at a place 11° east of Greenwich, it is 1.30 P.M. at another place. Find the longitude of the latter place.

7. A train ran from New York to San Francisco, 3,313.5 miles, in 3 da. 12 hr. 17 min. How many miles per hour did it average?

8. If for $6 I can have 1,200 pounds carried 36 miles, how many pounds can I have carried 24 miles for the same money?

9. At 80¢ per ounce, what is the value of 36 ingots of silver, each weighing 2 lb. 10 oz. 15 pwt.?

10. Find 30 per cent of 27 yards 8 inches.

11. The solid contents of a block 12 feet 6 inches wide and 3 feet 9 inches thick are 27 cubic yards 1 cubic foot 810 cubic inches. Required its length.

12. A farmer sold 237 bushels 3 pecks of wheat, which was 48 per cent of his crop. How many bushels, pecks, etc., did he have left?

(48% is given; you have to find what %? What part of 48% added to itself will give the required per cent? Do not find the whole crop.)

13. How many spoons, each weighing 2 ounces 12 penny-weights, can be made from 4 pounds 4 ounces of silver?

14. A man travels due west, on the 45th parallel of latitude, 34 miles per hour for 24 hours. How many degrees has he traveled, the length of a degree being 48.96 miles?

REVIEW.

1003. Oral Problems.

1. A puts $600 into business; B, $400; the profits are $500. What is the share of each?

2. Two boys hire a camera for 26 weeks, paying $5.20. How much should be paid by the boy that uses it 12 weeks?

3. New Orleans is 90° west of Greenwich. When it is 2 P.M. at the latter place, what is the time at New Orleans?

4. Find the discount, at 6%, on a note for $300, that has 48 days to run.

5. What will be the cost of 84 yards of muslin at 49¢ a yard?

6. Two men hire a pasture for $84. One puts in twice as many head of cattle as the other. What should each pay?

7. A and B have together 360 acres of land. A owns 50 acres more than B. How many does B own?

8. How long is the day, when the sun rises at 20 minutes to 5 and sets at 21 minutes after 7?

9. After $\frac{1}{3}$ of the contents is taken from a cask, and then $\frac{1}{3}$ of the remainder, there are left 4 gallons. How many gallons did the cask contain originally?

10. A farmer has 16 days' food for 6 horses. If he should purchase 2 more horses, how long would it last?

1004. Questions from Examinations for N. Y. State Certificates.

1. A, B, and C are associated in a business venture, A furnishing $200 of the capital, B $500, and C $700. They gain $472.22, which is divided among them in proportion to their respective investments. Required each man's share of the gain.

2. Find the weight of one dozen silver table-spoons, each weighing 1 oz. 5 pwt.

3. Multiply the sum of $\frac{3}{4}$ and $\frac{4}{5}$ by their product, and reduce the result to a decimal.

4. For how long must $450 be at interest, at five per cent per annum, to amount to $481.62?

5. In how many days of 10 hours each will seven men build 70 rods, if 12 men, in 15 days of 8 hours each, build 90 rods?

6. Express in common fractions the value of $\frac{1}{4}$ per cent, $6\frac{1}{4}$ per cent. Express as per cents $\frac{3}{4}$, $\frac{5}{8}$.

7. Divide the difference between $4\frac{7}{8}$ and $5\frac{3}{8}$ by their sum.

8. Required the amount of $550, for 2 yr. 5 mo. 11 da., at 6% per annum.

9. A bin 6 ft. long and 4 feet wide has a capacity of 75 bushels. How deep is it? (Cancel.)

10. Subtract sixteen ten-millionths from 27 millionths, and multiply the remainder by 20.5.

11. A piece of land is 120 feet wide and 150 feet long. What part of an acre does it contain?

12. Find the difference in time corresponding to a difference in longitude of 7° 42′ 30″.

13. Find the cost of 11,723 feet of lumber, at $19.25 per M.

14. A note is made and dated Sept. 11, 1888, and is made payable in 90 days. Find the date of maturity.

15. Add 137 days to Aug. 14, 1892, and give the resulting date.

16. Divide 320 acres of land among A, B, and C, so that A shall have 15 acres more than B, and C shall have 27 acres more than B.

17. From 10 bu. subtract the sum of 3.64 bu., $\frac{9}{16}$ bu., and 1 bu. 3 pk. 6.52 qt.

18. At 90¢ a yard, what will be the cost of a carpet for a flight of stairs of 18 steps, each $7\frac{1}{2}$ inches high and 10 inches wide?

19. By what number must $30\frac{5}{8}$ be multiplied to produce $604\frac{1}{5}$?

20. Reduce to common fractions .0125, $.06\frac{1}{4}$, $.067\frac{7}{9}$, .006325.

21. Reduce to decimals and find their sum: $\frac{3}{4}$, $\frac{3}{64}$, $\frac{13}{40}$, $\frac{91}{512}$.

22. Why does the value of a fraction remain unchanged when both terms are multiplied by the same number?

23. Required the height of a tower that casts a shadow 21 ft., when under the same conditions of time and place a staff 10 ft. high casts a shadow 28 in.

24. At an election A and B were the candidates for sheriff, and B received 1,211 majority. If the whole number of votes cast was 9,891, how many votes did each receive?

25. A note for $125, made Jan. 4, 1887, is paid May 3, 1888, with interest at 5 per cent per annum. Find the amount paid.

BILLS OF EXCHANGE.

Exchange for £180 17*s.* 6*d.* NEW YORK, Dec. 14, 1895.

Sixty days after sight of this First of Exchange (Second unpaid), pay to the order of John W. Moran & Bro., One Hundred Eighty pounds sterling, seventeen shillings, six pence,

Value received, and charge the same to account of

To JAMES LENNON & Co.,
London.
No. 39.

PETER COMERFORD & SON.

1005. Slate Exercises.

1. Find the cost of the above bill at $4.87 per £.

£200 = $974.00
20 = _____
£180 = $
10*s.* = 2.435 £½
5*s.* =
2*s.* 6*d.* = ____
$

2. What would be the cost of a cable transfer of £251 11*s.* 9*d.*, at $4.88½ per £?

£250 = $1,221.25 ¼ of £1,000
1 =
10*s.* =
1*s.* =
6*d.* =
3*s.* = _____

The newspapers give quotations of foreign exchange for sight and 60-day bills, also for cable transfers.

1006. The New York quotations for French exchange give the number of francs for $1.

Paris cable transfers	$5.16\frac{1}{4}$ @ $5.15\frac{3}{4}$.
Paris bankers' 60 days	$5.18\frac{3}{4}$ @ $5.18\frac{1}{4}$.
Paris bankers' sight	$5.16\frac{7}{8}$ @ $5.16\frac{1}{4}$.

1007. The quotations for German exchange give the value in U. S. money of 4 Reichmarks (or marks).

Reichmarks (4) 60 days	$95\frac{1}{4}$ @ $95\frac{1}{4}$.
Reichmarks (4) sight	$95\frac{3}{8}$ @ $95\frac{1}{4}$.

3. Find the cost of a sight bill on Paris for 1,000 francs, at $5.16\frac{1}{4}$ francs for $1.

4. Find the cost of a 60-days bill of exchange on Berlin for 1,874.35 marks, at $95\frac{1}{4}¢$ for 4 marks.

5. What will be the face in marks of a sight bill of exchange on Berlin that can be bought for $1,000, at $95\frac{1}{2}¢$ for 4 marks?

6. A New York merchant pays $1,637.50 for a 60-days bill on Paris. What is the face of the bill, the rate of exchange being $5.18\frac{1}{2}$ francs for $1?

7. At $4.88 per £, what will be the face of the sight bill on London that can be bought for $1,500?

$$\frac{1500.00}{4.88} = \frac{18750}{61}$$

61)18750 (£307 7s., etc.
450
£23 remainder
20
460s., new dividend.

8. Bought goods in London amounting to £437 5s. 10d. less 4%. How much will I have to pay in Boston for a sight bill of exchange at 4.88\frac{1}{2}$, to settle the account?

9. What will be the cost in Chicago for a 60-day bill on Paris, that will pay for the following articles? Rate, 1 franc = $19\frac{1}{2}¢$.

18 pieces silk, 44 meters each, at 25 francs per meter, less $7\frac{1}{2}$%.
3 pieces of cloth, 50 meters each, at 20 francs per meter, less 5%
Packing charges, 60.50 francs.

10. I wish to send a sight bill of exchange on Berlin in payment of the following invoice:

4 cases musical instruments amounting to 3,598.60 marks, less 10, 5, and $2\frac{1}{2}\%$.

Freight to Hamburg, 165 kilos, at 4.80 marks per kilo.

At $95\frac{7}{8}¢$ for 4 marks, what will be the cost of the bill of exchange?

CHAPTER XIII.

PARTIAL PAYMENTS. — RATIO AND PROPORTION. — SQUARE ROOT. — SURFACES AND VOLUMES.

PARTIAL PAYMENTS.

1008. **U. S. Rule.**

LOUISVILLE, KY., Jan. 5, 1889.

On demand, I promise to pay to the order of Owen McGee Three Hundred Dollars, value received, with interest at 7 per cent.

$\$300\frac{00}{100}$. J. RANDOLPH PAGE.

Endorsements: May 20, 1889, $100; Oct. 30, 1889, $100; March 6, 1890, $50.

How much was due Jan. 5, 1891?

Find amount of $300 Jan. 5, 1889, to first payment May 20, 1889, 4 mo. 15 da. (by compound subtraction).[1]	$307.88
Deduct first payment,	100.00
Balance May 20, 1889,	$207.88
Interest on $207.88 to Oct. 30, 5 mo. 10 da.,	6.47
Amount,	$214.35
Less second payment,	100.00
Balance Oct. 30, 1889,	$114.35
Interest on $114.35 Oct. 30 to March 6, 4 mo. 6 da.,	2.80
Amount,	$117.15
Less third payment,	50.00
Balance March 6, 1890,	$67.15
Interest on $67.15 March 6 to Jan. 5, 9 mo. 29 da.,	3.90
Due Jan. 5, 1891,	$71.05

1009. Slate Exercises.

NOTE. — Find time by compound subtraction.

1. How much is due June 3, 1896, on a demand note for $1,200, with interest at 6%, dated June 3, 1893, bearing endorsements of payment of $500, Sept. 18, 1894; $600, Jan. 3, 1895?

2. A demand note for $600, bearing interest at 5%, was given Feb. 18, 1892. A payment of $250 was made May 28, 1893; one of $150 was made Oct. 8, 1893. How much is due Jan. 23, 1895?

3. Note for $2,000; interest, 7%; dated April 15, 1891. Endorsements: $50, Sept. 20, 1891; $100, May 26, 1892; $1,000, June 20, 1893. How much is due Dec. 27, 1894?

1010.

Face of note,	$2,000.00
Interest from April 15 to Sept. 20, 1891, 5 mo. 5 da.,	60.27
Amount due Sept. 20, 1891,	$2,060.27
If the $50 payment were deducted, and interest computed on the balance, $2,010.27, the maker would be charged interest on $10.27 more than the face of the note, and this the law does not allow. Interest is taken on $2,000 until next payment, May 26, 1892, 8 mo. 6 da.,	95.66
Amount due May 26, 1892,	$2,155.93
As the two payments are not large enough to meet the interest now due, the interest is again calculated on the original $2,000 from May 26, 1892, to June 20, 1893, 1 yr. 24 da.,	149.33
Amount,	$2,305.26
Less $50 + $100 + $1,000 (three payments),	1,150.00
Balance due June 20, 1893,	$1,155.26
Interest on $1,155.26 to Dec. 27, 1894, 1 yr. 6 mo. 7 da.,	122.87
Due Dec. 27, 1894,	$1,278.13

1011. By the United States rule for partial payments, the amount of the principal is found to the time when the payment, or the sum of two or more payments, equals or exceeds the interest.

From this amount deduct the payment or sum of payments.

Use the balance then due as a new principal, and proceed as before.

4. ALBANY, N.Y., March 5, 1893.

One year after date, I promise to pay John Harrigan, or order, Nine Hundred Dollars, value received, with interest at six per cent.

$900\frac{00}{100}$ ANDREW T. SULLIVAN.

Endorsed as follows: June 5, 1893, $10; Sept. 5, 1893, $50; Jan. 5, 1894, $120.

What was due March 8, 1894?

5. Find the amount due April 16, 1893, on a note for $1,000 drawn June 1, 1891.

Endorsements: $150 Sept. 16, 1891;
$50 Sept. 16, 1892.

Interest 6%.

1012. In the United States courts, and in those of some of the states, interest for a portion of a year is taken by days, upon the basis of 365 days to the year.

To make the work easier for the pupils, however, the year of 360 days should be used in the examples given, and the time between dates should be found by compound subtraction.

1013. U. S. Civil Service. State Department Examination.

1. Express one hundred billion in figures, both according to the American and to the English method. (Art. 1014.)

2. Express in figures, both in the form of decimal fractions and of common fractions, the following amounts: Four millionths, twenty-one thousandths, one hundredth, and one tenth.

3. Divide 1.92 by .16.

4. Add $\frac{3}{5}$ of $\frac{5}{7}$ to $4\frac{1}{3}$, and from the sum subtract $2\frac{1}{3}$.

5. A note dated May 20, 1886, for $1,650, is payable March 4, 1887, with interest at 6 per cent. October 12, 1886, $625 was paid on the note. How much will be due, principal and interest, at its maturity?

6. An agent of the Government was sent $7,650 to purchase coal at a coaling station; his commission was 2 per cent on the purchase, which he was to deduct from the money sent. Required: (1) his commission, and (2) the amount expended for coal.

7. If the consular fees collected by the U. S. consul at Liverpool in a year amount to £4,000, and his salary and expenses are $7,591.74, what percentage of the fees can be paid to the United States after deducting the salary and expenses, the estimated value of the pound sterling being $4.8665?

8. Express the value of $3,000,000 in French money, the value of the franc being 19.3 cents.

9. Express according to the Roman system of notation the date 1887.

10. Express in words the number DCLXXXIX.

1014. NOTE. — By the English method of notation a billion is a million million; by the French method, the one usually followed in the United States, a billion is a thousand million.

1015. St. Paul Public Schools. Examination Questions.

1. A man drew out of the bank $\frac{2}{5}$ of his money, and expended 30% of 50% of this for 728 bu. of wheat at 1.12\frac{1}{2}$ a bu. What sum had he left in bank?

2. A grocer bought 40 gal. of maple syrup at the rate of 4 gal. for $6, and sold it at the rate of 5 gal. for $8. What was the whole gain, and the gain per cent?

3. What is meant by trade discount? Why is the discount generally made? What is meant by profit and loss? On what is each estimated?

4. Write an example in compound interest, the time not to be less than 2 yr. 4 mo., and solve it.

5. Two pictures were sold for $99 each; on one there was a gain of 10%, on the other a loss of 10%. Was there a gain or a loss on the sale of both, and how much?

6. Write the analysis of the fifth.

7. NEW YORK, Jan. 1, 1874.

One year after date I promise to pay Noyes Bros. & Cutler Eight Hundred Dollars for value received, with interest.

800\frac{00}{100}$. JAMES M. SCOTT.

Endorsed as follows: Apr. 1, 1874, $10; July 1, 1874, $35; Nov. 1, 1874, $100.

What was due Jan. 1, 1875?

8. What is the difference on a bill of $780, between a discount of 40%, and a discount of 35 and 5% off?

9. A house that cost $14,500 rents for $1,189. What per cent does it pay on the investment? Analyze.

10. How many cords in a pile of wood 42 feet long, 12 ft. high and 8 ft. wide? Find its cost at $6.35 per cord.

PRESENT WORTH AND TRUE DISCOUNT.

1016. Problems are frequently met with in books, in which the "present worth" is asked of a sum of money payable at a future date.

1017. **1.** What is the present worth of $150 payable in 1 year 6 months, interest 6%?

By this is meant what sum at 6% interest will amount to $150 in 1 year 6 months? Or,

Given the amount ($150), rate 6%, time 1 yr. 6 mo. to find principal.

$$x + (x \times \tfrac{6}{100} \times 1\tfrac{1}{2}) = 150.$$

1018. By "true discount" is meant the difference between the sum payable at a future time and its "present worth."

2. What is the "true discount" of $150, payable in 1 year 6 months, interest 6%?

The amount $150, rate 6%, time 1 yr. 6 mo. are given. Find the interest.

Let x = principal

Amount $= x + (x \times \frac{6}{100} \times 1\frac{1}{2}) = 150$

Interest = amount $- x$

1019. Worcester Public Schools. Examination Questions.

1. What principal, on interest for 2 yr. 6 mo. at 4%, will gain $850?

2. What is the cost of insuring a house, worth $25,000, for $\frac{3}{4}$ of its value at $1\frac{1}{2}$%, including $1.00 for the policy?

3. If Boston is 71° 3′ 30″ W. L. and Washington is 77° 2′ 48″ W. L., what is the difference in time between the two cities?

4. At 9¢ a cubic foot, what will be the cost of a block of stone 9 ft. long, 4 ft. wide, and 5 ft. 6 in. thick?

5. If a steeple 150 ft. high casts a shadow of 275 ft., how long a shadow will be cast by a man 6 ft. tall, at the same time of day?

6. If 4 men dig a ditch 24 rods long in 20 days, how long a ditch can 5 men dig in 8 days?

7. What is the true discount of $756, due in 1 yr. 4 mo., at 6%?

(See Art. 1018.)

8. The tax to be raised in a certain town is $1,350. The taxable property is valued at $108,000. What is the tax on one dollar?

9. For what sum must a 60-days note be written to yield $296.85 at a bank, discounting at 6%?

10. If an agent receives $5,616 to invest in silk, after deducting his commission at 4%, how many yards can he purchase at $1.50 per yard?

11. What will be the proceeds of a 60-days note for $500, dated June 4, 1879, and discounted at a bank July 1, 1879, at 6%?

12. At what rate will $142 gain $21.30 interest in 3 years?

13. Mr. Fox buys one-fifth of an acre of land for $21.78. For how much a square foot must he sell it to gain 20%?

14. What is the duty, at 50¢ a pound and 30% ad valorem, on 700 yards of French broadcloth, invoiced at $1.25 per yard, 1 yd. weighing $1\frac{1}{4}$ pounds?

15. What will be the amount, at compound interest, of $340, at 8%, for 1 yr. 3 mo., the interest compounded semi-annually?

REVIEW.

1020. Add:

1.	2.	3.
$184,675.29	$583.67	$34,653.58
1,564.85	6.42	768.57
74,392.68	76.45	67,685.65
439.85	856.78	7,858.74
98,678.98	5,293.58	359.27
7,643.49	421.86	5.64
2.76	15,927.64	87.65
.43	38,528.91	6,539.58
87.54	868,599.72	.48
8,527.00	15,485.35	42.71
796,480.09	84,334.87	3,658.12
5,070.70	6,543.55	86,176.47
3,457.84	465.64	7,018.28

4. £	s.	d.	5. bu.	pk.	qt.	6. rd.	yd.	ft.
5	5		4		4	8		
	3	7		1	3	3	1	
1	8	9	8	2		4	2	1
		4	9	1	6		3	

Supply missing amounts:

7.	8.	9.
$561,876.25	$44,568.72	$74,178.53
3,431.78	7,417.85	854.71
12.94	38.54	92,846.15
769.21	671,928.46	
58,346.87	156.98	698,876.59
521.86	.87	19,328.84
43,419.68	65.91	765.43
287,643.41	9,328.85	8,788.49
	765.43	97,728.85
198.23	187,884.99	9,466.77
6,058.79	772.88	787,788.96
533.38	59,466.77	5,864.86
21,773.96		789.38
$1,234,567.89	$1,301,205.03	$2,000,000.00

1021. Boston Examination Questions. Mental.

1. What will a building lot 100 ft. long and 50 ft. wide cost at 50¢ a sq. ft.?

2. If a broker buys for me five shares of railroad stock whose par value is $100, what is his brokerage at $\frac{1}{4}\%$?

3. If I have 4 tons 1,000 pounds of coal, to how many persons can I give $\frac{1}{2}$ a ton each?

4. If a flagpole is in two parts, the longer, or mainmast, being 50 ft., the topmast 38 ft. long, how high above ground would the pole stand, if 7 ft. were below ground, and the topmast lapped on to the mainmast 2 ft.?

5. A horse was sold for $90, at which price $12\frac{1}{2}\%$ was gained. What per cent would have been gained by selling him for $100?

6. What per cent does a merchant lose by selling goods at $\frac{4}{5}$ of their cost?

7. If I sell 10 shares of railroad stock for $1,090, and gain 9% on the cost, what was the cost?

8. What is the interest of $660, for 3 mo., at 4%?

9. What is the premium for insuring $6,000 on my house at $1\frac{1}{4}$%?

10. How many quarts of peanuts in one bu. and three pk.?

11. What principal at 6% simple interest will gain $36 in 1 year and 6 months?

12. What per cent is gained on goods sold at double the cost?

13. What is 8% of fifty bushels?

14. How many cubic inches in a ten-inch cube?

15. What is the interest of $1,500, for 60 days, at 6%?

16. How many years will it take $20 to gain $20 at 5 per cent simple interest?

17. Bought 2 chairs at $1.25, 1 wash-tub for $1.50, 1 table for $3.00, and 5 doz. glasses at 48¢ a dozen. Gave a ten-dollar bill in payment. How much change did I receive?

18. $3,000 is $11\frac{1}{9}$% of my property. How much am I worth?

19. What is the interest on $700, for 15 days, at 6%?

20. Bank discount on a note for 65 days, $1,000, discounted at date?

21. At what rate will $2 gain $20 in 5 years?

22. Cost of 700 lb. of coal at $7 a ton?

23. My desk is $1\frac{1}{2}$ ft. long and 1 ft. wide. How many inches around it?

24. If a man spends 50¢ a day during April, May, and June, what does he spend in the three months?

25. A grocer bought 15 barrels of flour at $5 a barrel. At what price must he sell them to gain $30?

26. How many weeks will $4\frac{1}{5}$ tons of coal last Mrs. Bright, if she uses $\frac{3}{10}$ of a ton each week?

27. I can buy 2 pairs of shoes for 12 shillings. How many pairs at the same rate can I buy for £3?

28. Seven-eighths of James's vacation will be equal to seven-ninths of yours; yours will be 63 days. How many will his be?

29. If two-thirds of your age is eight years and four months, how old are you?

30. 5 quarts equal what decimal of a peck?

1022. Boston Examination Questions. Slate.

1. What is the cost of carpeting a room $16\frac{1}{2}$ ft. long, 12 ft. wide, with oil-cloth $1\frac{1}{2}$ yd. wide, at 75¢ a yard?

2. What is the present worth of $105.71, due in 4 yr., at 6%?

3. What must be the face of a note which, discounted at a bank, for 27 days and grace, would yield $95?

4. If I lose 10% by selling goods at 18¢ a yard, for what must they be sold to gain 20%?

5. I sold $24\frac{1}{2}$% of my estate, or $1,372 worth. I am worth, in addition to my real estate, $14,000. How much am I worth in all?

6. The edges of a large cubical box are 5 ft. long. How many sq. ft. of paper will cover the outside of the box?

7. Mr. Jones insured his house, worth $48,000, for one year for $\frac{5}{6}$ of its value, at $\frac{3}{4}$%. What would the insurance company lose if the house should burn?

8. If I lose 9% by selling land at $764.40 an acre, what shall I gain by selling it at $894.60 an acre?

9. A house lot containing 24,702 sq. ft. is 179 ft. long. How wide is it?

10. What is the value of a pile of wood 25 ft. × 8 ft. × 8 ft., at $3.87½ per cord?

11. I bought a horse for $250, paid $2.50 for shoeing, then sold him for $141.40. What per cent did I lose?

12. What is the value of a pile of wood 40 ft. long, 4 ft. wide, and 5 ft. high, at $5.30 a cord?

13. How many square yards in the walls of a room 42 ft. long, 15 ft. wide, and 9 ft. high?

14. By buying a cargo of coal at $6 per ton, and selling it at $8 a ton, I gained $198. How much did I pay for it?

15. Make out a receipted bill for the following: 325 yd. of silk at $2.25 per yd.; 296 yd. of lace at $1.50 per yd.; 480 yd. of ribbon at $0.50 per yd.; 45 doz. gloves at $15 per doz.

16. My dividend is 8¾, quotient 9⅘. What is the divisor?

17. I gave away ⅛ and ⅖ of 4½ bushels of chestnuts. What % was left?

18. The distance around a square field is 16 rods. What is the field worth, at 8⅓¢ a sq. ft.?

19. Sent a broker $467.40 to invest in cotton worth 7⅛¢ a pound, first deducting his commission of 2½%. How many pounds did he buy, and what was his commission?

20. Find the face of a three-months note, discounted at 6½%, to yield $856.

21. I sold 80 yards of broadcloth for $240, thereby losing 20% on the cost. For what should I have sold it per yard to have gained 15% on the cost?

22. A man bought 60 casks, of 65 gallons each, for $1,542; 80 gallons leaked out. For what must he sell the remainder per gallon to gain 12½% on the cost?

23. Each of two men sold his horse for $180. One made 20%, the other lost 20% on the cost. Cost of each horse?

24. What will it cost to plaster the four walls and the ceiling of a room 18 ft. long, 12 ft. wide, and 9 ft. high, at 40¢ per square yard? How many cords of wood would the above room contain if completely filled?

25. The sum of two numbers is $4\frac{1}{5}$, and their difference $\frac{4}{5}$. What are the numbers?

26. A man agrees to dig a cellar 30 ft. long, 24 ft. wide, and 6 ft. deep. What % of the work has he done when he has dug out 16 cu. yd.?

27. A man bought 672 yards of cloth at $1.25 a yard. He sold it immediately for $2.25 a yard, receiving in payment a 60-days note for the amount, which he had discounted at a bank at 7%. How much money did he make?

28. What will it cost to fill in a street 55 feet wide, 600 feet long, and $5\frac{1}{2}$ feet below grade, at 40¢ a cubic yard?

29. BOSTON, May 23, 1888.

Ninety days from date I promise to pay Geo. Baker, or order, Three Thousand Seven Hundred Fifty Dollars. Value received.

$3,750\frac{00}{100}$ JAS. DONOVAN.

Find the avails (proceeds) of the above note if discounted at 7%.

30. Add $754.60, $187.24, $536.84, $976.79, $878.29, $458.71, $549.96, $597.85.

1023. U. S. Civil Service. Departmental. Clerk Examination.

1. Express the following in figures, using such signs, monetary or otherwise, as may be necessary:

(*a*) Fifty million one thousand three and seven hundred five millionths; (*b*) two million fifty-four thousand twenty dollars, eight cents, seven mills; (*c*) twenty-seven pounds, twelve shillings, and six pence.

2. Express the following in words: (See Art. 444.)

(*a*) $\overline{\text{XCIX}}\text{LXXIV.}$; (*b*) \$83,010,036.069;
(*c*) 304,005,160.010003; (*d*) $\frac{10}{11}\frac{8}{0} \times \frac{1}{4}\frac{8}{7} \div \frac{4}{11}\frac{6}{1} - \frac{5}{6}$.

3. The quotient arising from the division of 6,985.473 by a certain number is 51, and the remainder is 68.853. What is the divisor?

4. What is the value of the following?

$$\frac{3\frac{3}{16} + 4\frac{4}{5} - 6\frac{5}{8}}{2\frac{4}{7} + 1\frac{3}{4} - 3\frac{1}{2}} \div \frac{\frac{3}{8} \times \frac{7}{10} \times \frac{2}{3}}{\frac{6}{7} \times \frac{3}{4} \times \frac{2}{8}}.$$

5. In going 1 mile 94 rods 2 yards 1 foot, a carriage wheel makes 526 revolutions. What is the circumference of the wheel?

6. What is the weight (in tons, cwt., qr., etc.) of the water which fills a cistern 9 feet 8 inches long, 9 feet 4 inches wide, and 6 feet 9 inches deep, a cubic foot of water weighing 1,000 ounces? (1 ton = 2,240 pounds.)

7. On a note dated Oct. 16, 1886, for \$2,650, with interest at 6 per cent, the following payments were made: Jan. 28, 1887, \$575; May 22, 1887, \$25; and Aug. 4, 1887, \$948. What was due Nov. 25, 1887?

8. An owner of 6 per cent bonds sells them at the market quotation of 118, and invests the proceeds in $4\frac{1}{2}$ per cent bonds. The latter investment yields him the same income as the former. What did he pay per hundred for the $4\frac{1}{2}$ per cent bonds?

9. A grocer pays 18¢ per lb. for coffee and roasts it, it losing 10 per cent of its weight in the process. What must he charge per lb. for the roasted coffee in order to make a profit of 20 per cent, allowing 4 per cent for bad debts?

10. Money being worth 8 per cent per annum, what is the true discount on \$450, which is due in 5 months and 21 days, without interest? (See Art. 1018.)

SURFACES AND VOLUMES.

1024. Slate Problems.

1. If a piece of cloth is 20 yards long and $\frac{3}{4}$ yd. broad, how broad is another piece of cloth 12 yards long that contains as many square yards as the former?

2. An iron beam 16 ft. long, $2\frac{1}{4}$ ft. broad, and 8 in. thick, weighs 1,280 lb. What is the length of a similar beam whose breadth is $3\frac{1}{4}$ ft., thickness $7\frac{1}{2}$ in., and weight 2,028 lb.?

3. What will it cost to carpet a room $22\frac{1}{2}$ ft. long by $15\frac{3}{4}$ ft. wide with carpet $2\frac{1}{4}$ ft. wide, costing \$1.50 per yd.?

4. What is the length of a box $6\frac{2}{3}$ ft. wide and $7\frac{1}{2}$ ft. high, that will exactly contain 12 boxes $4\frac{1}{6}$ ft. long, $3\frac{1}{3}$ ft. wide, and $2\frac{1}{2}$ ft. deep?

5. What is the value, at \$120 per acre, of a square field whose side is 35.25 chains?

10 sq. chains = 1 acre.

6. What is the area in square feet of a triangle whose base is 18 ft. 4 in., and whose altitude is 11 ft. 10 in.?

7. What is the area of a circle whose diameter is 7.5 feet, the area of the circle being .7854 times the area of the square that will just enclose it?

8. Find the capacity, in bushels, of a bin 22 ft. long, 14 ft. wide, 12 ft. high?

9. How many gallons will a tank hold, its dimensions being 4 ft. 1 in. by 3 ft. 8 in. by 2 ft. 3 in.?

10. How many square yards are there in the walls and the ceiling of a room 21 ft. long, 18 ft. wide, 12 ft. high? Make a diagram.

11. A tank $5\frac{1}{2}$ ft. by 6 ft. by 7 ft. can be emptied by two pipes, one of which discharges 9 gallons per minute and the

other 7 gallons per minute. How long will it take each to empty the tank? How long will it take both together?

12. A parlor is 18 feet long, 15 feet wide. Make a diagram showing how carpet 27 inches wide can be laid without cutting the carpet lengthwise. Which would be the better way to lay carpet 30 inches wide in the above room?

13. Calculate the number of running yards of carpet 30 in. wide needed for the floor of the above room, including $4\frac{1}{2}$ yards wasted in matching the pattern.

Find the cost of carpeting the room at 95 cents per running yard for carpet, 5 cents per square yard for lining, and 10 cents per running yard for sewing and laying.

14. A rug 18 feet long, 15 feet wide, is placed in the centre of the floor of a room 21 feet long, 18 feet wide. What is the width of the strip left uncovered? Find the area of the uncovered space?

15. A room is 18 feet wide, 24 feet long, 9 feet high. There are two doors 4 feet wide, $7\frac{1}{2}$ feet high; two windows 4 feet wide, 6 feet high; and a fire-place 5 feet square. How many square feet of plastering will there be on the walls and ceiling, deducting for a baseboard 12 inches wide? How many running feet of baseboard will be needed?

Draw "development" of the above room, showing the four walls and the ceiling, and locating the doors, the windows, and the baseboard.

Do not use baseboard where it is not required.

16. At the rate of $1,400 for a pile of lumber 25 ft. long, 20 ft. wide, 10 ft. high, what is the value of a pile 50 ft. long, 40 ft. wide, 20 ft. high?

17. If it costs $14 to paint the walls and the ceiling of a room 25 ft. long, 20 ft. wide, and 10 ft. high, what will it cost to paint the walls and the ceiling of a room 50 ft. long, 40 ft. wide, and 20 ft. high?

REVIEW.

1025. Slate Exercises.

Multiply:

1. $.0099 \times .0078$
2. $15.39 \times .0965$
3. $.0927 \times 4.86$
4. $89.64 \times .086$
5. $.00859 \times 8.29$
6. $.05094 \times .095$
7. $.0084 \times 75.8$
8. $3.785 \times .0099$
9. $2.007 \times .4096$
10. $.0596 \times .0008$

Divide:

1. $4.385 \div 6.4$
2. $88.476 \div .15$
3. $6.144 \div .04$
4. $235.08 \div .072$
5. $.128 \div .00256$
6. $.48 \div 7.5$
7. $.0152 \div 640$
8. $.1288 \div 11.2$
9. $971.7 \div .0123$
10. $5.483 \div .025$

1026. U. S. Civil Service. Departmental Clerk Examination.

1. Express the following in figures, using such signs, monetary or other, as may be necessary:

(*a*) Eight million eight ten-millionths.

(*b*) Eight million eight hundred eight ten-billionths.

(*c*) Seven million nine hundred seventy-four thousand dollars seventy-five cents five mills.

(*d*) Four hundred thirty-five pounds (sterling) eighteen shillings nine pence.

2. Express the following in words: (*a*) 7,988,800.00088; (*b*) \$97,970,120.058; (*c*) $\overline{\text{XXV}}\text{XLIX}$; (*d*) $\frac{25}{78}$ of $\frac{96}{1027} - \frac{9}{97}$ of $\frac{44}{7}$. (Art. 444.)

3. From the sum of 976,438, 2,526, 57,694, and 698,342 subtract 937,660, divide the remainder by 388,000, and multiply the quotient by 2.005.

4. Find the value of $\frac{2\frac{1}{9} + 4\frac{5}{12}}{1\frac{3}{4} \times 3\frac{1}{8}} \div \frac{1}{6}$ of $6\frac{5}{7}$.

5. The fence around the four sides of a square field measures $3.708\frac{1}{3}$ miles. What is the length in fur., rd., yd., and ft. of the fence on one side of the field?

40 rods = 1 furlong (fur.) 8 fur. = 1 mi.

6. A merchant buys 42 gallons of alcohol at $2.50 per gal., and keeps it for 3 years. He then finds that he has lost 7 gallons by leakage and evaporation. Estimating the value of money at 6 per cent per annum, how much per gallon must he charge, in order that he may realize a profit of $16\frac{2}{3}$ per cent on his outlay (including interest)?

7. At $62\frac{1}{2}¢$ per cu. yd., what will it cost to remove an embankment 240 ft. long, 38 ft. wide, and 9 ft. high?

8. What is the weight (in tons, cwt., etc.) of the water that fills a tank 4 ft. 10 in. long, 4 ft. 8 in. wide, and 5 ft. $4\frac{1}{2}$ in. deep, a cubic foot of water weighing 1,000 ounces?

1 ton = 2,240 lb.

9. What sum placed at interest at 8 per cent per annum will amount to $18,590 in 2 years 11 months and 27 days?

10. What is the difference between the interest and the true discount of $380, due in 2 years and 5 months, at 6%? (See Art. 1018.)

1027. Kansas City Public Schools. Examination Questions. Oral Problems.

1. By selling an article for $9.00, a man gained 25%. How many dollars would he have gained if he had sold the article at an advance of 50% over cost?

2. A man sold two cows for $30 each. On one he gained 25%; on the other he lost 25%. Did he gain or lose, and how much?

3. $12 is 4 per cent of A's and B's money. If A has twice as much as B, how much has each?

4. What is the interest of $60, for $5\frac{2}{3}$ years, at 6%?

5. What is the amount of $300 for 8 years 10 months, at 6%?

6. What principal, in 3 years and 4 months, at 6%, will give $40 interest?

7. What is the present worth of $52, due 5 years hence, at six per cent? (Art. 1017.)

8. In what time will $50, at 6%, give $18 interest?

9. At what per cent will $12, in 3 yr. 4 mo., amount to $14?

10. If I buy an article for $75 and sell it for $50, what is my loss per cent?

1028. Massachusetts Civil Service. Clerks. General Examination.

1. Write in figures the following number:

Five million thirteen thousand six.

Write in words the number expressed by the following figures:

3,001,014.37

2. Add these across, placing the totals in the space indicated; then add the totals.

						Totals.
32,849	79,541	23,642	34,685	68,423	26,439	
18,364	42,863	8,375	126,582	17,397	78,542	
27,262	29,988	26,837	23,776	35,601	8,785	
50,010	75,542	66,352	19,240	4,327	36,296	
235,065	15,637	12,364	23,262	35,496	62,053	

3. What number must be added to the sum of $\frac{4}{5}$, $\frac{7}{8}$, and $\frac{11}{12}$ to make $5\frac{89}{120}$?

4. A traveler walked $23\frac{1}{8}$ miles the first day, $3\frac{3}{4}$ miles more the second day than the first, and $3\frac{1}{8}$ miles more the third day than the second. How far did he walk in the three days?

5. Multiply 63.15 by 1.04; divide the product by 6.25, and subtract the quotient from 11.

6. If $21.63 be divided between two persons so that one shall receive one-third more than the other, how much will each receive?

7. How many bricks, 8 inches long and 4 inches wide, will be needed to make a sidewalk 26 feet long and 4 feet wide?

8. If it costs $10.24 to carry 1,500 lb. 356 miles, what will it cost to carry 2,700 lb. 890 miles?

9. If a post 11 feet in height casts a shadow 9 feet 2 inches long, what is the height of a house which casts a shadow 45 feet 10 inches long?

10. A house rents for $30 a month, and the owner pays $75 a year for taxes and repairs. What is the value of the house, if his net profit is 5 per cent per annum?

11. A loaned B a sum of money at $4\frac{1}{2}$ per cent interest per annum. At the end of 18 months, B paid the debt, principal and interest, in all $1,814.75. What was the sum borrowed?

12. Find the interest on $2,320, for 5 months and 21 days, at the rate of 7 per cent a year.

13. Find the interest on $640, from Sept. 3, 1888, to Oct. 30, 1889, at 6 per cent per annum.

14. At compound interest, what will $200 amount to, in 1 year and 3 months, at 6 per cent, interest compounded semi-annually?

15. If a 5-months note for $760, dated March 13, is discounted at a bank May 23, the rate being 7 per cent a year, what will be the proceeds?

SQUARE ROOT.

1029. Squaring a number is multiplying the number by itself. The square of $8 = 8 \times 8 = 64$.

1030. The square of a number is indicated by writing a small 2 a little to the right of the upper part of the number.

$$5^2 = 25,\ 12^2 = 144.$$

What is the square of 4? Of 6? Of 7? Of 9? Of 10? Of 11?

$$2^2 = ?\quad 3^2 = ?$$

Square 13. 15. 21. 16. 19. $14^2 = ?$ $17^2 = ?$ $24^2 = ?$ $33^2 = ?$

1031. The square of $25 = (20 + 5) \times (20 + 5)$.

$$\begin{array}{ll} & 20 + 5 \\ & \underline{20 + 5} \\ \text{Multiplying by } 20 & 20^2 + \quad 20 \times 5 \\ \text{Multiplying by } 5 & \qquad\quad \underline{20 \times 5 + 5^2} \\ & 20^2 + 2(20 \times 5) + 5^2 = 400 + 200 + 25 = 625. \end{array}$$

1032. The square of the sum of two numbers is equal to the square of the first + twice the product of the first by the second + the square of the second.

$$13^2 = (10 + 3)^2 = 10^2 + 2(10 \times 3) + 3^2 = ?$$
$$18^2 = (10 + 8)^2 = 100 + 160 + 64 = ?$$
$$27^2 = (20 + 7)^2 = 400 + 280 + 49 = ?$$

1033. Oral Exercises.

Square:

1. 14	**4.** 22	**7.** 51	**10.** 32	**13.** 24
2. 15	**5.** 31	**8.** 61	**11.** 42	**14.** 33
3. 21	**6.** 41	**9.** 23	**12.** 52	**15.** 43

1034. The square root of 4 is 2; of 9 is 3; of 16 is 4; of 25 is 5.

1035. Give the square root of 36. Of 64. Of 81. Of 121. Of 49. Of 100. Of 144.

1036. The sign of square root is $\sqrt{\ }$.

$\sqrt{81} = 9.$ $\sqrt{121} = ?$ $\sqrt{25} = ?$ $\sqrt{49} = ?$

1037. Find the square root of 169.

$10^2 = 100.$ $20^2 = 400.$ The square root is between 10 and 20; it is, therefore, 10 + a second number.

$$169 = 10^2 + 2(10 \times \text{second}) + \text{second}^2.$$
$$169 = 100 + 20 \times \text{second} + \text{second}^2.$$
$$20 \times \text{second} + \text{second}^2 = 69.$$

From this it appears that the second number is 3, since

$$20 \times 3 + 3^2 = 69.$$

1038. It may be shown in this way:

$$\begin{array}{rl} & \underline{10} \text{ (first number)} \\ & 169 \\ & \underline{10^2 = 100} \\ \text{Trial divisor — twice 10} & 20) \quad 69 (3 \text{ second number}) \\ & \underline{60} \\ & 9 \\ & 3^2 = \underline{9} \end{array}$$

Ans. 10 + 3 = 13.

1039. Find the square root of 2,116.

$$\begin{array}{rl} & \underline{40} \text{ (first number)} \\ & 2{,}116 \\ 40^2 & \underline{1{,}600} \\ 40 \times 2 = 80, \text{ trial divisor} &) \ 516 (6 \text{ second number}) \\ & \underline{480} \\ & 36 = 6^2 \end{array}$$

Ans. 46.

1040. Instead of multiplying the trial divisor by the second number, and then ascertaining whether the remainder is the square of the second number, the second number is added to the trial divisor and this sum is multiplied by the second number.

$$\begin{array}{rl} & \underline{40} \text{ (first number)} \\ & 2{,}116 \\ & \underline{1{,}600} \\ (2 \times 40) + 6 = 86) & 516 (6 \text{ second number}) \\ & 516 \end{array}$$

Ans. 46.

1041. In practice, the work is shortened by omitting the ciphers.

First, point off in periods of two figures, commencing at the right. Find the greatest square in the first period, and place the root in the quotient. Subtract the square from the first period. Bring down the next period. Multiply the first quotient figure by 2, and use it as a trial divisor. Place the second figure in the quotient. Affix it also to the trial divisor. Multiply the two figures in the trial divisor by the second quotient figure.

$$\begin{array}{r} 4\ 6 \\ \hline 21'16 \\ 16 \\ \hline 86)\ 516 \\ 516 \end{array}$$ *Ans.*

1042. Slate Exercises.

Extract the square root:

1. 196	**6.** 1,296	**11.** 2,809	**16.** 5,625
2. 256	**7.** 1,225	**12.** 2,916	**17.** 6,889
3. 324	**8.** 1,764	**13.** 3,721	**18.** 7,056
4. 576	**9.** 1,936	**14.** 3,969	**19.** 8,281
5. 676	**10.** 2,601	**15.** 5,184	**20.** 9,025

REVIEW.

1043. Slate Exercises.

Divide (Arts. 385, 616):

1. 4,270,978,096 ÷ 564,347	**6.** 2,171,008,895 ÷ 721,985
2. 4,375,621,423 ÷ 856,789	**7.** 86,409,429,120 ÷ 876,008
3. 4,518,821,072 ÷ 752,134	**8.** 57,681,954,968 ÷ 768,437
4. 3,817,832,184 ÷ 607,432	**9.** 40,333,410,989 ÷ 568,709
5. 3,462,706,614 ÷ 567,843	**10.** 53,531,676,960 ÷ 678,432

Write answers (Art. 385):

1. $\frac{450,000}{86,432}$	**4.** $\frac{700,000}{59,084}$	**7.** $\frac{901,020}{98,642}$
2. $\frac{500,000}{72,356}$	**5.** $\frac{683,427}{67,805}$	**8.** $\frac{385,093}{76,057}$
3. $\frac{583,217}{64,587}$	**6.** $\frac{701,380}{58,437}$	**9.** $\frac{673,217}{85,607}$

1044. An Invoice (English).

Invoice of 3 bales Linen Goods forwarded by rail to Glasgow, for shipment thence per S.S. Anchoria to New York, to order, and for account and risk of Messrs. Robinson & Co.

			yd.				
[R] Co.	#2	30 pcs. Bord. Crash	1500	$1\frac{7}{8}$ *d.*	£11	14	$4\frac{1}{2}$
		30 " " "	1500	2 "			
	#3	60 " Checked G. C.	3000	$1\frac{7}{8}$ "			
	#4	60 " " "	2889	$2\frac{1}{12}$ "			
					£		
		Less $2\frac{1}{2}\%$ disct.			£		

1. Find the duty in U. S. money at 50% ad valorem.

£ = $4.8665.

2. What is the cost in English money of crockery amounting to £166 13*s.* 4*d.* less a discount of 5 and 5%?

RATIO.

1045. Ratio is the relation which one number has to another of the same kind.

1046. The first term of the ratio is called the antecedent; the second, the consequent.

The ratio of 3 to 6, $9 to $18, 15 cows to 30 cows may be expressed $\frac{3}{6}$, $\frac{9}{18}$, $\frac{15}{30}$. They are each equal to $\frac{1}{2}$.

1047. Oral Exercises.

Express the ratio in lowest terms:

1. 175 to 700

$\frac{175}{700} = \frac{1}{4}$

2. $19 to $95

3. $36.50 to $18.25

4. 3 quarts to 4 gallons

NOTE. — The denominations must be the same.

3 quarts to 16 quarts = $\frac{3}{16}$

5. 6 pecks to 5 bushels
6. 20 mills to 1 dollar
7. 7 tenths to 3 fifths
8. 1 gallon to 500 cu. in.
9. 1 mark (23.8¢) to 1 franc (19.3¢)
10. 1 shilling (24.33¢) to 1 dollar

1048. Sight Exercises.

1. $\frac{3}{16}=\frac{?}{64}$
2. $\frac{18}{37}=\frac{36}{?}$
3. $\frac{15}{13}=\frac{?}{65}$
4. $\frac{1 \text{ pk.}}{3 \text{ bu.}}=\frac{\$?}{\$24}$
5. $\frac{3 \text{ qt.}}{1 \text{ gal.}}=\frac{30¢}{?¢}$
6. $\frac{17}{21}=\frac{51}{?}$
7. $\frac{18}{?}=\frac{36}{70}$
8. $\frac{?}{24}=\frac{57}{72}$
9. $\frac{\$16}{?}=\frac{7 \text{ marks}}{21 \text{ marks}}$
10. $5 \div 22 = ? \div 88$
11. 6 horses $\div$? horses $= \$600 \div \$900$
12. 1 ft. $\div$? yd. $= 15¢ \div 90¢$
13. 1 qt. 1 pt. $\div$ 1 pt. $= ?¢ \div 4¢$
14. $1\frac{1}{4} \div \frac{3}{4} = \frac{?}{8} \div \frac{6}{8}$
15. $2.8 \div .4 = .14 \div x$

1049. Oral Problems.

1. One line is a rod long, another is $5\frac{1}{2}$ ft. long. What is the ratio of the first to the second?

2. What is the ratio of 7 hours to 1 day?

3. A pound of coffee costs 30¢, of sugar 6¢. What is the ratio of their respective prices?

4. A walks in 4 hours as far as B in 5. What is the ratio of A's speed to B's?

5. E earns in 6 days as much as D earns in 8 days. Find the ratio of E's daily earnings to D's.

6. One wheel makes 300 revolutions in 2 minutes, the second requires only $1\frac{1}{2}$ minutes to make the same number. Find the ratio of the number of revolutions made by the first wheel in 1 minute to the number made by the second wheel in the same time.

7. A circle whose diameter is 1 ft. has a circumference of $3\frac{1}{7}$ ft. What is the ratio of the diameter to the circumference?

8. One train goes 40 miles an hour, a second goes 45 miles an hour. What is the ratio of the speed of the first to that of the second?

9. A window is 6 ft. 4 in. high by 4 ft. 2 in. wide. What is the ratio of the height to the width?

10. A father is 36 years old, his son is 9. What was the ratio 6 years ago of the father's age to that of the son?

1050. Slate Problems.

(Be sure your answer is correct.)

1. One line is 3 rods 4 yards long, the length of another is 5 rods 1 ft. Find the ratio of the first to the second.

2. One candle lasts 4 hours 20 minutes, another lasts 3 hours 15 minutes. Find the ratio of the first to the second.

3. A pound of coffee costs $25\frac{7}{8}$¢; 1 lb. of sugar costs $5\frac{3}{16}$¢. What is the ratio of price of sugar to that of coffee?

4. M walks in 1 hour 47 min. as far as N walks in 2 hours 3 minutes. What is the ratio of M's speed to N's?

5. P earns in $19\frac{5}{7}$ days as much as Q in $18\frac{3}{4}$ days. What is the ratio of Q's daily earnings to P's? Of P's to Q's?

6. One wheel makes 600 revolutions in $8\frac{4}{7}$ seconds; a second makes 300 revolutions in $3\frac{1}{3}$ seconds. What is the ratio of the speed of the first wheel to that of the second?

7. The circumference of a circle is 12.5664 ft., and its radius is 2 ft. What is the ratio of the diameter to the circumference?

8. One train goes 40 miles in 50 minutes, another goes 24 miles in a half hour. What is the ratio of the speed of the second to that of the first?

9. One window is 6 ft. 8 in. × 4 ft. 2 in.; a second is 4 ft. 8 in. × 2 ft. 1 in. What is the ratio of the area of the second to that of the first?

10. A mother is now 35 years old, and her son is 3 years and 6 months old. Fourteen months ago, what was the ratio of the mother's age to that of her son?

1051. Newark, N. J., Public Schools. Examination Questions.

1. Sold 60 bushels of oats at $0.42, 40 bbl. of flour at $8.50, 56 bushels of corn at $0.58. Make out the bill, add 1 per cent for cartage, and deduct 3 per cent for cash, and receipt the bill.

2. A purchased a farm 156½ rods long and 124.6 rods wide at $75½ per acre. Paid one-half cash, gave a mortgage for the balance. At the close of one year, he paid $500 on the mortgage. How much remains due, money being worth 5 per cent?

3. Analyze: A farmer sold 2 barrels apples at $3¾ per barrel, spent ⅔ of the money for tea at ⅝ of a dollar a pound, and the balance for coffee at ⅓ of a dollar a pound. How many pounds of each did he buy?

4. What will it cost to dig and wall the cellar of a house 41¼ feet long, 33 feet wide, and 8 feet deep, the wall to be 1½ feet thick? The digging will cost $0.50 per cubic yard, and the laying of the wall $15 per 100 cubic feet.

5. A merchant got a shipment of 500 bbl. of flour insured for 80 per cent of its cost at 3¼ per cent, paying $107.25 premium. What did the flour cost him per bbl.?

6. Find the maturity, term of discount, and proceeds of the following note:

$1,250. NEWARK, N. J., June 12, 1889.

Six months after date, I promise to pay James Jones, or order, Twelve Hundred and Fifty Dollars, with interest at 5 per cent.

Discounted at the bank Nov. 15.

7. A demand note for $1,000, dated April 17, 1884, has the following endorsements: June 5, $253; Aug. 20, $274.50; Nov. 17, $420. What is due Jan. 1, 1885, interest at 6 per cent?

8. A and B, contractors, received $857.50 for grading a roadway. A furnished 5 men 20 days, and 6 others for 15 days; B furnished 10 men for 12 days, and 9 others for 20 days. What was each contractor's share?

9. How many men will be required, working 12 hours a day for 250 days, to dig a ditch 750 feet long, 4 feet wide, and 3 feet deep, if it requires 27 men, working 13 hours a day for 62 days, to dig a ditch 403 feet long, 3 feet wide, and 3 feet deep?

10. I sent to my agent in St. Louis $1,508.80 to invest in flour. After deducting his commission at 2½ per cent, how many barrels of flour can he buy at $5¾ per barrel?

INTEREST AND DISCOUNT.

1052. Slate Exercises.

(Solve the first ten by aliquot parts.)

Find the amount:

1. $1,875.25, 3 yr. 5 mo. 15 da., 4½%.
2. $487.50, 1 yr. 10 mo. 25 da., 6%.
3. $1,206.84, 2 yr. 1 mo. 16 da., 5%.
4. $595.00, 7 yr. 7 mo. 7 da., 7%.

7 mo. = $\frac{7}{12}$ of 7 yr. 7 da. = what part of 7 mo.?

5. $763.25, 8 mo. 11 da., 4%.

6. $685.70, 19 mo. 5 da., 3½%.

7. $1,563.00, 3 mo. 20 da., 5%.

8. $998.45, 87 da., 4%.

9. $2,575.50, 149 da., 3%.

10. $693.27, 214 da., 6%.

1053. Find the value of x:

11. Principal, $240; rate, x; interest, $32.04; time, 2 yr. 11 mo. 18 da.

12. Principal, x; rate, 6%; amount, $717.40; time, 3 yr. 3 mo. 4 da.

13. Principal, $360; rate, 3%; interest, $48.87; time, x.

14. Principal, $288; rate, 2½%; amount, $307.22; time, x.

15. Principal, x; rate, 6%; interest, $13.10; time, 4 mo. 11 da.

16. Principal, $270; rate, x; amount, $273.27; time, 3 mo. 19 da.

1054. Distinguish between "term" and "time." Term of a 90-day note is 93 days. (See Arts. 939 and 941.)

17. Term, x; face, $600; discount, $6.30; rate, 6%.

18. Term, 33 days; face, x; proceeds, $397.80; rate, 6%.

19. Time, 90 days; face, $300; proceeds, x; rate, 6%.

20. Term, 21 days; face, $600; discount, $2.45; rate, x.

21. Time, 4 mo.; face, $200; discount, x; rate, 6%.

22. Term, 132 days; face, x; proceeds, $2,689.50; rate, 6%.

23. Term, x; face, $150; proceeds, $147.75; rate, 6%.

24. Term, x; face, $1,650; discount, $4.95; rate, 6%.

25. Time, 69 days; face, x; proceeds, $469.30; rate, 6%.

REVIEW.

1055. Find products:

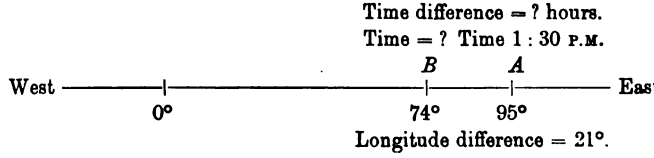
1.	$648 \times \frac{7}{8}$	11.	$1{,}864 \times 250$
2.	$976 \times \frac{15}{16}$	12.	983×51
3.	$1{,}648 \times 87\frac{1}{2}$	13.	$1{,}576 \times 62\frac{1}{2}$
4.	$2{,}592 \times 9\frac{15}{16}$	14.	$176 \times 23\frac{3}{4}$
5.	$2{,}416 \times 875$	15.	$1{,}128 \times 375$
6.	874×99	16.	$895 \times 44\frac{4}{5}$
7.	848×125	17.	864×486
8.	375×999	18.	975×318
9.	792×25	19.	$37\frac{2}{3} \times 6\frac{1}{9}$
10.	457×16	20.	$48\frac{3}{4} \times 4\frac{2}{3}$

LONGITUDE AND TIME.

1056. NOTE. — Making diagrams, as shown below, may assist the pupil to solve the problems.

1. Given the longitude of A as 95° east, and that of B as 74° east, and the time at A as 1 : 30 P.M., to find the time at B.

Since the latitude of B has no bearing upon its time, both places may be located upon the same line running east and west.

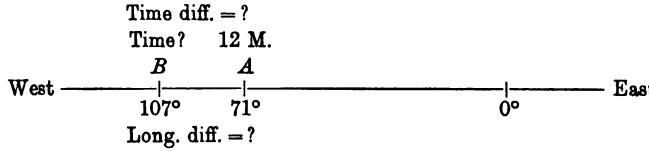


Locate the prime meridian (that of 0°), then the meridians of 74° and 95° east. Mark above the last two the names of the places, B and A. Write above A its given time, 1 : 30 P.M.

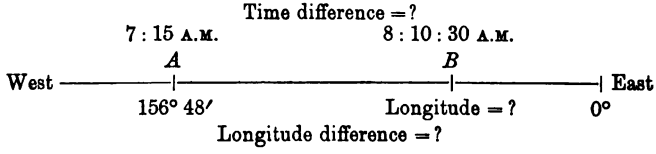
To find the time at B, we must find the difference of time between B and A. The difference in longitude is $95° - 74° = 21°$. The difference in time is $(21 \div 15)$ hours.

NOTE. — Remember that the more easterly of the two places has the later time.

2. A is situated in 71° west longitude, B in 107° west longitude. What time is it at B, when it is noon at A?

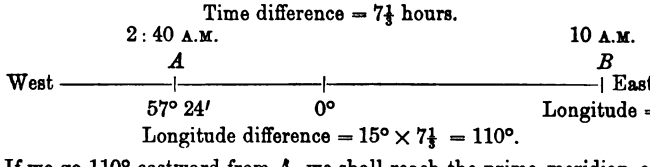


3. Find the longitude of B, whose time is 8 : 10 : 30 A.M., when it is 7 : 15 A.M. at A, whose longitude is 156° 48′ west.



Since B has the later time, its location is east of A. The difference in time, being nearly an hour, shows the difference in longitude to be nearly 15°. Find the exact difference. Is it to be added to 156° 48′ or subtracted from it, to give the longitude of B?

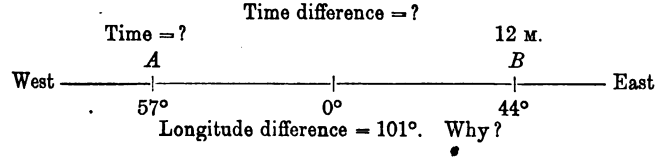
4. When it is 2 : 40 A.M. at A, in 57° 24′ west longitude, it is 10 A.M. at B. Find the longitude of B.



Longitude difference = 15° × 7⅓ = 110°.

If we go 110° eastward from A, we shall reach the prime meridian after going how many degrees and minutes? How many more degrees and minutes must we travel to reach B? Is B in east or in west longitude?

5. When it is noon at B, what is the time at A, the former being in longitude 44° east, and the latter in longitude 57° west?



Longitude difference = 101°. Why?

1057. Find the longitude or the time:

	Longitude of *A*.	Time at *A*.	Longitude of *B*.	Time at *B*.
6.	63° east	9 A.M.	54° east	?
7.	57° 25′ east	?	83° 20′ east	1 : 45 P.M.
8.	156° 48′ west	3 : 15 P.M.	?	4 : 10 P.M.
9.	?	11 : 42 A.M.	56° 25′ west	1 : 27 P.M.
10.	2° 15′ west	6 : 53 A.M.	67° 48′ east	?
11.	27° 10′ east	?	27° 10′ west	12 M.
12.	?	4 : 10 P.M.	18° 4′ east	11 : 30 A.M.
13.	74° 56′ west	3 : 50 A.M.	?	11 A.M.
14.	4′ 30″ east	8 : 47 A.M.	90° 15′ west	?
15.	?	10 : 30 P.M.	32° 30′ east	6 : 48 P.M.

SQUARE ROOT.

1058. Find the square root:

1.	$\frac{4}{25}$	5.	$\frac{49}{144}$	9.	$\frac{5929}{9025}$
2.	$\frac{9}{64}$	6.	$\frac{225}{1024}$	10.	$\frac{7921}{8649}$
3.	$\frac{169}{441}$	7.	$\frac{324}{2209}$	11.	$\frac{2304}{4225}$
4.	$\frac{36}{121}$	8.	$\frac{529}{3844}$	12.	$\frac{3025}{5041}$

NOTE. — Before extracting the square root of the following, reduce the mixed numbers to improper fractions.

13.	$12\frac{1}{4}$	16.	$4\frac{124}{225}$	19.	$156\frac{1}{4}$
14.	$11\frac{1}{9}$	17.	$6\frac{265}{324}$	20.	$264\frac{1}{16}$
15.	$2\frac{46}{49}$	18.	$10\frac{311}{625}$	21.	$102\frac{33}{64}$

1059. U. S. Civil Service. Pension Office.

1. Add the following:

79,654,321,908.25
47,776,013,705.30
92,773,331,693.25
7,774,910,236.15
44,297,793,329.37
6,105,723,266.57
132,173.63
8,857,367,397.45
42,233,001,764.86
63,347,476,074.03
3,335,602,047.90
173,827,764,501.77
96,878,956,489.68

Express in figures:

2. Six million thirty-one thousand five.

3. Five hundred million and one thousand one ten-thousandths.

Express in words:

4. $163,003,601.0044.

5. What will be the result if $\frac{3}{4}$ of $3\frac{1}{8}$ be multiplied by $\frac{5}{6}$, and the product divided by $\frac{1}{4}$?

6. John Jones agrees to pay Peter Jones one thousand thirty dollars and five cents, two years nine months and twelve days from date, and to pay interest thereon at 4 per cent. How much will be due, principal and interest, at maturity of the note? (No days of grace.)

7. What are the present worth and the true discount of $5,450, due in 9 months and twelve days, at seven per cent? (See Art. 1017.)

SPECIAL DRILLS.

1060. Find sums:

$11+55+99$	$92+18+48$	$25+84+58$	$45+56+68$
$46+22+88$	$36+71+57$	$66+15+96$	$75+34+86$
$33+76+37$	$89+28+64$	$27+19+87$	$41+65+59$
$66+42+54$	$53+47+35$	$69+73+38$	$52+39+78$

1061. Find answers:

$150-23-48$	$162-(26+61)$	$174-41-35$	$165-(28+47)$
$172-19-66$	$154-(36+44)$	$163-38-43$	$171-(32+33)$
$183-37-42$	$184-(39+35)$	$155-49-24$	$180-(18+28)$
$161-12-71$	$173-(57+17)$	$181-47-33$	$153-(45+31)$

1062. Find products:

36×31	54×51	72×71	90×89
36×29	54×49	72×69	90×91
45×41	63×61	81×81	99×99
45×39	63×59	81×79	101×99

1063. Find quotients:

$576\div18$	$448\div14$	$600\div24$	$225\div12\frac{1}{2}$
$693\div21$	$533\div13$	$675\div75$	$150\div6\frac{1}{4}$
$608\div19$	$924\div22$	$825\div75$	$825\div37\frac{1}{2}$
$848\div16$	$943\div23$	$525\div75$	$750\div62\frac{1}{2}$

1064. Find answers:

$315\times1\frac{14}{15}$	$32\times39\frac{7}{8}$	$7\frac{1}{2}\times7\frac{1}{2}$	$18\frac{2}{5}\times5\frac{1}{3}$
$378\times1\frac{17}{18}$	$45\times19\frac{14}{15}$	$8\frac{1}{2}\times8\frac{1}{2}$	$12\frac{1}{2}\times6\frac{1}{5}$
$97\frac{1}{2}\div3$	$105\frac{1}{5}\div12$	$155\frac{1}{6}\div7$	$19\frac{1}{5}\div2\frac{2}{5}$
$85\frac{1}{4}\div11$	$109\frac{1}{4}\div4$	$200\frac{2}{3}\div18$	$25\frac{2}{3}\div3\frac{1}{3}$

1065. Oral Problems.

1. Paid 92¢ for coffee, 48¢ for butter, and 18¢ for lard. How much was my bill?

2. I had $150. Spent $23 for a suit of clothes and $48 for tools. How much was left?

3. What is the area of a field 36 yd. by 31 yd.?

4. 600 hours = how many days?

5. What is the cost of a cow if I pay $630 for 15?

6. How many ounces in $29\frac{1}{2}$ lb.?

7. $109\frac{1}{4}$ lb. sugar are divided among 4 people. What is the share of each?

8. At $1\frac{9}{10}$¢ per lb., how many lb. iron can I get for $5.70?

9. What will be the cost of 51 tons iron at $17 per ton?

10. What will be the average age of 9 boys, each 12 years old, and 6 boys, each 10 years old?

11. At 42 miles per hour, how long will it take a train to go 882 miles?

12. At 25¢ per hour, what will a man earn in 18 days of 10 hours?

13. What will be the net price of an article whose catalogue price is $20.00, the discount being 90 and 10%?

14. A man had $181 in bank. What will be his balance after taking out $47 and $33?

15. How many feet in 14 rods?

16. 77 yards = how many rods?

17. How many sq. yd. are there in a floor $10\frac{2}{3}$ yd. long and $6\frac{1}{2}$ yd. wide?

18. Cost of 372 eggs at 15¢ per doz.

19. A man owns 3 farms containing 65 acres, 86 acres, and 98 acres, respectively. How many acres does he own?

20. What is the area of a piece of glass measuring $8\frac{1}{2}$ by $6\frac{1}{4}$ inches?

21. What is the value in U. S. money of 50 marks at $23\frac{8}{10}$ cents?

22. How many francs will a calf cost, if 18 are worth 630 francs?

23. A man spends $1,740 per year. What is the average amount spent per month?

24. What will be the cost of 16 lots at $1,250 each?

25. At $87\frac{1}{2}$ each, how many horses can be bought for $5,600?

TABLE.

1066. The following shows the value of goods exported to Europe from the United States during 1891.

Find the missing value, and the per cent received by each country (2 places of decimals):

	Countries.	Value.	%
1.	Great Britain and Ireland,	$441,599,807	
2.	Germany,	91,684,981	
3.	France,	59,826,739	
4.	Belgium,	26,694,150	
5.	Netherlands,	23,816,814	
6.	Spain,	14,607,893	
7.	Russia in Europe,	7,764,012	
8.	Portugal,	4,986,909	
9.	All others,		
		$697,614,106	100%

1067. Brooklyn Civil Service. Examination for Clerks.

1. Add the following amounts:

421,874.33
99,887.76
10,371,289.44
4,875,643.21
8,945.01
765,432.10
37,965.57
8,573,749.68
8,846,357.11
6,328,573.76
469.12
23,475,386.25

2. From $2,736,870.23 subtract $1,867,760.34.

3. Multiply 8,632,092 by 60,780.

4. Divide 7,326,543 by 7,428.

5. Write in words: $13,341,083.91; $\frac{64}{250}$; and $6\frac{2}{17}$.

6. Write in figures: three billion, eight million, four hundred thirty-five thousand twenty-eight dollars, twenty-four cents and seven mills.

7. (*a*) What would be the tax upon property assessed at $39,250, at the rate of twenty-eight dollars twelve cents and eight mills per thousand dollars?

(*b*) The tax you have just computed was payable December first, but was not overdue until January first. If it had been paid during December, a rebate of $7\frac{3}{10}\%$ per annum, to be computed from the date of payment to January first, would have been allowed. If it had been paid after January first, interest at the rate of 9% per annum, to be computed from December first to date of payment, would have been added. What would have been the amount actually paid if the tax had been paid on December 21st? What, if paid on January 15th?

8. Compute simple interest on $78,532.16, at $5\frac{1}{2}\%$, from Feb. 3, 1889, to May 15, 1891.

9. Reduce $\frac{4}{5}$ of $\frac{5}{12}$ of $\frac{3}{7}$ to a simple fraction.

10. Make out a receipted bill for the following articles, sold by Richard Roe to John Doe on May 1, 1891:

$25\frac{2}{3}$ lb. of butter @ $12\frac{1}{2}¢$ per lb.
$12\frac{1}{2}$ bbl. of flour @ $4.75 per bbl.
$35\frac{1}{2}$ lb. of coffee @ $25\frac{2}{3}¢$ per lb.
$10\frac{2}{5}$ lb. of rice @ $3\frac{1}{2}¢$ per lb.

PROPORTION.

1068. A ratio is generally expressed by the sign (:). This is another form of the division sign (÷).

1069. Two equal ratios form a *proportion.*

$3 \div 9 = 13 \div 39$;
or, $3 : 9 = 13 : 39$;
or, $3 : 9 :: 13 : 39$.

1070. Supply missing term:

1. $\frac{1\frac{3}{4}}{6\frac{2}{3}} = \frac{21}{?}$
2. $3\frac{1}{2} \div 16 = \frac{7}{8} \div x$
3. $5 : 7 :: 12\frac{1}{2} : x$
4. $\frac{14}{45} = \frac{x}{90}$
5. $3 \div x = 12 \div 20$
6. $? : 19 :: 28 : 76$
7. 1 lb. 1 oz. : 2 lb. 4 oz. : : $17¢ : x¢$.
8. 3 qt. 1 pt. ÷ 1 gal. = $x¢ \div 80¢$.
9. 4 bottles : x bottles = 6 pints : 15 pints.
10. x men : 9 men = 16 acres : 36 acres.

1071. The first and the last term of a proportion constitute the *extremes;* the second and the third, the *means.*

$$5 : 15 :: 9 : 27$$

5 and 27 are the extremes. 15 and 9 are the means. The proportion is read: 5 is to 15 as 9 is to 27.

1072. The proportion $3 : 4 :: x : y$ may be written $\frac{3}{4} = \frac{x}{y}$. Clearing of fractions, we have $3y = 4x$; *i.e.*, the product of the extremes is equal to the product of the means.

Solve the following:

1073. Make the product of the extremes equal to the product of the means, after canceling any factor common to an extreme and a mean.

1. $x : 5 :: \overset{3}{\cancel{12}} : \cancel{4}$ $\quad x = 15$, *Ans.*
2. $3 : x :: \cancel{5} : \overset{6}{\cancel{30}}$ $\quad x = 18$, *Ans.*
3. $3 : \cancel{4\frac{1}{2}} :: x : \overset{4}{\cancel{18}}$
4. $3 : 4\frac{4}{5} :: 5 : x$
5. $x : 15 :: 4 : \frac{8}{7}$
6. $22\frac{1}{9} : x :: 18 : 13$
7. $\frac{5}{9} : \frac{11}{21} :: x : \frac{11}{24}$
8. $x : \frac{1}{2} :: 2 : 7$
9. $\frac{2}{5} : x :: \frac{1}{4} : \frac{3}{5}$
10. $\frac{1}{9} : \frac{3}{9} :: x : 2\frac{1}{4}$
11. $1 : \frac{5}{8} :: 1\frac{3}{5} : x$
12. $x : \frac{5}{9} :: 11 : 3\frac{1}{8}$
13. $x : 9 :: 4 : x$

1074. Oral Problems.

1. If 9 eggs cost 25¢, what will 3 dozen cost?
2. If 7 lb. flour cost 23¢, what will be paid for 49 lb.?
3. For $5 I can get 12 straw hats. How many can I get for $20?
4. A wheel makes 75 revolutions in 5 minutes. How many does it make in an hour?

5. $100 principal gives $6 interest. How much will be the interest of $450 principal?

6. A merchant pays 75¢ freight for 125 lb. of merchandise. How much will be the freight on 1,000 lb. at the same rate?

7. A locomotive goes 3 miles in 4 minutes. How far does it go in an hour?

8. 4 horses can eat a certain quantity of hay in 10 months. How long will it last 20 horses?

9. 12 men can do a piece of work in 15 days. How long will 36 men require?

10. 15 kilos cost 270 francs. What will be the cost of 5 kilos?

1075. Slate Problems.

NOTE. — Solve by proportion or in any other way.

1. If 9 cows cost $267, what will be the cost of 36 at the same rate?

In solving such examples by proportion, we say

$$9 \text{ cows} = \$267,$$

$$36 \text{ cows} = \$x.$$

The ratio of the cost, $267 : x$, must be the same as the ratio of the number of cows, $9 : 36$. Making the proportion, we have

$$9 : 36 :: 267 : x.$$

Canceling, $x = \$1{,}068.$

2. 7 bbl. sugar cost $104.32. Find the cost of 42 bbl.

3. A wheel makes 248 revolutions in 5 minutes. How many does it make in 1 hour 20 minutes?

4. A locomotive goes 1.8 kilometers in 4 minutes. How far does it go in an hour?

5. From 9 kilos (kilogrammes) of yarn are made 42 meters of cloth. How many meters of cloth can be made from 165 kilos of yarn?

How many kilos of yarn are needed for 196 meters of cloth?

6. If 17 men receive $357 for a week's work, how much should 24 men receive?

7. If 17 men take 27 days to finish some work, how long would it take 54 men?

17 men take 27 days

54 men take x days

17 or 54 men ? : 17 or 54 men ? :: 27 days : x days.

8. When a sum of money is divided among 48 persons, each receives $27.50. How much would each receive if the same sum were divided among 16 persons?

9. For $85 I can purchase 238 yards of dress goods. How many yards can I purchase for $5?

10. A can do a piece of work in 6 days, B can do it in 7 days. If B's wages are $2.10 per day, how much should A receive per day?

COMMERCIAL DISCOUNT.

1076. Oral.

When the list price is $1, what is the net price after the deduction of each of the following discounts?

1. 30 and 20%
2. 40 and 10%
3. 25 and 40%
4. 50 and 10%
5. 40 and 20%
6. 10 and 5%
7. 20 and 20%
8. $33\frac{1}{3}$ and 10%
9. 20 and 15%
10. 30 and 15%

1077. What single discount is equal to each of the following double discounts?

11. 30 and 30%
12. 20 and 25%
13. 25 and 20%
14. 15 and 30%
15. 40 and 30%
16. 30 and 10%
17. 40 and 5%
18. 50 and 20%
19. 40 and 15%
20. 50 and 15%

1078. Slate Exercises.

Which is the better discount for the buyer?

21. 30 and 20%, or 40 and 10%.
22. 50 and 10%, or 40 and 20%.
23. 20 and 20%, or 30 and 10%.
24. 20 and 15%, or 30 and 5%.
25. 30 and 15%, or 25 and 20%.
26. 30 and 30%, or 50 and 10%.
27. 40 and 30%, or 20 and 50%.
28. 40 and 5%, or 30 and 15%.
29. 20 and 50%, or 60 and 10%.
30. 40 and 15%, or 30 and 25%.

1079. Find the value of x:

1. List price, \$250; selling price, \$$x$; rate, 40 and 10%.
2. List price, \$800; selling price, \$684; rate, x and 5%.
3. List price, \$$x$; selling price, \$90; rate, $33\frac{1}{3}$ and 10%.
4. List price, \$600; selling price, \$378; rate, 30 and x%.
5. List price, \$16; selling price, \$$x$; rate, 30 and 20%.

6. List price, x¢; selling price, 27¢; rate, 50 and 10%.

7. List price, \$5; selling price, \$3.20; rate, x and 20%.

8. List price, \$$x$; selling price, \$73.50; rate, 30 and 30%.

9. List price, \$200; selling price, \$$x$; rate, 25 and 20%.

10. List price, \$1.50; selling price, 60¢; rate, 50 and x%.

SQUARE ROOT.

1080. Find the square root of 425,104.

	6 5 2	
	42'51'04	
12|5	6 51	
130|2	26 04	*Ans.* 652.

1081. In finding any figure of the root after the first, we multiply the other figure or figures by 2 for a trial divisor.

1082. Find the square root of 20,857,489.

	4 5 6 7	
	20'85'74'89	
8|5	4 85	
90|6	60 74	
912|7	6 38 89	*Ans.* 4,567.

1083. Find the square root of

1. 64,516
2. 734.41
3. 1.8769
4. 718.24
5. 14.1376
6. .702244
7. 264.7129
8. .23775376
9. .093636
10. .004761

1084. Slate Problems.

1. What is the profit on 9 boxes of oranges, each containing 20 dozen, bought at \$1.10 per hundred and sold at the rate of 18 for 25¢?

2. How long will it take a train to go 176 miles at the rate of 3,520 feet per minute?

3. If .0375 of an acre of land is worth \$9, what is $\frac{5}{32}$ acre worth?

4. At £1 1*s.* 7*d.* per barrel, how many barrels of flour can be bought for £161 17*s.* 6*d.*?

5. A, B, and C buy a house for \$7,500. A furnishes \$2,000; B, \$2,500; C, the remainder. The yearly rent, less expenses, is \$576. To what amount is each entitled?

6. If 580 tiles, each 6 inches square, will cover a certain area, how many tiles, each 4 inches long and 3 inches wide, will be needed to cover the same area?

7. A man receives \$1,500 commission on his yearly sales. What is the amount of his sales, if he is allowed $\frac{1}{4}$ per cent commission?

8. At what rate per cent will \$360 produce \$3.06 interest in 2 months 12 days?

9. Find the square root of 25.00400016.

10. What will be the capacity, in gallons, of a tank 9 feet long, 6 feet 8 inches wide, and 6 feet 5 inches deep?

11. What decimal multiplied by 312.5 will give the sum of $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, .09375, and 2.46?

12. A dealer bought a lot of coal at \$4.95 per ton. What was the total cost if he gained \$142.50 by selling it at \$5.25 per ton?

13. Find the value of $\frac{2\frac{1}{3}+4\frac{5}{12}}{1\frac{3}{4}\times 3}-\frac{1}{6}$ of $6\frac{5}{7}$.

14. The front wheel of a wagon measures 13 feet in circumference. What is the distance traveled in miles, rods, yards, etc., when the wheel has made 527 revolutions?

15. Write in words .349, 300.049, $\frac{340}{9000}$, $300\frac{40}{9000}$.

16. If a bar of silver weighing 4 lb. 6 oz. 12 pwt. is worth £13 8*s*. 4*d*., what is the value (in English money) of a similar bar weighing 7 lb. 9 oz. 12 pwt.?

17. A and B form a partnership. A furnishes \$5,000; B, \$10,000. During the year A draws \$1,500 of the profits and B draws \$1,000. At the end of the year the entire business is disposed of for \$20,000. What amount should each receive?

18. What per cent is gained on an article bought for 20 per cent less than its value and sold for 20 per cent more than its value?

19. A person loans \$750 to M and \$1,200 to N at the same rate. From the latter he receives half-yearly \$9 more interest than from the former. What is the annual rate of interest?

20. A 4-months note for \$375, drawn March 19, was discounted at a bank June 4. Find the proceeds. Rate, 6%.

21. M can do a piece of work in 4 days, N can do it in 5 days, O in 6 days. How long will it take the three together to do the work?

REVIEW FRACTIONS.

1085. Slate Exercises.

1. Divide the sum of $6\frac{3}{7}$ and $1\frac{7}{8}$ by the difference between $2\frac{1}{5}$ and $3\frac{1}{8}$.

2. What is the difference between the sum of $\frac{3}{7}$ and $\frac{5}{8}$ and the product of $\frac{4}{5}$ and $\frac{7}{12}$?

3. What is the product of the sum and the difference of $4\frac{1}{4}$ and $6\frac{1}{3}$?

4. Subtract $\frac{2}{3}$ of $\frac{1}{4}$ from $\frac{11}{18}$; and find the value of $\frac{3}{11}$ of 16*s.* 6*d.*

5. Add $7\frac{5}{9}$, $\frac{3}{4}$ of $\frac{7}{12}$, $\frac{5}{8}$ of $7\frac{2}{3}$, and $\frac{19}{24}$.

6. Reduce $\frac{3}{5}$ of a sq. rod to the fraction of an acre, and find the value of $\frac{7}{64}$ of a ton in pounds and ounces.

7. Reduce $\frac{696}{18005}$ to its lowest terms, and $\frac{3\frac{1}{3}-2\frac{1}{5}}{3\frac{1}{3}+2\frac{1}{5}}$ to its simplest form.

8. Add $\frac{1}{2}$, $\frac{5}{9}$, $\frac{3}{4}$, and $\frac{1}{3}$; multiply the sum by $\frac{9}{22}$; and subtract the product from 1.

9. Find the value of $9\frac{4}{11}$ meters at $4\frac{5}{9}$ francs per meter.

10. Divide $2\frac{1}{4}$ by $3\frac{1}{2}$, and add the quotient to $\frac{5}{14}$.

11. Multiply $2\frac{2}{13}$ by $16\frac{5}{7}$, and divide the result by $1\frac{1}{2}$ of $2\frac{2}{5}$.

12. Reduce 7*s.* 6*d.* to the fraction of a £, and 7 hr. 12 min. to the fraction of a day.

13. Reduce to its simplest form $\frac{2+\frac{1}{6}\text{ of }5\frac{1}{7}}{\frac{5}{7}\text{ of }\frac{1}{2}}$.

14. Add together £$\frac{5}{8}$ and $\frac{1}{7}$ of $5\frac{5}{9}$ shillings.

15. What fractional part of 7 A. 127 sq. rd. is 5 A. 81 sq. rd.?

16. What must be added to $\frac{3}{7}$ of $\frac{5}{8}$ to make it equal to $\frac{9}{10}$ of $3\frac{3}{4}$?

17. $\frac{2}{5}$ of a number is 148. What is the number?

18. If $\frac{5}{8}$ of a field is worth $325, what is the field worth?

19. If $\frac{2}{3}$ of a house is worth $4,900, what is the value of $\frac{1}{8}$?

20. If $\frac{3}{16}$ of a ship is worth £1,273 2*s.* 6*d.*, what is $\frac{5}{32}$ worth?

$\frac{3}{16} = \frac{6}{32} =$ £1,273 2*s.* 6*d.*
Deduct $\frac{1}{32} =$ _____
$\frac{5}{32} =$

1086. Questions from U. S. Civil Service Examinations.

1. A merchant imported from Bremen 32 pieces of linen of 32 yards each, on which he paid for the duties, at 24 per cent, $122.16, and other charges to the amount of $40.96. What was the invoice value per yard, and the cost per yard after duties and charges were paid?

2. A man sold his house and lot for $12,500, payable $4,000 cash, $3,500 in 9 months, $2,600 in 18 months, and the balance in 28 months, with 6 per cent simple interest. What was the whole amount paid?

3. A garrison of 1,200 men is provisioned for 100 days. At the end of 30 days, 600 men are withdrawn, and at the end of 60 days, 900 men are added. How long will the provisions last?

4. The duty on certain colored cottons is $5\frac{1}{2}¢$ per square yard and 20 per cent *ad valorem*, or upon the value. What would be the duty on 267 pieces of such cottons, 30 inches wide, each piece containing 37 lineal yards, and costing $7¢$ per lineal yard?

5. What will be the result, if $\frac{1}{5}$ of $\frac{3}{4}$ of $3\frac{1}{8}$ be multiplied by $\frac{1}{2}$ of itself, and the product be divided by $\frac{1}{2}$?

6. A collector of internal revenue deposited in the treasury $762,742.50, retaining $2\frac{1}{2}$ per cent of the amount collected. What amount did he collect?

7. What will it cost to transport 1,000 pounds of mail matter 1,000 miles, at $1.00 per 100 pounds per 100 miles?

8. What is the duty on 25 tons 2 cwt. 3 qr. of iron at $40 per ton? (1 ton = 2,240 lb.)

9. An importer sold a part of a cargo of tea at 30 cents a pound and made a profit of 20 per cent. What per cent did he make on the remainder of the cargo, which he sold at 40 cents a pound?

DOMESTIC EXCHANGE.

1087. Slate Exercises.

Find the value of x.

(Do not allow days of grace in the case of "sight" drafts.)

	Face of Draft.	Rate of Exchange.	Term.	Rate of Interest.	Cost of Draft.
1.	\$100	20¢ per M. premium	sight		x
2.	x	par	6 days	6%	\$499.50
3.	\$1,800	x	60 days	6%	\$1,778.85
4.	x	$\frac{1}{4}$% premium	sight		\$701.75
5.	\$200	\$1 per M. discount	30 days	6%	x
6.	\$600	$\frac{3}{8}$% premium	x	6%	\$598.95
7.	\$1,000	75¢ per M. discount	x	6%	\$999.25
8.	\$1,200	par	93 days	x	\$1,178.30
9.	\$800	x	24 days	6%	\$796.80
10.	\$400	\$2 per M. premium	9 days	x	\$400.30

APPLICATIONS OF SQUARE ROOT.

1088. Slate Problems.

1. How many inches in the side of a square table top containing 529 square inches?

2. The surface of a square piece of board contains 3 sq. ft. 97 sq. in. What is the length of one side in feet and inches?

(Reduce area to square inches.)

3. How many rods long is a square field containing 90 acres? How many yards of fence would be needed to enclose it?

4. Land surveyors use a measure called a chain. What is its length in feet, 10 square chains being equal to an acre?

It is subdivided into 100 "links." Find the length of a link in inches and decimal.

5. The surface of the six equal faces of a cube is 1,350 sq. inches. What is the length of the cube?

6. Carefully construct a right-angled triangle, base 4 inches, perpendicular 3 inches. Measure the hypotenuse.

Take the square of the length of each side, and endeavor to show the relation between the square of the hypotenuse and the squares of the other two sides.

7. Construct a right-angled triangle, base 3 in., perpendicular $1\frac{1}{4}$ in. Measure the hypotenuse, and see if the relation between this hypotenuse and the other two sides of this triangle is the same as that found in the other triangle.

8. A right-angled triangle has a base 12 inches long; its perpendicular is $3\frac{1}{2}$ inches. What is the length of the hypotenuse?

9. The hypotenuse of a right-angled triangle is 25 inches; its perpendicular is 7 inches. What is the base?

10. The base of a right-angled triangle is 12 feet; the hypotenuse is 13 ft. Find the perpendicular.

1089. Draw a right-angled triangle (Fig. 1). Upon each side construct a square (Fig. 2). From the upper portion of the

Fig. 1.

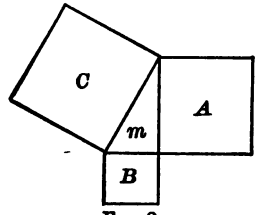


Fig. 2.

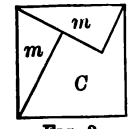


Fig. 3.

Fig. 4.

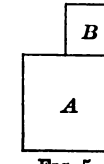


Fig. 5.

largest square, C, cut a right-angled triangle of the same dimensions as those of the original triangle m. Cut another triangle of the same dimensions from the left-hand portion (Fig. 3). Place one of these triangles below the remainder of the square C, and the other at the right, as in Fig. 4, and the resulting polygon will be exactly equal in surface to the two squares A and B (Fig. 5).

1090. Wilmington, Del., Public Schools. Examination Questions.

1. What is the bank discount on a 60-days note for $800?

2. What is the amount of $520, for 3 years and 6 months?

3. A note for $850 was given January 1, 1880. The payments on it were: July 1, 1880, $100.62; December 1, 1880, $15.28; August 13, 1881, $175.75. What was due January 1, 1882, by the U. S. rule?

4. A note of $900, dated February 28, 1887, was endorsed as follows: July 12, 1887, $500; August 30, 1887, $200; February 28, 1889, $25. What was due September 1, 1889?

5. A 60-days note was made and discounted at a bank on the 10th of June; the proceeds of this note were $340. What was the face of it?

6. What is the cost of a 60-days draft for $500, at a discount of 2 per cent?

7. What is the compound interest of $325, at 5 per cent, for 3 years and 6 months?

8. What is the exact interest of $200, for 73 days, at 5 per cent? (Take 365 days to the year.)

Express five-eighths of one per cent by a common fraction in its lowest terms, then by a pure decimal, and again with the sign of per cent.

9. Write a proportion. Then write some of the principles of proportion, and show that they are true of the proportion you have written.

Solve by proportion, giving reasons:

10. If $\frac{2}{3}$ of an acre of land cost \$100, how much land will \$750 buy?

If 5 men can do a piece of work in 13 days, in how many days can 31 men do it?

COMPOUND INTEREST.

1091. Slate Exercises.

1. Find compound interest of \$2,048, for 3 years, at 5%, interest compounded semi-annually.

	\$2,048	Do not use unnecessary figures. $\frac{1}{10}$ each 6 mo.
first ½ year	51.20	
	\$2,099.20	
second ½ year	52.48	Carry to only 4 places of decimals.
	\$2,151.68	
etc.	etc.	

2. Compound interest of \$1,864, at 4%, for 2 years, interest compounded quarterly.

3. Compound interest of \$1,500, at 6%, for 3 years, interest compounded semi-annually.

4. Amount of \$1,000 at compound interest for 5 years, at 5%, interest compounded annually.

5. Amount of \$800 at compound interest, for 3 years, at 3%, interest compounded semi-annually.

		\$800.00
first ½ year	1%	8.00
	½%	4.00
		\$812.00
second ½ year	1%	8.12
	½%	4.06
		\$824.18
third ½ year	1%	8.2418
	½%	4.1209
		\$836.5427
	etc.	etc.

STOCKS AND BONDS.

1092. Slate Problems.

Brokerage is calculated on the *par* value. The dividends are based on the *par* value.

1. Find the cost of 240 shares mining stock, par value \$10, at 87¾, brokerage ⅛%.

2. Paid \$11,460 for 120 shares R. R. stock, par value \$100, brokerage ¼%. What was the value of the stock per share?

3. Bought 150 shares canal stock at 87½, brokerage ¼%, paying for it \$5,265. What is the par value per share?

4. How much brokerage is paid by the buyer of 275 shares bank stock, par value \$100, brokerage ⅛%?

5. A broker sells for a customer 200 shares stock, par value \$25, at 102¼. If he retains ⅛% brokerage, how much does he pay over to the former owner of the stock?

6. A man buys 60 shares bank stock, par value \$100, at 450, no brokerage. If the annual dividend is 18%, what is his income therefrom? What per cent does he receive on his investment?

7. A manufacturing corporation makes \$20,000 a year over all expenses. The stock consists of 4,000 shares, par value \$50. What rate of dividend can be declared?

What per cent on his investment does a man receive who has bought his stock at 175, no brokerage?

8. A railroad company's stock consists of 10,000 shares, par value \$100. Its profits for the year are \$47,500, out of which must be paid the interest for the year on \$200,000 worth of bonds, at 5%. What rate of dividend can be paid the stockholders?

9. A capitalist bought 360 shares stock, par value \$25, at 168½. He paid therefor, including brokerage, \$15,176.25. What was the rate of brokerage?

10. A broker sold 250 shares, par value \$100, at $107\frac{1}{4}$. He deducted brokerage, and paid over the proceeds, amounting to \$26,875. Find the amount of the brokerage and the rate per cent.

1093. St. Paul Public Schools. Examination Questions.

1. Define a corporation. Name two facts a charter specifies. Define stock, share, shareholders, stock broker, dividend, and assessment.

2. A, having a farm of 109 acres, which rents for \$681.25, sells the same for \$125 per acre, and invests the proceeds in U. S. 6's @ $108\frac{3}{4}\%$, brokerage $\frac{1}{4}\%$. Will his yearly income be increased or diminished, and how much? (U. S. 6's are 6% bonds.)

3. I loaned a man a certain sum of money; at one time he paid me \$59.57, which was $16\frac{2}{3}\%$ of the whole sum loaned to him. What was the sum? Analyze.

4. A man was offered \$3,675 in cash for his house, or \$4,235 in 3 years, without interest; he accepted the latter offer. How much did he lose, money being worth 7%? (Art. 1017.)

5. Which is the better investment: U. S. 5's @ $98\frac{1}{4}\%$, or U. S. 6's @ $108\frac{3}{4}\%$, brokerage $\frac{1}{4}\%$ in each case? Analyze.

6. If a traveler finds his watch one hour slow, in which direction is he traveling? How far has he traveled?

7. If the distance between two places is 75°, and it is 12 M. at the eastern point, what is the time at the western point?

8. What are the proceeds of a note of \$2,400, dated June 20, due in 3 mo., @ 9%, and discounted Aug. 8 at 6%?

9. What is the face of a 60-days note, whose proceeds when discounted at date, at 6%, equal \$4,749.60?

10. Prove the ninth.

SQUARE ROOT.

1094. Slate Exercises.

Find square roots to three decimal places:

1. 7	**3.** 38	**5.** 350	**7.** 807	**9.** 1,874
2. 14	**4.** 74	**6.** 758	**8.** 1,384	**10.** 4,000

1095. Oral Exercises.

What is the square of .1? Of .3? Of .11? Of .12?

1096. How many decimal places in the first two answers? In the last two?

$\sqrt{.01} = ?$ $\sqrt{.09} = ?$ $\sqrt{.0121} = ?$ $\sqrt{.0144} = ?$

1097. Slate Exercises.

Find square roots to three decimal places:

1. .10	**5.** .4	**9.** 3.6	**13.** 4.9	**17.** .121
2. .40	**6.** .9	**10.** 1.60	**14.** 6.4	**18.** .144
3. .90	**7.** 1.6	**11.** 2.50	**15.** 8.1	**19.** .169
4. .1	**8.** 2.5	**12.** 3.60	**16.** 10.0	**20.** .196

1098. U. S. Civil Service. Inspectors' Examination. Post-Office Department.

1. Express the following in figures, using such monetary signs as may be necessary:

Ninety-six million seven hundred six dollars six cents and two mills.

2. Express the following in words: (*a*) MCCCXCII. (*b*) $530,763,120,167.

3. Multiply 540.101 by 10.95, and divide the product by .365.

4. Add the following numbers:

$72{,}756\frac{1}{2}$; $63{,}429\frac{1}{4}$; $173{,}287\frac{4}{5}$; $55{,}418\frac{14}{20}$; and $677\frac{2}{16}$.

Subtract from their sum 193,540.

5. The revenues of a post-office in 1884 were \$1,260. In 1885 they were \$1,146.60. What was the per cent of decrease?

6. The following stock of postage stamps, stamped envelopes, etc., was found on hand at Clinton (Neb.) post-office at close of a quarter:

29 sheets 1¢ stamps }
25 " 2¢ " } 100 stamps to each sheet.
33 " 4¢ " }

62 " 10¢ " }
37 " 15¢ " } 50 " " " "

38 boxes 1¢ postal cards, 500 cards in each box.

12 " newspaper wrappers, 250 in each box, @ \$11.00 per thousand.

12 boxes stamped envelopes, 500 in each box, and 368 envelopes (part of a box), @ \$21.80 per thousand.

State the total value of this stock.

7. An inspector found that the postmaster at —— had, on the 5th day of July, 1881, unlawfully paid \$467.50 to one AB, out of the moneys of the government. This sum was recoverable with interest at 6 per cent from the time it was received until time of payment. On the 19th of December, 1885, the inspector demanded restitution of this sum unlawfully paid, and AB, to avoid suit, paid the amount due. What sum must the inspector collect and deposit as postal revenue to satisfy the demand?

8. Five registered letters, containing, respectively, \$150, \$202, \$78, \$104, and \$116, were rifled of their contents by a postal clerk. When arrested, he made restitution of \$523.25, which an inspector was required to apportion pro rata among the senders of the letters. What amount should he pay to each out of the sum recovered?

9. An inspector, being entitled to a salary of $1,400 per annum, was on duty twelve days. He was entitled to $4 per diem for his traveling expenses, and to reimbursement for $12.50 paid as stage fare, $17.40 steamboat fare, and $3.25 for telegraphing on government business during that time. State his account for services per diem, and expenses for the twelve days. It is assumed that the twelve days' service was rendered in the month of November, occurring in a quarter which has ninety-two days, the quarterly pay being $350.

10. Under the law the salary or compensation of postmasters at fourth-class offices consists of —

(1) All the box rents, if there are any.

(2) Commissions on the amount of postage-due stamps, ordinary postage stamps, stamped envelopes, postal cards, and newspaper and periodical stamps canceled on matter actually mailed at their offices, and on amounts received from sales of waste paper, dead newspapers, printed matter, and twine, at the following rates: On the first $50 or less, 100 per cent; on the next $100 or less, 60 per cent; on the next $200 or less, 50 per cent; on the excess above $350, 40 per cent.

In no case is the compensation of a postmaster of this class to exceed $250 in any one quarter.

During the year, a postmaster's account was as follows:

Quarter.	Box rent.	Cancellation of 2-cent stamps.	Cancellation of 1-cent stamps.	Cancellation of postal cards.	Cancellation of stamped envelopes.	Sales of waste paper, etc.	Sales of twine.
First	$77.00	$46.00	$22.00	$6.17	$47.00	$0.67	$3.12
Second	93.00	71.55	13.09	7.18	39.47	1.69	2.04
Third	87.19	53.04	17.06	9.49	27.96	.74	3.19
Fourth	85.15	39.45	18.32	8.27	33.40	.95	4.15

What was his compensation each quarter? What was his salary for the year?

MEASUREMENTS.

1099. Slate Exercises.

Find the missing side of each of the following ten right-angled triangles:

1. Base, 15; perpendicular, 8; hypotenuse, x.
2. Base, 35; perpendicular, x; hypotenuse, 37.
3. Base, x; perpendicular, 15; hypotenuse, 39.
4. Base, 20; perpendicular, 21; hypotenuse, x.
5. Base, x; perpendicular, 45; hypotenuse, 53.
6. Base, 56; perpendicular, x; hypotenuse, 65.
7. Base, 55; perpendicular, 48; hypotenuse, x.
8. Base, x; perpendicular, 14; hypotenuse, 50.
9. Base, 63; perpendicular, x; hypotenuse, 65.
10. Base, 112; perpendicular, 15; hypotenuse, x.

1100. Slate Problems.

11. One parallel side of a trapezoid measures 160 yd., the other measures 200 yd., the area is 32,400 sq. yd. Find the perpendicular.

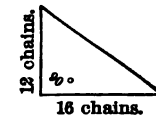
12. One parallel side of a trapezoid is 20 rods, the perpendicular is 15 rods, the area is 225 sq. rods. Find the length of the other parallel side.

13. One parallel side of a trapezoid measures x rods, the other measures $x + 6$ rods, the perpendicular is 10 rods, the area is 150 sq. rods. Find the length of the parallel sides.

14. Find the area in acres of a right-angled triangle, the length of the sides being 24 rods, 7 rods, 25 rods.

15. A court yard 84 ft. by 36 ft. is to be paved with flagstones measuring 6 ft. by 3 ft. How many stones will be needed? What will be the cost of the work at $1.25 per sq. yd.?

16. How many rods of fence will be needed to enclose the field shown in the diagram?

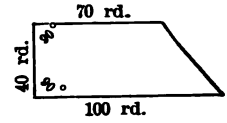


1 chain = 66 feet.

NOTE. — A right angle contains 90 degrees.

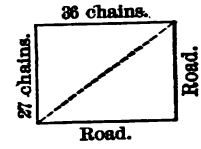
17. Find the length of the fourth side of the following piece of ground.

How many yards of fence are needed to enclose it? How many acres does it contain?



18. What is the length of the diagonal of a rectangular field 90 yd. wide, 120 yd. long?

19. The dotted line in the accompanying diagram indicates a path through the field. How many yards are saved by taking the path instead of following the road?



20. Find the length (in rods and a decimal) of the diagonal of a square 40-acre field.

REVIEW.

1101. Oral Problems.

To the following ten problems the wrong answers are very frequently given.

1. Sold a horse for $250, losing $50. What is the loss per cent?

2. If 3 boys solve 3 problems in 3 minutes, how long will it take 6 boys to solve 6 problems?

3. Two boys go fishing; one brings 7 cakes for lunch, the other brings 5 cakes. A third boy joins them at noon, and pays 12¢ for his share of the dinner. How should the first two divide the money received?

4. If 100 per cent is gained by selling an article for $1, how much would be gained by selling it for $2?

5. A boy had a slate 5 inches by 7 inches. He buys one twice as large. Give the dimensions of the new slate.

6. A man wishes to put up on the front of his lot a fence 30 feet long. If the posts are 6 feet apart, how much will they cost at 25¢ each?

7. One-half the money received by a newsboy is profit. What per cent does he make?

8. 50 per cent of a number multiplied by 30 per cent of the same number equals 60. What is the number?

9. Three-fourths per cent of a number is 90. What is the number?

10. An importer receives some cases of goods numbered consecutively. How many cases are there, if the number of the first is 28, and of the last 75?

1102. Slate Problems.

1. If a bar of silver weighing 2 lb. 3 oz. 6 pwt. is worth £6 13*s.* 7*d.*, what is the value in English money of a similar bar weighing 15 lb. 7 oz. 4 pwt.?

2. A quantity of provisions would last a ship's crew 20 days, allowing each man 2 lb. 4 oz. daily. What should each man be allowed so as to make the provisions last 4 days longer?

3. If 40 men are able to do a piece of work in 10 hours, how many extra men must be employed to finish it in 8 hours?

4. If it requires 40 yd. carpet 2 ft. 9 in. wide to cover a floor, how many yards of carpet 2 ft. 6 in. wide would be needed?

5. How long will it take a train to go 112 miles, at the rate of 46 miles in 1 hour 20 min. 30 sec.?

6. Change 1,759 yards to rods, yards, etc.

7. If a beam 5 ft. 6 in. long, 10 in. wide, and 8 in. thick, weighs 924 lb., find the length of another beam of the same material which weighs 3,024 lb., and whose end is a square foot.

8. A field 110 yd. long and 44 yd. wide contains an acre. What is the area of a field 220 yd. long and 88 yd. wide? Of one 440 yd. long and 176 yd. wide?

9. A ship with a crew of 32 men has provisions that will serve for 45 days, at a daily allowance of 3 lb. for each man. If it then picks up another vessel's crew consisting of 16 men, what must be the daily allowance, to make the provisions last for 40 days?

10. If a steel bar 12 ft. long, 4 in. broad, and $2\frac{1}{2}$ in. thick, weighs 480 lb., what is the weight of another steel bar 18 ft. long, 3 in. broad, and 2 in. thick?

11. If 8 horses eat 13.5 bushels of oats in 9 days, how many days will 15.75 bushels last 14 horses?

12. A person deposits in two banks \$750 and \$1,200, respectively, at the same rate. The latter sum draws \$18 more interest per year than the former. What is the rate per cent?

13. Two men have saved \$2,000 each. One has loaned \$1,400, at 4%, and the remainder at 5%. What rate must the other man receive for his money in order to get the same interest?

14. I owe \$8,625, payable in 3 years 4 months. I have at present \$7,500. What rate of interest must I receive to pay my debt at maturity?

15. A person loans $\frac{1}{2}$ of his capital at 5%, and the other half at 4%. He receives annually \$40 more interest from the former than from the latter. What is his capital?

16. A certain sum loaned at 4% produces \$30 less interest than a sum \$400 greater, loaned at 5%. How much is loaned at each rate?

17. A capitalist has placed $\frac{4}{5}$ of his money at 4%, and the remainder at 5%. His income is $2,940 per year. What is his capital?

18. Change 13,576 inches to rods, etc.

19. Three men buy a lot for $600. After selling it A receives $220 as his share of the proceeds, B receives $280, and C $300. How much did each invest originally?

20. D receives $\frac{1}{5}$ of a sum of money, E $\frac{1}{3}$, and F the remainder. E's share is $90 more than D's. What is the share of F?

21. A man receives $593.70 as the proceeds of his note. 63 days thereafter he pays the bank $600. What rate of interest has the bank charged on the $593.70 loaned?

What is the rate of bank discount on the $600 note?

22. A tank is fed by two pipes, one of which can fill it in 2 hours, and the other in 3 hours. A third pipe can empty it in 1 hour. If, when the tank is full, the supply pipes and the exhaust pipe are all set to work, in what time will it be emptied?

1103. San Francisco Public Schools. Examination Questions.

1. (*a*) March 15, 1890, John Doe hired of Richard Roe $300, and gave his note payable on demand, with interest at 7 per cent a year. Write a promissory note for John Doe and sign his name to it.

(*b*) John Doe paid this note Jan. 1, 1892. Amount paid?

2. Multiply 2.5 by 2.5; divide the product by .5; multiply by .002; multiply by .025; divide by .000005; divide by .2; find $\frac{1}{4}$ of 1 per cent of the last quotient.

3. (*a*) Add $\frac{5}{7}$, $\frac{2}{3}$, and $\frac{4}{5}$.

(*b*) Add $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{12}$, and $\frac{3}{16}$.

(*c*) 144 is $\frac{4}{5}$ of what?

(*d*) Divide $\frac{3}{4}$ by the decimal .02.

(*e*) Multiply $5.00 by $5.00.

4. You sell to John Smith, to-day, the following:

25 lb. Coffee	@	$18\frac{1}{2}$ cents,
100 lb. Sugar	@	$4\frac{5}{8}$ cents,
15 lb. Tea	@	45 cents,
200 lb. Flour	@	$2\frac{3}{4}$ cents,
200 lb. Potatoes	@	$1\frac{1}{4}$ cents.

Make out your bill, and receipt it.

5. Find one-half of 3 miles, 7 furlongs, 15 rods, 7 feet, and 5 inches. (1 mi. = 8 furlongs.)

6. Find the Greatest Common Divisor and the Least Common Multiple of 18, 24, 30, 36.

7. What is the diagonal of a square lot $137\frac{1}{2}$ feet square?

8. Sold a house and lot at a loss of $162.50, thereby losing $2\frac{1}{2}$ per cent of the cost. Find the cost price.

9. If a horse dealer buys a span of horses at 10 per cent less than their value, and sells them at 10 per cent more than their value, what per cent does he make?

10. If a boy buys peaches at the rate of 5 for 2 cents, and sells them at the rate of 4 for 3 cents, how many must he buy and sell to gain $4.20?

EXACT INTEREST.

Exact interest is used by the Government in its calculations. 365 days are taken to the year.

1104. Slate Exercises.

1. Find the exact interest of $280 from April 14 to Sept. 6 at 4%.

Time 145 days. *Ans.* $\$280 \times \frac{4}{100} \times \frac{145}{365}$.

2. Find the exact interest on $76.65 from March 4 to Dec. 15 at 6 per cent.

3. On $384 at $7\frac{3}{10}$ per cent for 75 days.

4. On $438 at 5% from Jan. 1 to March 15.

5. On $109.50 at $4\frac{1}{2}$% for 87 days.

6. On $847.60 at 5% from April 29 to Sept. 20.

7. On $584 at $3\frac{3}{4}$% from May 16 to Dec. 1.

8. On $1,000 at 3.65% from Jan. 4, 1893, to June 2, 1894.

9. On $1,460 at 6% from April 1, 1892, to Feb. 15, 1893.

10. On $1,200 at 5% from Aug. 19, 1892, to March 4, 1894.

1105. Unless "exact" or "accurate" interest is specified, use 360 days to the year.

1106. N. Y. State Examinations for Second Grade Certificates.

1. William Wilson sold goods to the amount of $1,000. One-half of his sales showed a profit of 25 per cent on the cost, and the remaining half a loss of $16\frac{2}{3}$ per cent on the cost. Required the total cost of the goods.

2. Find the amount March 4, 1890, of $187.40 at interest from Nov. 3, 1889, at 4 per cent per annum.

3. If .01125 of the value of a piece of property is $348.75, what is the whole value?

4. If I sell $\frac{2}{3}$ of my farm for $\frac{3}{4}$ of what the farm cost me, what is my gain per cent?

5. Reduce 240 rd. 4 yd. 1.2 ft. to the decimal of a mile.

6. In what time will $726 amount to $764.35 at 6 per cent per annum?

7. Which is the higher rate of freight on wheat, $.16 per hundred or $.10 per bushel (60 lb.) and what per cent?

8. Write in words:

(*a*) .267; (*b*) 200.067; (*c*) $\frac{260}{7000}$; (*d*) $200\frac{60}{7000}$.

9. Find the interest on $865 for 13 days at $5\frac{1}{2}$ per cent per annum.

10. If 40 per cent of the selling price of an article is profit, what is the per cent of gain on the cost?

11. What number added to $4\frac{1}{2}$ times itself will equal $60\frac{1}{2}$?

12. Divide $\frac{1}{8}$ by $.00003\frac{1}{3}$.

13. Reduce to lowest terms (*a*) $\frac{126}{1037}$; (*b*) $\frac{357}{1785}$.

14. A hotel is insured for $\frac{2}{3}$ of its value, at $1\frac{1}{2}$ per cent premium, and the premium amounts to $150. Find the valuation of the hotel.

15. If 4 men eat 64 pounds of bread in 2 weeks, how many pounds will 16 men eat in 7 weeks at the same rate?

16. Divide .75 of $17\frac{5}{8}$ by $\frac{4}{5}$ of .035.

17. Make and solve a problem in which avoirdupois weight is changed to troy weight.

18. Find the cost of 3,846 pounds of hay at $15 per ton.

19. Multiply $17\frac{4}{5} + 13\frac{2}{3}$ by $\frac{3}{4}$ of 27.

20. Find the cost of plastering the walls and the ceiling of a hall 72 ft. long, 50 ft. wide, and 22 ft. high, at $18\frac{3}{4}$ cents a square yard, allowing 972 sq. ft. for openings and baseboards.

21. A certain quantity of paper will make 4,000 copies of an octavo book (8 pages to the sheet). How many copies of a 12mo book (12 pages to the sheet) will the same paper make?

22. In what time will a note for $200, drawing $5\frac{1}{2}$ per cent interest, double itself?

23. A reservoir 8 ft. $\times$ 4 ft. $\times$ 3 ft. 9 in., holds how many barrels of water? (1 bbl. $= 31\frac{1}{2}$ gal.)

24. Find the diagonal of a square park containing 20 acres.

1107. Newport, R. I., Public Schools. Examination Questions.

Mental.

1. (*a*) How many sq. ft. are there in a figure 7 ft. square?

(*b*) 3 desks are bought at $10 each and sold for $45. Find the rate of gain.

(*c*) A wheelman sells his old bicycle for $100, and loses $16\frac{2}{3}\%$. How much did it cost him?

(*d*) How much does an agent get for buying 5 bales of goods at $400 each, if he receives 3% for his services?

(*e*) 10% of 200 is $\frac{2}{3}$ of what number?

Slate.

2. BANGOR, MAINE, June 24, 1887.

On demand, I promise to pay Peter Fogg, or order, Two Thousand Five Hundred Fifteen Dollars, with interest, value received.

$2,515\frac{00}{100}$. AARON ROCK.

$1,541.01 was paid Jan. 1, 1888. Find the amount due Aug. 15, 1888.

3. (*a*) Who is the maker of the above note? (*b*) Define dividends. (*c*) A ton of coal costing $3.50 is sold for $8. Find the gain per cent.

4. How much does it cost annually to insure the "City of Paris" for $1,525,000, if $2\frac{1}{4}\%$ is paid for the insurance?

5. Find the amount of $75.18, from June 10, 1888, to June 8, 1891, at 5%.

6. A merchant in this city buys of a New York firm 85 barrels of sugar @ $18.50 per barrel, giving in payment a 90-days note, dated June 20, 1890. If the New York firm has the note discounted July 2, what will be the proceeds?

7. $150 is paid an agent for purchasing 1,200 barrels of flour on a commission of $2\frac{1}{2}\%$. How much was paid for the flour?

8. 5 houses are sold for \$62,425, and \$7,425 is gained. Find (*a*) cost of 3 houses, and (*b*) the gain per cent.

9. Which will cost the more, and how much more, 15 shares of N. Y., N. H., & H., at 85, or 13 N. Y. & N. E., at 102, if the brokerage in each case is $\frac{1}{4}\%$? (Par value per share \$100.)

10. An agent receives \$2,562.50 to invest in land at \$62.50 per acre, after taking out a commission of $2\frac{1}{2}\%$. How many acres can he buy?

MEASUREMENTS.

1. What is the area of a triangle whose sides measure 15, 16, and 17 inches, respectively?

15
16
17
2)48
24 − 15 = 9
24 − 16 = 8
24 − 17 = 7

From the half sum of the three sides subtract each side separately. The square root of the continued product of the half sum and the three remainders will be the area.

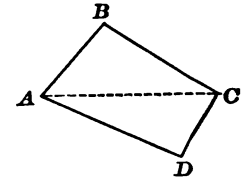
$\sqrt{24 \times 9 \times 8 \times 7} =$

$\sqrt{12{,}096} = 109.98$ sq. in. *Ans.*

2. Find the area in square feet of a triangle whose sides measure 35 ft., 84 ft., 91 ft.

3. Find the area of a triangle whose sides measure 21, 28, and 35 rods, respectively.

4. In the following field, AB measures 39 rods; BC, 52 rods; CD, 25 rods; AD, 60 rods; and the diagonal, AC, 65 rods. Find the area of the field in square rods.



5. Find the area of an isosceles triangle whose base is 30 yards, its equal sides measuring 25 yards.

6. What is the altitude of an isosceles triangle, base 96 ft., equal sides 64 ft.? Find its area.

7. Find the area of an equilateral triangle, each side being 6 ft.

8. Find the area of a right-angled triangle, base 42 ft., hypotenuse 70 ft.

9. Find the area of an isosceles triangle, altitude 48 ft., equal sides 50 ft.

10. Place two equilateral triangles, sides 2 inches, base to base, making a rhombus. Find its area, also the length of each diagonal.

11. Find the radius of a circle whose circumference is 132 ft. (3.1416 × diam. = circum.)

12. Find the area of a circle whose radius is 4 inches. (Area = $\frac{1}{2}$ circumference × $\frac{1}{2}$ diameter.)

13. Find the area of a circle whose radius is x inches.

14. Find the radius of a circle whose area is 314.16 sq. in.

15. Find the area of a circle whose circumference is 15.708 ft.

PARTIAL PAYMENTS.

1108. Merchants' Rule.

BROOKLYN, N.Y., June 19, 1894.

On demand, I promise to pay William R. Budd, or order, Two Thousand Four Hundred Fifty-four $\frac{75}{100}$ Dollars, value received, with interest at 6 per cent.

$2,454\frac{75}{100}$. ARTHUR TOWNSEND.

The following payments are endorsed on the note:

July 5, 1894, $200.
July 29, 1894, $450.
Sept. 18, 1894, $700.
Oct. 25, 1894, $300.

Find the amount due Jan. 2, 1895.

If no payments had been made, there would be due	$2,454.75
And interest from June 19 to Jan. 2, 197 days,	80.60
Total due,	$2,535.35
The credits are: Payment July 5, 1894,	200.00
Interest on $200, July 5 to Jan. 2, 181 days,	6.03
Payment July 29, 1894,	450.00
Interest on $450, July 29 to Jan. 2, 157 days,	11.78
Payment Sept. 18, 1894,	700.00
Interest on $700, Sept. 18 to Jan. 2, 106 days,	12.37
Payment Oct. 25, 1894,	300.00
Interest on $300, Oct. 25 to Jan. 2, 69 days,	3.45
Balance due,	$851.72

1109. By the merchants' rule, interest is calculated on the face of an interest-bearing note from its date until settlement, and interest is allowed on all credits from their payment until settlement.

1110. Slate Exercises.

1. A note for $500, with interest at 6%, is dated July 25, 1893. Payments are made: $100, Sept. 18; $200, Feb. 5, 1894. How much is due April 1, 1894?

2. Find amount due Sept. 15, 1894, on a demand note for $1,875, with interest at 6%, dated Jan. 18, 1894. Payments of $1,000 and $500 were made March 30 and June 17, respectively.

3. June 12, 1892, Robert Colgate bought goods amounting to $600. Dec. 31, 1892, he paid $300; April 5, 1893, $200; June 1, 1893, he settled the account. How much did he pay on that date, if he is charged 6% on the purchase from its date, and is allowed 6% interest on his payments?

4. T. J. Minturn loaned Chas. A. Dorsey $500, Sept. 1, at 6%. Payments of $200 each were made Oct. 1 and Nov. 1. How much is due Dec. 1?

Dr. WILSON T. JONES. *Cr.*

1893.					1893.				
Feb.	5	To merchandise,	840	00	Mar.	9	By cash,	500	00
Dec.	31	To interest to date,			Sept.	13	By cash,	200	00
					Dec.	31	By interest to date,		
					"	"	By cash,		

5. Find the amount paid in settlement of the foregoing account, Dec. 31, 1893. Interest 6%.

6. A merchant's books show the following debits: Feb. 13, merchandise, $725.00; April 14, merchandise, $603.00. The credits are: April 5, cash, $600; Aug. 29, cash, $300. How much is due Oct. 5, interest 6%?

1111. The merchants' rule is frequently used where the transactions all take place within a year. The exact number of days is taken, and the interest is calculated on the basis of 360 days to the year.

MEASUREMENTS.

1112. Slate Exercises.

Find the areas of the following triangles, their sides being, respectively:

1. 17 ft., 10 ft., 21 ft.
2. 41 yd., 15 yd., 52 yd.
3. 37 rods, 15 rods, 44 rods.
4. 13 inches, 14 inches, 15 inches.
5. 113 ft., 132 ft., 25 ft.
6. 17 ft., 120 ft., 113 ft.
7. 25 yd., 39 yd., 56 yd.

8. 39 rods, 44 rods, 17 rods.

9. 65 rods, 34 rods, 93 rods.

10. 13 chains, 37 chains, 40 chains.

1113. Examination for State Scholarships, Cornell University.

1. Reduce the fraction $\dfrac{8\frac{1}{2} \times \frac{7}{18}}{\frac{1}{12} \text{ of } \frac{3\frac{1}{2}}{\frac{4}{5}}} \div \dfrac{\frac{1}{8}}{\frac{3}{4} \text{ of } 7\frac{7}{11}}$.

2. Divide $\frac{7}{8}$ by 2.5, to the quotient add the divisor, and from that sum subtract the dividend. Give the fractional part of the answer in a decimal.

3. A rectangular field, containing $7\frac{1}{2}$ acres, is 30 rods wide. How much will it cost to build a fence from one corner to the diagonally opposite corner at $2 a rod?

4. If the interest on $300, for 1 yr. 8 mo., is $36, find what would be the interest on $212.50, for 3 yr. 4 mo. 24 da., at the same rate.

5. Find the proceeds of a bankable note for $500, at four months, when discounted at six per cent.

6. Reduce .0468 T. to a compound number (lb. and oz.).

7. Divide $87\frac{3}{4}$ by $\frac{9}{18}$ of $25\frac{11}{14}$.

8. Find the amount of $635, from Nov. 13, 1888, to June 6, 1889, interest at 5% per annum.

9. Define notation; numeration.

Write with figures two million, forty-three thousand, eighty. Write in words 60,030,016.

10. Find the prime factors of 20,930.

CHAPTER XIV.

EQUATION OF PAYMENTS. — MENSURATION OF SURFACES AND VOLUMES. — BOARD MEASURE. — ANNUAL INTEREST. — GOVERNMENT LANDS. — METRIC SYSTEM.

EQUATION OF PAYMENTS.

1114. Oral Problems.

1. A friend loans me $800 for 6 months without interest. How long ought I to loan him $400 to cancel the obligation?

2. In what time would the interest on $450 be the same as the interest for 3 months on $600?

3. W borrows from X $200 for 5 months and $400 for 2 months. How much money should W loan X for one month in return for the accommodation?

4. A man offers a lot for $600, payable $300 in 2 months, and $300 in 4 months. How much credit should be given to a buyer who wishes to pay the $600 at one time?

5. Mr. Jones has bought $600 worth of goods on 6 months' credit, and $300 worth on 3 months' credit. For what time should he give a note (without days of grace) for the whole amount, $900?

1115. Slate Problems.

1. In what time would the interest on $1,000 be the same as the total interest on the following amounts:

$100 for 1 month, $200 for 2 months, $300 for 3 months, $400 for 4 months?

Interest on	$100	for	1	month	=	Interest on	$100	for	1	month
"	"	200	"	2	months	=	"	"	400	"	1	"
"	"	300	"	3	"	=	"	"	900	"	1	"
"	"	400	"	4	"	=	"	"	1600	"	1	"
"	"	1000	"	?	"	=	"	"	3000	"	1	"

2. A person owes $400 payable in 4 months, and $500 payable in 13 months. What would be the average time for the payment of the whole indebtedness of $900?

The debtor is entitled to the use of $400 for 4 months, which is equal to the use of $1,600 for 1 month. He is also entitled to the use of $500 for 13 months, which is the same as $6,500 for 1 month. He is entitled, in all, to the use of $8,100 for 1 month, which is equal to the use of $900 for how many months?

1116. By equation of payments is meant a method of ascertaining at what time several debts due at different times may be settled by a single payment. The time thus found is called the *average* time, or the *equated* time.

3. Find the average, or equated, time for the payment of the following:

$600 due in 2 years
$500 due in $1\frac{1}{2}$ years
$300 due in 1 year
$400 due in 9 months

4.

$250 due in 8 months
$450 due in 6 months
$500 due in 3 months
$600 payable in cash

$250 \times 8 =$
$450 \times 6 =$
$500 \times 3 =$
$600 \times 0 = 0$
$1800 \times ? =$

5. $200 due in 15 days
$300 due in 30 days
$400 due in 45 days

6. $840 to be paid in four equal installments in 1, 2, 3, and 4 months, respectively.

7. $960 to be paid $\frac{1}{8}$ in 2 months, $\frac{1}{6}$ in 4 months, $\frac{1}{4}$ in 5 months, and the remainder in 6 months.

8. A debt to be paid $\frac{1}{10}$ in 2 months, $\frac{1}{5}$ in 3 months, $\frac{1}{6}$ in 4 months, and the balance in 12 months.

9. $6,000; $\frac{1}{3}$ to be paid in cash, $\frac{1}{4}$ of the remainder in 3 months, another fourth in 6 months, and the balance in 9 months.

10. On what date should the following account be paid in full?

Bought, July 1, goods to the amount of $300 payable in cash, to the amount of $800 payable in 30 days, and to the amount of $1,000 payable in 60 days.

1117. Miscellaneous.

11. A farmer sold 300 bu. wheat at $92\frac{1}{2}$¢ per bushel, 100 bu. at 90¢, 400 bu. at 95¢, 200 bu. at $1. What was the average price?

12. Three men hire a pasture for $84. One puts in 15 cows for 12 weeks, the second puts in 20 cows for 6 weeks, the third puts in 18 cows for 10 weeks. What amount should each pay?

13. A and B form a partnership. A furnishes $2,000, B $3,000. After a year A furnishes an additional $1,000. At the end of 2 years the business is disposed of for $7,100. How much should each receive?

SUGGESTION: A receives his $3,000 and how much of the profits? Should he receive as much as B, who had $3,000 in the business the whole time?

14. How many bushels of bran worth 40 cents per bushel should be mixed with bran worth 30 cents per bushel to make 100 bushels worth 36 cents a bushel?

x = number of bushels at 40¢

$100 - x$ = number of bushels at 30¢

$40x$ = value (in cents) of one kind

$30(100 - x)$ = value of other kind

Total value = how many cents?

15. How many bushels of corn worth 60¢ per bushel should be mixed with 80 bushels of corn worth 50¢ per bushel to make a mixture worth 52¢ per bushel?

16. A can do a piece of work in 20 days, B can do it in 30 days. They work together and receive $5 per day as the wages of both. What should be A's share of the total amount received?

How long does it take both together to do the work? What would A receive per day if he did the work alone?

17. A partnership is formed between A with a capital of $1,500 and B with a capital of $2,500. Six months thereafter, they take in C with a capital of $4,000. How should a profit of $3,500 be divided at the end of the year?

18. Three merchants shipped a cargo of iron by sea. A sent 180 tons, B sent 105 tons, C sent 315 tons. During a storm the sailors were obliged to throw overboard 180 tons to save the vessel. What portion of the loss should each merchant sustain?

19. If pure milk is reduced in value from 24¢ per gallon to 20¢ per gallon by the addition of water, how many quarts of water have been placed in a can that contains 40 quarts of the adulterated article?

1118. Brooklyn Public Schools. Examination Questions. Dictated Examples.

1. Add 84,359; 9,876; 798; 6,007; 17,388; 8,597; 584; 3,679; 25,080; 7,569; 888; 3,096; 12,817; 5,835; 68.

2. Multiply 6,789$\frac{4}{7}$ by 5,087.

3. Divide 65,281,172 by 789.

1119. Oral Problems.

4. How old December 1, 1892, was a boy born November 25, 1876?

5. A man has $1,000 in bank. What will remain after he has taken out $478?

6. How many hours in the month of January?

7. In how many years, months, and days, will $100 amount to $111, at 5%, simple interest?

8. A and B contract to haul a pile of lumber for $105. A furnishes 3 teams, and B 4 teams. How much does B receive?

9. A farmer makes 675 gallons of cider. He has but 12 barrels, each of 45 gallons' capacity, to store it in. How many more such barrels does he need?

10. What will be the cost of 36 yards of cloth at $2.75 per yard?

1120. Slate Problems.

1. A man paid $999 for the rent of a house from June 29, 1889, to May 5, 1891. What was the rent per year?

2. What per cent of 3 pounds 7 ounces is 7 pounds 9 ounces?

3. At 50 cents per running yard, what will be the cost of fencing a square field containing 10 acres?

4. At the rate of 20 problems an hour for A, and 15 in 55 minutes for B, in what time can both together solve 100 problems?

5. What sum will amount to $1607.39 in 2 years 5 months 11 days, at 4$\frac{3}{4}$ per cent, simple interest?

6. Find the entire surface of a cube, whose edge measures 15 inches.

7. A dealer buys books at $1.50 each, less $33\frac{1}{3}$ and 10 per cent. At what price per copy must he sell them to gain $43\frac{1}{3}$ per cent?

8. Abraham Lincoln died at the age of 56 years 2 months 3 days, after serving as president 4 years 1 month 11 days. Give the date of his birth, the date of his inauguration being March 4, 1861.

9. A dealer buys 150 barrels of flour. He sells one-third of it at $4.50 per barrel, losing 10 per cent. The remainder he sells at a profit of 6 per cent. What is his net gain or loss?

10. Sixty per cent of $66\frac{2}{3}$ per cent of a number equals 810. What is the number?

1121. St. Paul Public Schools. Examination Questions.

1. Bought $\frac{4}{7}$ of a box of candles, and having used $\frac{7}{8}$ of them, sold the remainder for $\frac{17}{25}$ of a dollar. How much would a box cost at the same rate?

2. Reduce $\frac{3}{32}$, $.62\frac{1}{2}$, $.37\frac{1}{16}$, $\frac{3}{8}$, to decimals, and find their sum.

3. A ladder 40 feet long is so placed in a street, that without being moved at the foot, it will reach a window on one side 33 feet, and on the other side 21 feet from the ground. What is the breadth of the street? Illustrate.

4. A grocer has a false balance by which he gives but 12 ounces to the pound. What is the real value of a barrel of sugar that sells for $28? (Solve by proportion.)

5. Explain partnership, telling what it is, its purpose and its advantage.

6. Four men hired a coach for $13, to convey them to their respective homes, which were at distances from the place of starting as follows: A's 16 miles, B's 24 miles, C's 28 miles, and D's 36 miles. What ought each to pay?

7. Analyze the fourth.

8. Extract the square root of $\frac{7056}{9216}$.

9. The gray limestone of Central New York weighs 175 pounds a cubic foot. What is the weight of one solid yard. Analyze.

10. Give the tables of avoirdupois and troy weight. Of lineal and square measure.

1122. Questions from U. S. Civil Service Examinations.

1. If a garrison of 3,600 men eat a certain quantity of bread in 35 days, at 24 ounces per day to each man, how many men, at the rate of 14 ounces per day, will consume twice as much in 45 days?

2. What is the interest on $378.42, for 1 year 5 months and 3 days, at 7 per cent?

3. What is a pile of wood 8 feet long, 7 feet wide, and 5 feet high worth, at $4.50 per cord?

4. When bank stock sells at a discount of $7\frac{1}{2}$ per cent, what amount of stock, at par value, will $3,700 purchase?

5. Add the following numbers: Five tenths; twenty-five hundredths; five thousandths; seven tenths; seven thousand seventy-two ten-thousandths; thirty-four ten-thousandths; forty-three hundred-thousandths.

6. The nominal value of drafts, etc., found in dead-letters in the year 1871, was $3,075,869.23, and in 1872 it was $3,320,300.38. What was the per cent of increase in 1872 over 1871?

7. How long a time will it take $1,560, at 5% simple interest, to gain 426.83\frac{1}{3}$?

8. The pound sterling is worth $4.8665. How much U. S. coin would it require to pay a debt of £780 18*s.* 11*d.*?

9. Suppose a tree 100 feet in height to be broken off by the wind, and that the top of the tree strikes the ground 40 feet from its base, while the other end of the part broken off rests on the top of the stump. Required the length of the part broken off.

10. A log of wood is 15 inches broad and 11 inches thick. What length of it will make 10 cubic feet?

11. What rate per cent is required to make \$10,050 gain \$682.00$\frac{5}{12}$ interest in 11 months and 19 days?

12. How long will it take \$16,500 to gain \$2,117.50, at 7 per cent?

13. A merchant imported 120 tons of English iron, costing $1\frac{1}{2}$ pence per pound, on which he paid a duty of 20 per cent. The freight was 5 shillings sterling per ton. What was the total cost in U. S. currency? (1 ton = 2,240 lb. £1 = \$4.8665.)

14. Reduce the following complex fraction to its simplest form:

$$\left\{\frac{\frac{1}{8}}{\frac{1}{16}} \times \frac{8\frac{1}{3}}{3\frac{1}{8}}\right\} - \left\{\frac{\frac{6}{9} \times \frac{3}{2}}{\frac{4}{8} \times \frac{2}{\frac{1}{2}}}\right\}$$

15. Samuel Adams, a contractor, had the following dealings with the Treasury Department: He furnished, January 3, 1883, 2,575 lb. of twine at 12¢ a lb.; April 4, 1883, 25 doz. gold pens at \$25 a doz.; May 7, 1883, 645 reams letter paper at \$2 a ream; July 9, 1883, 45 doz. qt. Arnold's ink at \$3 a doz.; October 30, 1883, 1,000,000 envelopes, at \$2 a thousand; and December 5, 1883, 8 doz. inkstands at \$1.97 a doz. He was paid cash as follows: February 4, 1883, \$175; April 30, \$350; July 15, \$700; November 5, \$2,300; and December 31, 1883, he was allowed on settlement \$45 for cartage, and charged \$75 for breakage, and \$60 for shortage on envelopes. State his account in the blank below, with proper heading, and show the balance, if any, due him. (No interest.)

1123. Questions from Examinations for N. Y. State Certificates.

1. The large wheel of a bicycle is 15.7 feet in circumference. How many times must it revolve in running a mile?

2. How many rods of fence are required to enclose a square lot whose area is 5,184 sq. ft.?

3. Property worth $6,000 is insured for $\frac{3}{4}$ of its value, at $\frac{3}{4}$ of one per cent. What will be the loss, including premium, in case of total destruction by fire?

4. Required the cost of laying a pavement $5\frac{1}{3}$ rods long and 8 ft. 6 in. wide, at $1.40 per square yard.

5. The sum of two numbers is 12 bu. 1 pk. 3 qt., and their difference is 1 bu. 2 pk. 5 qt. What are the numbers?

6. Make and solve a problem illustrating how the principal is found when the interest, rate per annum, and time are known, using 1 yr. 1 mo. 1 da. as the time.

7. Make and solve a problem illustrating the application of square root in finding the side of a right-angled triangle.

8. How many acres of land, in the form of a square, may be enclosed by 160 rods of fence?

9. Find the square root of .441 correct to two decimal places.

10. Reduce 17 lb. 11 oz. avoirdupois weight to pounds, ounces, pennyweights, and grains, troy weight. (1 lb. avoirdupois = 7,000 troy grains.)

11. A note for $324.61, and interest, amounted to $384.13, 2 yr. 7 mo. 13 da. after date, when it was paid. Required the rate per cent per annum.

12. The premium on a fire insurance policy, at $\frac{7}{8}$ of 1 per cent, was $140. Required the face of the policy.

13. What is the length (in rods) of the diagonal of a square field containing 40 acres?

14. Reduce $\frac{15}{27}\frac{18}{50}$ to its lowest terms.

15. Find the solid contents of a cube, the area of one face of which is 256 square feet.

16. What elements are required to find the per cent of gain or loss in a business transaction? Illustrate by an original problem.

17. A car contains 21,643 pounds of wheat. Find the value of the load at 92¢ per bushel. (1 bu. = 60 lb.)

18. Find the area of a triangle whose base is 22 ft. 8 in., and altitude 19 ft. 9 in.

19. The list price of a certain stove is $38, and the retail dealer is allowed commercial discounts of 20 per cent, 5 per cent, and 3 per cent. What price does he pay for the stoves?

20. If 45 horses eat $1\frac{1}{2}$ tons of hay in 30 days, how many tons should last 84 horses 56 days?

21. If a ton of coal lasts a family 21 days, what will be the cost of coal used by it from Oct. 17, 1888, to April 25, 1889, exclusive of either day named, at $4.50 per ton?

22. A farmer sold 48.64 acres of his farm of 112 A. 96 sq. rd. What per cent of his farm had he left?

23. Find the cost of a pile of 4-foot wood, 27 ft. long and 6 ft. high, at $5.50 per cord.

24. Find the interest on $1, for 3 yr. 1 mo. 29 da., at 5 per cent per annum.

25. Four men hire a pasture field together. The first pastures 4 cows 18 weeks; the second, 5 cows $12\frac{4}{7}$ weeks; the third, 11 cows $6\frac{1}{2}$ weeks; the fourth, 9 cows 16 weeks. What part of the rent should each pay?

26. How many rods of fence will be required to enclose a field in the form of a right-angled triangle, whose area is $13\frac{1}{2}$ acres and whose base measures 48 rods?

MENSURATION OF PLANE SURFACES.

1124. Slate Exercises.

1. Find the circumference of a circle whose radius is x.

(Diameter $\times$ 3.1416.)

2. Find the area of a circle whose radius is x.

($\frac{1}{2}$ circumference $\times$ $\frac{1}{2}$ diameter.)

3. Find the area of a circle whose diameter is x.

4. Find the area of a circle whose circumference is x.

5. What is the area of a circle whose diameter is 36 feet?

6. What is the radius of a circle whose area is 153.9384 sq. yd.?

7. What is the circumference of a circle whose area is 198.95 sq. rods?

8. Find the area of a square whose diagonal is x.

9. Find the area of a square whose diagonal is 150 rods.

10. Find the area of an isosceles triangle, its base being 56 meters, equal sides 100 meters.

11. Find the area of an equilateral triangle whose side is 12 feet.

12. Find the area of a triangle whose sides are 50 yd., 60 yd., 70 yd.

13. What is the area of a circle whose circumference is 10 feet?

(The square of the circumference $\times$ what = area?)

14. Find the area of the rhomboid, Fig. 1.

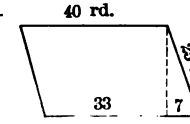


FIG. 1.

15. Of the rectangle, Fig. 2. **16.** Of the rhombus, Fig. 3.

17. Of the trapezoid, Fig. 4.

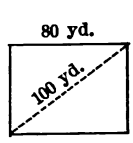


FIG. 2.

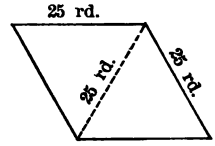


FIG. 3.

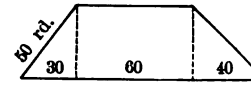


FIG. 4.

18. Of the trapezium, Fig. 5. **19.** Of the rhombus, Fig. 6.

20. Find the altitude, AB, of the following triangle, Fig. 7:

(First find the area.)

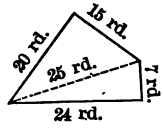


FIG. 5.

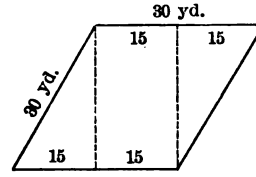


FIG. 6.

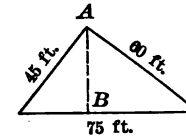


FIG. 7.

21. Find the diagonal (in rods) of the square whose area is 5 acres.

22. Find the area of a hexagon, composed of six equilateral triangles, each side being 6 inches. Fig. 8.

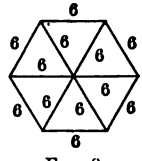


FIG. 8.

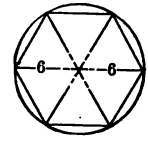


FIG. 9.

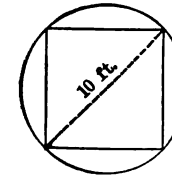


FIG. 10.

23. What is the area of the circle circumscribed about the above hexagon, Fig. 9?

24. What is the area of the square inscribed in a circle whose diameter is 10 feet, Fig. 10?

25. What is the area of the sector, Fig. 11, the radius of the circle being 10 inches, and the arc of the circle 72°?

(A sector is a portion of the surface of a circle inclosed between two radii.)

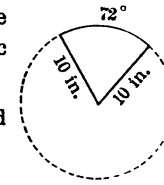


FIG. 11.

1125. Newport, R. I., Public Schools. Examination Questions. Oral Problems.

1. What would be the cost of 120 books at $66\frac{2}{3}¢$ each?

2. Change $\frac{8}{11}$ to a decimal of three places.

3. How much wood in three piles containing, respectively, $\frac{1}{2}$ of a cord, $\frac{1}{8}$ of a cord, and $\frac{1}{4}$ of a cord?

4. What is the percentage of gain in case of railroad stock bought for $80 per share and sold for $90 per share?

5. Change 66,321 mills to dollars.

Slate Problems.

1. Find the sum of the answers of the following examples:

30,872 ÷ 1.79 = ? 30.872 ÷ .0179 = ?

.30872 ÷ .179 = ? 308.72 ÷ 17.9 = ?

2. Define underwriter, policy, and premium.

3. I paid $33.75 for insuring $2,700 on my house, for 5 years. What was the per cent of the yearly premiums?

4. What is the balance of a bill of $64.50, after two discounts have been made; the first of 20% on the $64.50, the other of 5% on what then remained?

5. What is the accurate interest of $2,190, at 5%, from Dec. 28, 1890, to March 8, 1891?

(365 days to 1 year.)

6. Find the date when the following note becomes due, the time for which it was discounted, the bank discount, and the proceeds: A 4-months note for $225, dated December 10, 1890, and discounted January 1, 1891, at 7%.

7. What is the amount of $261, for 3 years, at 6%, compounded annually?

8. There was shipped to Liverpool from New York in one week $6,870,205 in specie. What amount of English currency could be bought with it? (£1 = $4.8665.)

9. How much money must be invested in the U. S. 4½'s to yield a quarterly income of $225, bonds selling at 105¼? (The bonds pay 4½% interest on the face value.)

10. I send to my agent at Havana $1,224. What part of this sum will remain to invest in sugar, after deducting his commission of 2% on what he lays out?

1126. Kingston, N. Y., Public Schools. Examination Questions.

1. If $\frac{4}{7}$ of a lot cost $1,200, what is the value of $\frac{3}{8}$ of it?

2. What is the freight on 9,860 lb. iron at $1.75 per ton?

3. What is the value of 10 lb. 7 oz. 16 pwt. of gold at $.75 a pwt.?

4. How many days from March 16 till Dec. 25 of this year?

5. The dividend is 6,171, the quotient 17, the remainder 102. What is the divisor?

6. Divide the L. C. M. of 132 and 156 by their G. C. D.

7. The product of three numbers is .0728; one of them is 1.3, another .07. Find the third.

8. What is the greatest length of boards that I can use in fencing my garden, the sides of which are 168 ft., 182 ft., 280 ft., and 252 feet?

(Find G. C. D. of the given numbers.)

9. If 5 men can make 38 rods 5 yards of fence in a day, how much can they build in 30 days?

10. The distance from New York to New Haven being 73 mi. 8 rd., at what rate does a train run per hour to cover the distance in 2 hr. 10 min.?

11. When it is 12 o'clock M. at St. Paul, 93° 10′ W., what is the time at Richmond, 77° 27′ W.?

12. What will it cost to carpet a hall 4 yd. wide by 12 ft. long, with carpet $\frac{7}{8}$ yd. wide at $3 per yd.?

13. Reduce 4 da. 4 hr. 48 min. to the decimal of a week.

14. What is the value of a pile of wood 19.5 ft. long, 8.4 ft. wide, and 6.3 ft. high, at $4.25 per cord?

15. My room is 24 ft. long; its width is .50 of its length. How many yards of carpeting, $\frac{3}{4}$ yd. wide, will it require?

16. After 4 per cent of a flock of sheep had been killed by dogs, and 68 had been sold to a butcher, four-sevenths of the original flock were left. Required the number of sheep in the flock at first.

17. Define percentage, rate, base.

18. A farmer sold 175 bu. corn @ $.79 a bu.; 179 bu. of wheat @ $1.19 a bu.; 50 bu. oats @ $.60 a bu.; and 3 tons hay @ $14.75 a ton. Make a receipted bill.

19. A room measures 14 ft. by 16 ft., and is 8 ft. high. How many rolls of paper $1\frac{1}{2}$ ft. wide will cover the walls, there being 8 yd. in a roll, and one-eighth allowed for openings?

1127. Paterson, N. J., Public Schools. Examination Questions.

1. Six men bought a ship worth $45,268, for which A paid $\frac{1}{4}$ of the whole, B $\frac{1}{5}$, and the others paid the remainder equally. How much did each of the latter pay?

2. In what time will $800, at 8%, gain $150?

3. A note of $1,250, dated July 5, 1888, was paid June 6, 1890, with interest at 8 per cent. What was the amount paid?

4. Two men hire a pasture for $420. A puts in 300 sheep for 5 weeks, and B puts in 450 sheep for 6 weeks. What should each pay?

5. A man agrees to dig a cellar 30 feet long, 24 feet wide, and 6 feet deep. What per cent of the work has he done when he has removed 16 cubic yards?

6. How many boards 16 ft. long, and 4 in. wide, are required to floor a room 48 ft. long, and 32 ft. wide?

7. What is the length of one side of a square field containing 15 A. 1 sq. rd.?

8. What is the present value or worth of $1,609.30, due in 10 months 24 days, when money is worth 5 per cent? (See Art. 1017.)

9. How much walking does a man save by crossing diagonally a field 28 rods long, and 21 rods wide, instead of going along the end and side?

10. In order to have an annual income of $2,500, what sum must be invested at 5%?

1128. Elmira, N. Y., Public Schools. Examination Questions. Oral Problems.

1. At 10¢ a quart, what are 3 bu. 1 pk. 5 qt. of chestnuts worth?

2. How many yards in 288 inches?

3. What decimal of a number is $\frac{2}{5}$ per cent of it?

4. $120 is $\frac{4}{5}$ per cent of what number?

Slate Problems.

1. An agent sold 5 tubs of butter, weighing respectively $46\frac{1}{2}$, 45, 48, $47\frac{1}{2}$, and 50 lb., at an average of 32¢ a pound. He paid

freight $1.80, and cartage $.40. His commission was 5 per cent. How much was his commission, and what were the net proceeds?

2. An agent's commission, at 2%, for selling wool was $250. What did the sale amount to?

3. An agent sold goods to the amount of $2,480, and received $74.40 commission. What was the rate?

4. What is ratio?

5. What are the terms of a ratio? Name them.

6. What is a proportion?

7. The extremes of a proportion are 16 and 25, and one of the means is 20. What is the other mean?

8. Change the following equation to the form of a proportion: $\frac{8}{50} = \frac{81}{1850}$.

9. Prove that the expression you have given is a true proportion.

10. If a quantity of flour will last 21 soldiers 16 weeks, for what time will it last 24 soldiers?

11. If 9 men can do a piece of work in 28 days, how many men can do the work in 12 days?

12. If a traveler walks 140 mi. in 8 da., walking 7 hr. a day, how many miles can he walk in 12 da. of 8 hr. each?

13. Find the cost of insuring property worth $18,000 for $\frac{2}{3}$ of its value, at $\frac{1}{8}$%.

14. At $\frac{1}{8}$ per cent, how much insurance can be effected on a store for $120?

15. A school house is to be built at a cost of $8,000. The property of the town is assessed at $1,000,000. What is the rate of tax, and what tax will a man pay whose property is assessed at $16,000?

SPECIAL DRILLS.

1129. Give sums:

112+91+85	43+131+61	95+144+79	68+56+174
129+62+98	26+172+81	63+117+97	63+34+186
182+13+67	75+193+23	91+126+32	91+59+165
114+21+49	132+94+77	63+143+24	78+39+183

1130. Give answers:

150−23+48	172+19−66	183−(72−37)	161+79−12
154−36+44	184+39−35	173−(57−18)	174+41−36
155−49+24	181+47−33	165−(47−28)	171+32−34
153−45+31	151+46−24	182−(48−33)	175+33−46

1131. Give products:

44×20	63×32	33×52	17×72
44×22	54×38	26×58	71×78
44×18	54×42	26×62	71×82
63×28	33×48	17×68	45×88

1132. Give quotients:

$676 \div 13$	$527 \div 17$	$704 \div 22$	$837 \div 27$
$602 \div 14$	$738 \div 18$	$966 \div 23$	$961 \div 31$
$690 \div 15$	$950 \div 19$	$768 \div 24$	$992 \div 32$
$672 \div 16$	$924 \div 21$	$975 \div 25$	$759 \div 33$

1133. Give results:

$84 \times 1\frac{5}{6}$	$211\frac{1}{4} \div 13$	$36 \times 49\frac{5}{6}$	$162\frac{1}{2} \div 25$
$48 \times 2\frac{7}{8}$	$214\frac{2}{3} \div 14$	$32 \times 59\frac{7}{8}$	$185\frac{7}{8} \div 31$
$36 \times 9\frac{1}{2}$	$130\frac{1}{5} \div 21$	49×49	$18\frac{3}{8} \div 3\frac{3}{8}$
$48 \times 19\frac{3}{8}$	$155\frac{1}{2} \div 22$	58×58	$17\frac{3}{4} \div 5\frac{3}{4}$

1134. Oral Problems.

1. A has 96 sheep; B has 28 sheep more than A. How many sheep have both?

2. There are 56 pupils in one class, 48 in a second class, and 52 in a third class. How many pupils are there in the three classes?

3. March 29 is what day of the year 1894?

4. How far is a man from his starting-point, if he travels due east 150 miles, due west 23 miles, due east again 48 miles?

5. A body falls 16 ft. in the first second, three times as far in the second second, five times as far in the third second. How far does it fall in three seconds?

6. The base of a right-angled triangle is 12 ft., the perpendicular is 16 ft. What is the hypotenuse?

7. At $35 per month, what will be the rent of a house for 16 months?

8. A field containing 169 square rods is 13 rods long. How many rods of fence will be needed to enclose it?

9. 25 packages of sugar weigh together $87\frac{1}{2}$ lb. How many pounds are there in each?

10. At 45 miles per hour, how many hours, minutes, and seconds will it take a train to go 230 miles?

11. How many years have elapsed since the invention of gunpowder, 1356?

12. What profit is made on an article bought for $175, less 12%, and sold for $200?

13. How many square rods in a field 71 rods long, 81 rods wide?

14. Assuming a kilo to be $2\frac{1}{5}$ lb., how many kilos will be equal to 143 lb.?

15. A degree of longitude in latitude 45° is about 70% of the length of a degree on the equator. Calling the latter length 69 miles, how long is a degree of longitude in latitude 45°?

16. At $44 per acre, how much land can be bought for $968?

17. A number of marbles divided among 29 boys gives each 16 marbles, and leaves a remainder of 26. How many marbles are there?

18. What is the monthly salary of a clerk who receives $1,500 per year?

19. How many revolutions in a mile, 5,280 ft., are made by a locomotive wheel 16 ft. in circumference?

20. How many feet of fence are there around a lot 49 ft. wide, 87 ft. long?

21. How many bricks 8 in. by 4 in. by 2 in. would make a cubic foot?

22. 13 is one factor of 1,001. Find the other two prime factors.

23. What are the three equal factors of 343?

24. What is the square root of 1,225?

25. At $4\frac{1}{2}$ miles per hour, how long will it take a man to walk $37\frac{1}{2}$ miles?

26. What will be the cost of 9 dozen hats at $\$1.33\frac{1}{3}$ each?

27. A 90-days note is drawn April 6. On what date is it due?

28. How many square yards are there in the entire surface of a cube whose solid contents are 8 cubic yards?

29. What are the proceeds of a 60-days note for $100 discounted at a bank the day it is drawn at 6%?

1135. An English Invoice.

LEEDS, Aug. 24, 1892.

Statement of certain goods, viz.: woolen mantlings, consigned by Johnson & Birmingham to Messrs. Hamburger Bros. of New York, and dispatched from Leeds via Liverpool, per S. S. Umbria.

H. B. 5,453	295 lb. net.			
48¼ yd.				
46¼ "	94½ yd.	2/3	£	
47¾ "				
47¾ "				
45¼ "	140¾ yd.	1/9	"	
61 "		3/—	"	
H. B. 5,454	351 lb. net.			
47½ yd.				
62 "				
66¼ "				
62½ "				
61 "				
48¾ "	yd.	1/9	"	
			£	
	less 2½%		"	
			£	

Find the duty in U. S. money at 44¢ per lb. and 50% *ad valorem*. £1 = $4.8665.

1136. NOTE. — 2/3 = 2 shillings 3 pence. 1/9 = 1 shilling 9 pence.

MEASUREMENTS.

1137. Find the area of each of the following triangles and its altitude.

When the area of a triangle is known and the length of the base, how can its altitude be calculated?

1. Base, 51 ft.; other sides, 20 ft. and 37 ft.

2. Base, 21 yd.; other sides, 13 yd. and 20 yd.

3. Base, 148 rods; other sides, 39 rods and 113 rods.

4. Base, 28 chains; other sides, 17 chains and 25 chains.

5. Base, 75 inches; other sides, 20 inches and 65 inches.

1138. Find the areas of the following quadrilaterals:

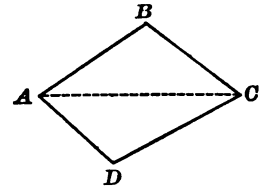
6. Given AB, 17; BC, 10; CD, 20; DA, 13. $AC=$ 21.

7. Given AB, 25; BC, 39; CD, 34; DA, 50. $AC=$ 56.

8. Given AB, 37; BC, 15; CD, 39; DA, 17. $AC=$ 44.

9. Given AB, 111; BC, 45; CD, 25; DA, 113. $AC=132$.

10. Given AB, 113; BC, 17; CD, 39; DA, 111. $AC=120$.



1139. Slate Problems.

1. A and B rented a field for a year for \$175. A put in 6 horses for the whole time, B put in 5 horses for 11 months and 3 horses for 5 months. How much of the rent had each to pay?

2. A bankrupt surrenders property worth \$1,287 for the benefit of three creditors to whom he owes \$750, \$1,125, and \$1,245, respectively. How much should each creditor receive?

3. Four persons rented a pasture for 26 weeks. K put in 50 sheep and L 60 sheep for the whole time, M put in 70 sheep for 20 weeks, and N 90 sheep for 22 weeks. How much of the rent, \$130, had each to pay?

4. A employs a capital of $2,500 in business, and at the end of 3 years takes into partnership B, who furnishes $4,000. Four years later they are joined by C, with a capital of $5,000. At the end of 12 years from the commencement of the business, the profits, amounting to $15,000, are divided. What amount should each receive?

A's money is in the business how many years? B's, how many years? C's, how many?

5. Four butchers rent a field, and pay for 6 months' rent $152.50. The first puts in 20 oxen for 10 weeks and 50 sheep for 8 weeks; the second, 25 oxen for 8 weeks and 30 sheep for 7 weeks; the third, 18 oxen for 10 weeks and 10 sheep for 12 weeks; the fourth, 30 oxen for 12 weeks. What share will each have to pay, counting 3 sheep equal to 1 ox?

6. A wall 700 yards long was to be built in 29 days. At the end of 11 days, 18 men had built 220 yards of it. How many extra men had then to be put to work, so that the wall might be completed in the given time?

7. If 5 needlewomen can do a piece of work in 11 days of 9 hours each, how long will it take 3 needlewomen to do two such pieces, supposing them to work $10\frac{1}{2}$ hours each day?

8. If 14 men can mow 168 acres in 12 days of 8 hours 15 minutes each, how many acres can 20 men mow in 11 days of 7 hours 48 minutes each?

9. If 12 men can do a piece of work in 20 days, what number of men will be required to do four times as much work in a fifth part of the time?

10. A ship sailed with a crew of 60 men, and provisions for 34 days, and 10 days afterwards, 12 persons were received on board from a sinking vessel. How long would the provisions last the 72 persons then on board?

How long would the provisions last the 60 persons at the time the sinking vessel was met?

10. Find the entire surface of a square prism, one side of whose base measures 4 inches, and whose altitude is 6 inches.

11. The convex surface of a square prism is 600 sq. ft., the altitude is 15 ft. What is the length of one side of the base?

12. The entire surface of a square prism is 1,650 sq. in. One side of the base measures 15 inches. What is its convex surface? What is its altitude?

13. Find the entire surface of a square prism whose convex surface is 540 sq. in., and whose altitude is 15 inches.

14. What is the entire surface of a cylinder whose base has a diameter of 1 foot, and whose altitude is 1 foot?

SURFACES OF PYRAMIDS AND CONES.

15. The convex surface of a square pyramid consists of how many equal triangles? Find the convex surface when one side of its base measures 4 inches and its slant height (AX) 6 inches.

Draw the development.

16. The convex surface of a pyramid is equal to the perimeter of the base multiplied by what?

17. Find the entire surface of the above pyramid.

18. Calculate the entire surface of a square pyramid whose slant height is 18 inches, the area of its base being 144 sq. in.

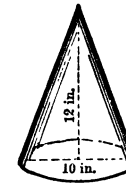
19. Find the entire surface of a triangular pyramid whose three convex faces and the base are equilateral triangles, each side measuring 2 inches.

20. Draw the developed convex surface of a cone, the diameter of whose base is 4 inches, and whose slant height is 6 inches.

Calculate the convex surface.

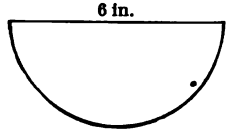
21. How many square inches of paper would be required to cover the side and the base of a cone 6 inches in diameter at the base, and having a slant height of 10 inches?

22. Calculate the slant height of a cone whose altitude is 12 inches, the diameter of its base being 10 inches. What is its convex surface?

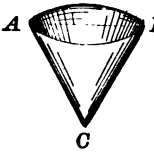


23. What is the entire surface of a cone, the diameter of whose base is 6 inches, and its slant height 10 inches?

Draw the development.



24. A semi-circular piece of paper 6 inches in diameter is folded into a hollow cone (without overlapping). What will be the diameter AB of the mouth of the cone (the base)? What will be the slant height BC?



1142. Cambridge, Mass., Public Schools. Examination Questions.

1. Last year 22 per cent of the whole number of pupils in the primary schools lost on an average 138 sessions each. How many sessions were lost by these pupils, the whole number of pupils being 4,300?

2. There are 40 weeks in a school year. There are usually 10 sessions in each week. There are 36 lines in the "Psalm of Life." If pupils commit to memory one line each session, how many poems containing as many lines as the "Psalm of Life" will they learn at school during the year, the school being closed 1 per cent of the 40 weeks?

3. If to-day (June 21, 1890) Mr. Chas. P. Emerson, of Cambridge, is to give you his note for $240, without interest, on three months' time, how should the note be written? What would be the avails (proceeds) of the note at a bank July 21, 1890?

4. A man invested some money in bonds, at par, bearing 6 per cent interest, and received $300 semi-annually. What was the sum invested?

5. At $2 a rod, what is the difference in the cost of fencing a lot of land 20 rods square, and another lot containing the same area which is 40 rods long?

6. A man has a straight fence made of three lengths of wire. The three wires are supported by 11 posts, the distance between the centers of the posts being 10 feet. These posts are each 6 inches in diameter, with holes through which the wires pass. The wires are fastened on the outside of the end posts by twisting them around spikes driven into the posts, and it took 3 inches of wire to fasten each end of each wire. How many feet of wire are required for the fence?

1143. Boston Examination Questions. Mental.

1. A room is $\frac{5}{6}$ as wide as it is long. Its length is 20 ft. How many sq. ft. are there in the floor?

2. What will 5 tons of granulated sugar cost at $6\frac{1}{4}¢$ per lb.?

3. If 5 yd. of cloth cost 90¢, what will $\frac{5}{9}$ of a yd. cost?

4. What will be the cost of 1 pk. 1 qt. 1 pt. of nuts at 10¢ per qt.?

5. What is the value of an acre of land at 10¢ per sq. ft.?

6. An agent insured a house for me at a commission of $\frac{1}{2}\%$. His commission was $15. How much was the house insured for?

7. A gold-digger who had 3 lb. of gold dust, lost 9 oz. What per cent was left?

8. What is the interest of $50, for 3 yr. 7 mo. 12 da., at 6%?

9. How many days from May 16 to July 5?

10. Add 379 and 297.

11. What is the bank discount on a sixty-days note for $400, at 6%?

12. A dealer sold flour at a profit of $2.00 a barrel, and gained 25%. What was the cost?

13. In what time will \$50 double itself at 8%?

14. What would 51 lb. of butter cost at $33\frac{1}{3}¢$ a lb.?

15. Mrs. Allen bought 7 chairs at \$4 apiece, 2 tables at \$9 apiece, and a carpet for \$33. She paid two \$50 bills. How much change was due her?

16. If \$1 is paid for insuring a piano worth \$500, what is the rate of insurance?

17. In what time will any sum of money double itself, at 6%?

18. Into how many lots, containing $\frac{2}{3}$ of an acre each, can eight acres be divided?

19. A man lends \$1,200 at 6%, and \$1,500 at 5%. What is the difference in the amount of yearly interest due on each?

20. A man owning $\frac{3}{4}$ of a ship, sold $\frac{2}{3}$ of his share. What part of the ship did he still own?

21. Find the sum of the prime numbers as far as 12.

22. $2\frac{1}{2} + 3\frac{1}{2} + 4\frac{1}{2} = ?$

23. Interest of \$1,234, for 30 days, at 6%?

24. Interest of \$1,234, for 6 months, at 4%?

25. How many rings, each 2 pwt. 12 gr., can be made from $\frac{1}{4}$ lb. of gold?

26. How far apart are two places whose difference in time is two hours and three minutes?

27. Find the number of sq. in. on the surface of a block 10 in. $\times$ 4 in. $\times$ 3 in.

28. How much would you pay for $3\frac{3}{4}$ yd. of cloth at $37\frac{1}{2}¢$ a yard?

29. Oil is worth $37\frac{1}{2}¢$ a pint. How many pints can I buy for \$6?

30. Sold oranges for $\frac{1}{2}¢$ apiece, gaining 50%. How much did they cost apiece?

1144. Boston Examination Questions. Slate.

1. Find the interest of $67.90 from June 24, 1871, to April 1, 1874, at 5½%.

Find the amount of $389.75 from May 27, 1881, to February 12, 1885, at 8%.

2. If a man owning 45% of a steamboat sells ⅛ of his share for $5,860, what is the value of the whole boat?

3. A farmer having 6 bu. 8 qt. of cranberries, lost by decay 7 pk. 7 qt. What % had he left?

4. Sold tea for 114% of its cost, and made a profit of 7¢ a lb. Find selling price.

5. In ⅔ of an acre of land how many building lots, each 60 ft. by 121 ft.?

6. What is the balance of a bill of $64.50 after two deductions have been made: the first, 10% on the $64.50; the other, 5% on what remained?

7. I bought a store for a certain sum, and after paying a tax of 2½% on the cost and ½% more for insurance, I sold it for $7,828, which exactly covered the cost, tax, and insurance. What was the cost?

8. I send to my agent $4,488.75, of which he is to lay out what he can in land at $15.00 an acre, after deducting his commission of 5% on the cost of the land. How many acres can he buy, and what is his commission?

9. Find the interest of $2,500, for 2 yr. 3 mo. 7 da., at 7½%.

Find the bank discount and the proceeds of a 4-months note for $450, dated June 10, 1889, and discounted July 25, at 7%.

For what sum must a 90-days note be drawn so that when it is discounted at a bank at 6%, the proceeds shall be $886.05?

10. John Johnston, of Vermont, sent to Smith Bros., of Boston, to be sold on commission, the following goods: 25 tons of hay, 2

tons of butter, 1,500 lb. of maple sugar, 75 gallons maple syrup. Smith Bros. sell the hay at $18 a ton, the butter at 20¢ a lb., the sugar at 7¢ a lb., the syrup at 90¢ a gallon.

Smith Bros. charge 2% commission. How much do they send to John Johnston?

11. How much will a granite block weigh which is 7 ft. long, 2 ft. 6 in. wide, 3 ft. 4 in. high? (12 cu. ft. of granite weigh a ton.)

12. Some men mowed $16\frac{1}{4}$ acres of grain in $7\frac{1}{2}$ days. At the same rate, how many acres could they mow in $9\frac{3}{4}$ days?

13. Change $\frac{9}{25}$ and $\frac{13}{65}$ to decimals, and add them.

Divide six hundredths by two ten-thousandths, and multiply the quotient by four millionths.

14. Lucy's hoop was 9 feet in circumference. How many yards would it travel in making 48 revolutions?

15. Bought 1 pk. 2 qt. 1 pt. of berries for $2\frac{1}{2}$. At what price per quart must they be sold to gain $12\frac{1}{2}$%?

16. I bought $\frac{3}{4}$ of an acre of land for $480, and sold it at a gain of $12\frac{1}{2}$%. What did I receive for it by the square foot?

17. A coal dealer bought 350 tons of coal, weighing 2,240 pounds each, at $3.50 a ton. He sold the coal at $4.25 a ton, each ton weighing 2,000 pounds. What was his profit?

18. A man paid $5,085 for his house, and $\frac{2}{3}$ as much for his lot. The cost of both amounted to $\frac{2}{3}$ of all his money. How much had he?

19. Mrs. Burns buys 40 yards of carpet $\frac{3}{4}$ of a yard wide. She uses 10% of it for a rug, and the remainder to carpet a floor. How many square yards does she use for the floor?

20. Mr. Burns sold his carriage for $224, which was $\frac{8}{7}$ of its cost. What per cent would he have gained if he had sold it for $210?

21. What is the difference between four thousand nine and seven hundred eighty-six ten-thousandths, and four hundred thousand nine and seven hundred eighty-six millionths.

If $\frac{5}{7}$ of a lb. of coffee costs $\$\frac{3}{8}$, what will 1 lb. cost?

Discover a fraction which, multiplied by $\frac{4}{5}$, equals $\frac{2}{3}$.

22. A box is 10 ft. long, 5 ft. wide, and 4 ft. high. How many yards of carpeting one yard wide will it take to cover the box on all sides?

23. A note for $\$1{,}750$, with interest at 5%, is given May 10, 1887. How much money would pay it June 20, 1890?

What should I get June 10 for a 90-days note for $\$375$, dated May 5, and discounted at 6%?

24. What % of $\frac{1}{2}$ of $\frac{5}{4}$ of $\frac{8}{3}$ is $\frac{1}{3}$?

25. What must be the face of a note, given for 125 days, to obtain $\$1{,}244$ from a bank, discount being at 8%?

26. Franklin Park contains 525 acres of land. What is it worth at $\$0.37\frac{1}{2}$ a sq. ft.?

27. A owns 12 A. 13 sq. rd. 2 sq. ft., and B $\frac{3}{5}$ of an acre of land. How much is it all worth at 10¢ per sq. ft.?

28. How many inches in $\frac{1}{16}$ of a mile?

29. Bought a horse for $\$90$, and sold him for $\$95$. What per cent of gain? Bought another horse for $\$95$, and sold him for $\$90$. What per cent was lost?

30. Bought land at $\$62.50$ per acre, and sold it again at $\$75$ per acre, thereby making $\$846{,}875$. How many acres were bought?

VOLUMES OF PRISMS AND PYRAMIDS. OF CYLINDERS AND CONES.

1145. Slate Exercises.

SUGGESTION. — Have the pupils construct of cardboard a hollow square prism of convenient size, and a pyramid having base and altitude respectively equal to those of the prism. Let them use sand or water to ascertain how

many times the contents of the pyramid must be taken to exactly fill the prism.

Volume of prism or cylinder = area of base × altitude.

Volume of pyramid or cone = area of base × $\frac{1}{3}$ altitude.

1. Find the volume of a square pyramid, the area of the base being 9 square feet and the altitude 6 feet.

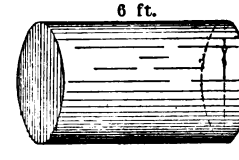
2. What is the volume of a square pyramid whose altitude is 12 inches, one side of the base being 10 inches?

3. The base of a prism is a triangle whose sides measure 3, 4, and 5 inches, respectively. Find the solidity, its altitude being 10 inches.

4. The base of a prism 19 feet high is a rectangle whose sides are 9 feet and 13 feet. How many cubic yards does it contain?

5. Find the volume of a prism whose bases are equilateral triangles, each side being 4 ft., and the height of the prism being 12 ft.

6. How many cubic feet are there in a stone roller 6 ft. long, 8 ft. in circumference?



7. Find the volume of a cone whose altitude is 18 meters, diameter of base 6 meters.

8. How many gallons of oil (231 cu. in.) will fill a cylindrical tank $5\frac{1}{2}$ ft. high, radius of base 3 ft.?

9. How many pounds of water would the above tank hold? (1 cu. ft. = 1,000 oz.)

10. Measure accurately the interior dimensions of a quart or a pint cup, and calculate its volume.

NOTE. — How many cubic inches in a quart, liquid measure?

11. Measure the interior dimensions of a peck or a bushel, and calculate its volume.

12. Pour a quart or a pint of water into a paper box having a rectangular base, and calculate the number of cubic inches of water in the box.

What would be the depth of a quart of water in a box whose base measures $5\frac{1}{2}$ by 3 inches?

13. Find the volume of a cone whose altitude is 18 yd., circumference of base 25.1328 yd.

14. Find the slant height of a cone whose altitude is 12 inches, the radius of the base being 5 inches.

15. Calculate the convex surface, the entire surface, and the volume of a square pyramid whose altitude is 24 inches, one side of the base being 14 inches.

1146. U. S. Civil Service Examination for Clerkships.

1. According to the census of 1880, the value of manufactured products of Boston was \$130,531,993; Brooklyn, \$177,223,142; Chicago, \$249,022,948; New York, \$472,926,437; Philadelphia, \$324,342,935; St. Louis, \$114,333,375. What was the total value of manufactured products of the cities named?

Express in figures the following numbers:

2. One million one thousand one hundred one.

3. Two hundred two million one hundred thousand one, and six hundred-thousandths.

Express in words the following numbers:

4. 103,004,601.00025.

5. 10,010,011,206.

6. The cost price of beef is $10\frac{3}{8}¢$ per pound, and of flour $3\frac{7}{8}¢$ per pound. A ration consists of 1 lb. 4 oz. of beef, and 1 lb. 6 oz. of flour. What will be the cost of 10,840 rations at the above rates?

7. From 1,000 grams of pure gold may be coined 279 of the 10-mark pieces of Germany. One gram is equivalent to 15.432349 troy grains. The U. S. gold dollar contains $23\frac{22}{100}$ troy grains. What is the equivalent in U. S. dollars of the 10-mark piece, decimally expressed?

8. Of an importation of wool weighing 42 tons 19 cwt. 3 qr. 20 lb., 21 tons 4 cwt. 1 qr. 19 lb. are sold, and one-half the remainder is lost by fire. How much is left? (The cwt. equals 112 lb.)

9. Two money counters in the Treasury were given packages of redeemed U. S. notes to count. The first received 100 $100 notes, 200 $50 notes, 300 $20 notes, and 400 $10 notes; and of each denomination there were 10 notes discounted $\frac{3}{10}$ each. The second counter received 50 $100 notes, 150 $50 notes, 250 $20 notes, and 350 $10 notes; and of each denomination there were 20 notes discounted $\frac{2}{10}$ each. What was the total face value of all the notes when issued, the total discount, and the cash value of the notes redeemed?

10. The whole amount of fractional currency issued was $368,724,079.45, and the amount outstanding unredeemed June 30, 1883, $15,354,425.31. What amount had been redeemed at that time, and what per centum was it of the whole amount issued?

11. At the close of business July 31, 1884, the interest-bearing debt of the United States was as follows: Bonds at 3 per cent, $237,453,250; bonds at $4\frac{1}{2}$ per cent, $250,000,000; bonds at 4 per cent, $737,954,700. What was the total annual interest charge; the average rate of interest the total debt bore (decimal carried to four places); and the amount that would have been saved in interest per annum if the entire debt had been refunded at $2\frac{1}{2}$ per cent?

12. The market rate of a 5 per cent stock is $85\frac{1}{2}$ per cent. If the purchaser pays brokerage (at $\frac{1}{4}$ per cent on par value), what rate of interest does he receive on his investment?

13. How much gold at $111\frac{5}{8}$ can be bought for $8,930 in currency?

14. Richard Wells, a contractor, furnished to the Interior Department, January 1, 1882, 645 barrels of flour at $9.45 per barrel; January 16, 1,912 bushels of oats at 57¢ per bushel; April 4, 9,231 pounds of bacon at 9¢ per pound; May 3, 8,264 bushels of corn at 74¢ per bushel; and June 20, 325 barrels of pork at $12.65 per barrel. January 31, 1882, he was paid cash, $885; February 5, $450; April 11, $615.35; and May 30, $4,162.15. On inspection, June 25, 345 pounds of bacon and 35 barrels of pork were condemned and rejected; and on settlement, June 30, 1882, he was charged $75 as penalty for failure to deliver goods in time, according to the terms of the contract, allowed a credit of $65 for cartage, and paid the balance due him in cash. State in the form below, with proper headings, Wells's account with the Interior Department.

LUMBER MEASURE.

1147. Lumber is measured in *board feet.* A board foot is 1 foot long, 1 foot wide, 1 inch thick.

A board 16 feet long, 1 foot wide, 1 inch thick, contains 16 board feet.

A board 16 feet long, 9 inches wide, 1 inch thick, contains $(16 \times \frac{3}{4})$ board feet, or 12 board feet. A board of the same length and width, 2 inches thick, contains (12×2) board feet, or 24 board feet.

In practice, the term *board foot* is seldom used, the word *foot* alone being generally employed.

1148. Find the number of feet (board feet) in each of the following boards and planks:

1. 16 feet long, 12 inches wide, 1 inch thick.
2. 14 " " 6 " " 1 " "
3. 12 " " 8 " " 1 " "
4. 14 " " 12 " " 1 " "
5. 16 " " 3 " " 1 " "
6. 12 " " 9 " " 2 " "
7. 14 " " 3 " " 2 " "
8. 16 " " 9 " " 2 " "
9. 14 " " 6 " " 2 " "
10. 12 " " 8 " " 2 " "
11. 18 " " 9 " " 3 " "
12. 9 " " 12 " " 3 " "
13. 20 " " 6 " " 3 " "
14. 15 " " 4 " " 3 " "
15. 12 " " 3 " " 3 " "

16. What is the cost, at $30 per thousand feet, of 15 planks, each 16 feet long, 9 inches wide, 3 inches thick?

17. Find the number of (board) feet of lumber required to floor a dock 36 feet long, 17 feet 6 inches wide, the planks being $2\frac{1}{2}$ inches thick.

18. Find the duty, at $1 per thousand feet, on the following lumber imported from Canada:

150 boards, 13 feet long, 8 inches wide, 1 inch thick;

60 planks, 14 feet long, 9 inches wide, 2 inches thick;

40 scantlings, 15 feet long, 5 inches wide, 4 inches thick.

19. At \$18 per thousand, what will be the cost of the boards necessary to enclose a field 160 yards long, 120 yards wide, with a fence 4 boards high, each board 6 inches wide, 1 inch thick?

20. The owner of a lot, 25×100 feet, wishes to enclose it by a tight board fence 6 feet high. He pays 25¢ each for the posts, which are placed $6\frac{1}{4}$ feet apart. The boards, costing \$25 per M., are 1 inch thick, and are nailed to two continuous horizontal strips of scantling, 3 inches wide, 2 inches thick, costing \$18 per thousand. How much is paid for the material, no allowance being made for waste?

1149. St. Paul Public Schools. Examination Questions.

1. (*a*) Write a negotiable note requiring endorsement. (*b*) When a man is unable to write his name, in what way does he sign a note?

2. Write equivalent fractions for each of the following, and find their sum: $\frac{1}{3}$, $\frac{2}{7}$, $\frac{14}{25}$, $2\frac{3}{14}$, $\frac{143}{286}$.

3. How many building lots, each 75 ft. by 125 ft., can be laid out on 1 A. 46 P. (sq. rd.) $18\frac{1}{2}$ sq. yd.? Analyze.

4. A young man inherited a fortune, $\frac{1}{4}$ of which he spent in 3 mo., and $\frac{3}{7}$ of the remainder in 10 mo., when he had only \$2,524 left. What was the inheritance?

5. If I lend my neighbor \$200 for 6 mo., how long ought he to lend \$1,000 to balance the favor?

6. What number is that which being increased by its half, its third, and 18 more, will be doubled?

7. Two ships sail from the same port; one goes due north 128 miles, and the other due east 72 miles. How far are the ships from each other? Illustrate.

8. B and C, trading together, find their stock to be worth \$3,500, of which C owns \$2,100. They have gained 40% on their first capital. What did each put in?

9. What is the difference between the true discount and the interest of $487, due in 2 yr. 4 mo., at 5%? (Art. 1018.)

10. What is brokerage, insurance, duty, tax, interest, discount, longitude, percentage?

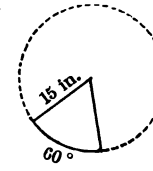
MENSURATION.

1150. Slate Problems.

Area of circle = ½ circumference × ½ diameter.
Area of sector = ½ arc × ½ diameter.

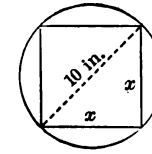
1. Find the area of a semicircle whose radius is 20 feet.

2. How many square inches are contained in a sector of 60°, the radius of the circle being 15 inches?



3. A square is inscribed in a circle 10 inches in diameter. Find its area.

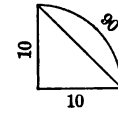
x = side of square, x^2 = area. Find x^2 from the right-angled triangle, without finding the value of x.



4. What is the difference between the area of a circle of 10 inches diameter and that of the inscribed square?

5. The sides of the above inscribed square are chords of arcs of 90°. Find the length of an arc of 90°, and of its chord.

6. A *segment* of a circle is that portion of the surface included between an arc and its chord. Find the area of a sector of 90° and the area of the segment, the radius of the circle being 10 inches.

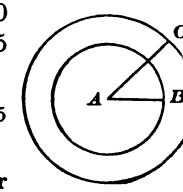


7. Calculate the area of a circle whose radius is 1 inch. Of a circle whose radius is 2 inches. What is the ratio of the two areas?

8. What is the ratio between the area of a circle whose radius is 1 inch and that of a circle whose radius is 3 inches?

The area of a circle = square of radius × ?

9. How many square yards are there in a circular walk, the radius, AB, of the inner edge of walk being 10 feet, and that of the outer edge, AC, being 15 feet?

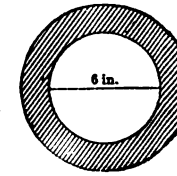


(Find the difference between the area of a circle of 15 ft. radius, and that of a circle of 10 ft. radius.)

10. A circular flower-bed 20 feet in diameter is surrounded by a walk 5 feet wide. How many square feet of surface does the walk contain?

(If you have to subtract 100 times 3.1416 from 225 times 3.1416, how can you shorten the work?)

11. How many square inches are there in the surface of a frame 3 inches wide, around a looking-glass 6 inches in diameter?

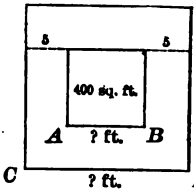


(Area = ? × 3.1416.)

12. What is the ratio between the surface of the above frame and that of the looking-glass?

(Indicate operations and cancel.)

13. What is the area of a walk 5 feet wide around the outside of a square plot containing 400 sq. ft.?



(What is the area of the large square, including the walk?)

14. The outer edge of a walk 5 feet wide, surrounding a plot of ground, measures 120 feet, the inner edge measures 80 feet. How many square feet does the walk contain?

(The "average" length of the walk is $\frac{120 + 80}{2} = 100$ ft.; that is, its length measured on a line along the center of the walk.)

15. Find the ratio between the area of a triangle whose sides measure 16, 30, and 34 feet, respectively, and the area of another whose sides are 32, 60, and 68 feet.

SURFACE OF SPHERE.

1151. Take a wooden hemisphere and drive a tack into the center of its curved surface. Commencing at the tack, carefully wind a waxed cord about the curved surface, in the way a boy winds a top. When this surface is exactly covered, cut the cord.

Wind the same cord around a tack driven into the plane surface of the base of the hemisphere, pressing it closely to the surface. When the latter is entirely covered, just one-half of the cord will be used.

If a sphere is cut through in any direction, the section made will be a circle. The section formed when the sphere is cut through the center is called a *great circle.*

The above experiment shows that the surface of the hemisphere is equal to that of two great circles of the same sphere.

1152. The surface of a sphere is equal to that of four great circles.

Since the surface of a great circle of the sphere is $\frac{1}{2}$ diameter $\times$ $\frac{1}{2}$ circumference, the surface of the sphere is $\frac{1}{2}$ diameter $\times$ $\frac{1}{2}$ circumference $\times$ 4 = diameter of sphere $\times$ the circumference.

Calling the radius of a circle R, and using the Greek letter π instead of 3.1416, we have

Diameter of circle $= 2R$.

Circumference of circle $= 2\pi R$.

Area of circle $= \pi R^2$. ($\frac{1}{2}$ of $2R \times \frac{1}{2}$ of $2\pi R$.)

Surface of sphere $= 4\pi R^2$.

1153. Slate Exercises.

16. Find the surface of a sphere whose radius is 1 inch.

Of a sphere whose diameter is 2 inches.

Of a sphere whose circumference is 6.2832 inches.

17. At 10 cents a square foot, what will be the cost of gilding a sphere 12 inches in diameter?

18. Find the ratio between the surface of a sphere 1 foot in diameter, and the convex surface of a cylinder 1 foot high, the diameter of the base 1 foot.

19. What is the ratio between the surface of the above sphere and the entire surface of the cylinder?

20. Find the surface of a sphere whose circumference is 20 inches.

1154. U. S. Civil Service. Bookkeeper's Examination.

1. The tax assessed by a certain town is $18,750; its property is valued at $1,250,000. What rate per cent is the tax, and how much will be paid by a man whose property is assessed at $6,000?

2. Add these across, placing the totals in the spaces provided, and find grand total:

			Total.
$496.001	$32.741	$18.90	$ .....
75.60	37.204	.17	.....
1,476.009	618.2045	84.29	.....
89.8045	777.09	4.221	.....
1,612.220	1.47	329.90	.....
77,624.00	6.196	91.19	.....
134.93	16.031	1,719.10	.....
16.676	414.80	5.125	.....
232.301	17.76	1,801.90	.....
88.74	242.001	1.29	.....
1.9011	758.23	96.427	.....
	Grand total.....		$ .....

3. What is the value in currency of 7,984 ounces of silver bullion at $19.8 per pound?

4. What must be paid for a $200 four-and-a-half per cent bond, in order to make it an 8 per cent investment?

5. G and H engaged in business as partners. G invested $10,000 and H $5,000, G sharing $\frac{2}{3}$ and H $\frac{1}{3}$ of the gains and losses. Their assets at the close of the year consisted of cash, $2,100; bills receivable, $4,400; merchandise, $13,000; and personal accounts, $8,000, 10 per cent of which are considered worthless. Their liabilities are bills payable, $1,625; personal accounts, $5,625. G drew out during the year $850 and H $1,075.

If H should retire from the firm, how much would he be entitled to receive?

CUBE ROOT.

1155. To cube a number is to employ it three times as a factor.

The cube of 4, written 4^3, is $4 \times 4 \times 4$, or 64.

Find the cube of 1, 9, 6, 3, 5, 8, 2, 7.

To find the cube root of a number is to find one of the three equal factors of the number.

The cube root of 343, written $\sqrt[3]{343}$, is 7.

The cube of 25, $20 + 5$, is equal to the following:

We have seen (Art. 1031) that

$$(20+5)^2 = 20^2 + 2 \times 20 \times 5 + 5^2$$

Multiplying by $20 + 5$ we have

$$\text{Product by } 20 = 20^3 + 2 \times 20^2 \times 5 + 20 \times 5^2$$

$$\text{Product by } 5 = 20^2 \times 5 + 2 \times 20 \times 5^2 + 5^3$$

$$(20+5)^3 = 20^3 + 3 \times 20^2 \times 5 + 3 \times 20 \times 5^2 + 5^3$$

which may be written in this way,

$$20^3 + [(3 \times 20^2) + (3 \times 20 \times 5) + 5^2] \times 5.$$

1156. Extract the cube root of 15,625.

We see by inspection that the cube root is between 20 and 30; that is, $20 + x$. Subtract from 15,625 the cube of 20, 8,000. The remainder, 7,625, is equal to the second number multiplied by the sum of three times the square of the first (1,200), etc. Using 1,200 as a trial divisor, the second number is seen to be 6 or less.

$$\begin{array}{rrrl} & & 20+5 & \\ & & \overline{15{,}625} & \\ (20)^3 = & & 8{,}000 & \\ 3\times 20^2 = & 1{,}200 & 7{,}625 & \text{remainder} \\ 3\times 20\times 5 = & 300 & & \\ 5^2 = & 25 & & \\ & \overline{1{,}525} & \underline{7{,}625} & \end{array}$$

Taking 5 as the second number, we add to the 1,200 three times the product of the first and second (300), and the square of the second (25), making a total of 1,525. Multiplying this sum by the second number, we get 7,625, which is equal to the difference between 15,625 and 8,000. The second number is, therefore, 5, and the cube root of 15,625 is 25.

$\sqrt[3]{110{,}592}$

$$\begin{array}{rrrl} & & 40+8 & \\ & & \overline{110{,}592} & \\ 40^3 = & & 64{,}000 & \\ 3\times 40^2 = & 4{,}800 & 46{,}592 & \\ 3\times 40\times 8 = & 960 & & \\ 8^2 = & 64 & & \\ & \overline{5{,}824} & \underline{46{,}592} & \end{array}$$

Ans. 48.

$\sqrt[3]{658{,}503}$

$$\begin{array}{rrrl} & & 8\ \ 7 & \\ & & \overline{658'503} & \\ 8^3 = & & 512 & \\ 3\times 80^2 = & 19\,200 & 146{,}503 & \\ 3\times 80\times 7 = & 1{,}680 & & \\ 7^2 = & 49 & & \\ & \overline{20{,}929} & \underline{146{,}503} & \end{array}$$

Ans. 87.

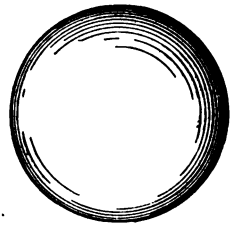
In the last example we point off three places, beginning at the right, and find the greatest cube in the first period, placing its cube root as the first figure of the answer.

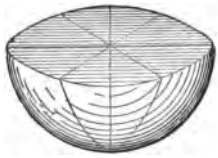
1157. Find the cube root of the following:

1. 2,197	6. 238,328	11. $\frac{8000}{12167}$
2. 9,261	7. 421,875	12. 3.375
3. 32,768	8. 551,368	13. $1\frac{217}{512}$
4. 68,921	9. $\frac{512}{729}$	14. $1\frac{397}{1331}$
5. 148,877	10. $\frac{1331}{2744}$	15. $5\frac{461}{729}$

VOLUME OF SPHERE.

1158. Cut up a sphere (a round potato, for instance) into a number of small pieces, passing the knife in each case through the center of the sphere.





Each piece is a solid, having for its base a portion of the surface of the sphere, and for its altitude the radius of the sphere.

When the pieces become very numerous, the base of each may be considered a plane, and the solid a pyramid. The volume of each pyramid is

equal to the base $\times \frac{1}{3}$ altitude; and the total volume of all, which is the volume of the sphere, is equal to the total surface of all the bases, which is the surface of the sphere, multiplied by $\frac{1}{3}$ altitude, that is, $\frac{1}{3}$ radius.

$$\text{Surface of sphere} = 4\pi R^2,$$

therefore, $$\text{volume of sphere} = 4\pi R^2 \times \tfrac{1}{3} R = \tfrac{4}{3}\pi R^3.$$

1159. Slate Exercises.

1. Find the volume of a sphere whose radius is 3 inches.

2. If the diameter of a sphere is 3 inches, what is its volume?

3. What is the ratio between the volumes of two spheres whose diameters are 1 foot and 2 feet, respectively?

4. Find the ratio between the volume of a sphere 1 foot in diameter, and that of a cube whose side is 1 foot.

5. The radius of a sphere is 18 inches. What is the circumference of a great circle? The surface? The volume?

6. What is the weight of an iron cannon-ball 12 inches in diameter, considering the weight of a cubic foot of water as 1,000 ounces, and considering iron 7.5 times as heavy as water?

7. Find the ratio between the volume of a sphere 4 inches in diameter, and that of a cylinder 4 inches in altitude, radius of base 4 inches.

NOTE. — *Indicate* the volume of each, and cancel.

8. A man has a cubical block of hard wood, its side measuring one foot, which he wishes made into a sphere one foot in diameter. What decimal part of the block is cut away?

The volume of the sphere is *about* what fraction of the volume of the cube?

9. How much would be cut away in making a cylinder one foot in diameter and one foot high, from the above cubical block?

About what fraction of the volume of the cube is the volume of the cylinder?

1160. Chicago Public Schools.

1. A general wished to remove 80,000 pounds of provisions from a fortress in 9 days. It was found that in 6 days 18 men had carried away but 18 tons. How many men would be required to carry away the remainder in 3 days?

2. A, B, and C enter into partnership. A puts in $500 for 4 months, B $400 for 6 months, and C $800 for 3 months; they gain $340. Find each man's share of the gain.

3. A school-room is 40 feet long, 30 feet wide, and 14 feet high. Find the difference between the length of a diagonal drawn on the floor and one drawn from the floor to the ceiling.

4. A merchant bought $6,500 worth of goods for cash, and sold them on 4 months' credit at 15% advance. He got the note discounted at 6% to pay the bill. How much did he make?

5. A merchant in Galveston paid $4,265 for a draft on St. Louis at 30 days' sight, exchange being $3\frac{1}{4}\%$ premium, interest 8%. Required the face of the draft.

6. Find the solid contents and the surface of a sphere 12 inches in diameter.

7. A man traveled from New York, lon. 74° 3′, to San Francisco, lon. 122° 27′, without changing his watch. On arriving at San Francisco was his watch too fast or too slow, and how much?

8. Find the entire surface of a cone whose altitude is 10 feet, and diameter of base 8 feet.

9. A, having a quantity of canal stock, sold 25% of it to B, who sold $33\frac{1}{3}\%$ of his purchase to C, who sold $37\frac{1}{2}\%$ of his purchase to D, who received 6 shares. How many shares had A at first?

10. A and B engaged in business for one year with the same capital. A increased his capital by $\frac{1}{5}$ of it, and B lost $\frac{1}{8}$ of his. The difference between their capitals then was $1,040. Find the capital of each at the beginning.

11. The number of copies in the first edition of the "Lady of the Lake" was 2,050, and was to the number in the second edition as 41 to 69. Find the number in the second edition.

12. At the end of 12 months A, B, and C, having a joint capital of $6,000, find that they have lost $625. A's capital of $2,500 has been in the business for 12 months, B's of $1,500 for 8 months, and C's of $2,000 for 4 months. Divide the loss among them.

13. Find the proceeds of the following note:

$1,050\frac{00}{100}$. CHICAGO, Feb. 13, 1885.

Six months after date I promise to pay to the order of Geo. Hall, One Thousand Fifty Dollars, with interest at 6 per cent.

HENRY SHAW.

Discounted at 8 per cent May 13.

14. Find the cost of a draft for \$1,450 payable in 60 days, when exchange is $\frac{1}{4}\%$ discount, and interest 5 per cent.

15. West Point is in longitude 74° 57′, and St. Louis in longitude 90° 15′. When it is 4 o'clock P.M. at West Point, what time is it at St. Louis?

16. If 108 men can build a fort in $12\frac{1}{2}$ days of $12\frac{1}{2}$ hours each, in how many days can 84 men build it by working $10\frac{1}{2}$ hours daily?

17. A can do $\frac{1}{8}$ of a piece of work in four days, and B can do $\frac{1}{4}$ of it in 5 days. In what time can they do the whole work together?

18. A man desires to secure an annual income of \$650 for his daughter. How much 5 per cent stock must he buy at par to do it?

19. A square is inscribed in a circle whose diameter is 84 inches. Find the area of the four segments of the circle outside of the square.

20. Find the difference between the volume of a cylinder whose diameter and height are 12 inches, and the volume of a sphere whose diameter is the same.

1161. New York Public Schools. Examination for Admission to the City College.

1. (*a*) Define a common fraction; (*b*) a compound fraction; (*c*) a proper fraction.

(*d*) Give the rule for the multiplication of one decimal fraction by another.

(*e*) What is discount?

2. Add the following fractions:

(*a*) $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{3}{10}$, $\frac{\frac{4}{5}}{\frac{7}{10}}$, $\frac{3}{8}$.

(*b*) Find the value of the following:

$$\frac{2\frac{1}{2}}{3} + \frac{1}{2} \text{ of } \frac{3}{8} - \frac{1\frac{1}{2} \times \frac{7}{8}}{\frac{7}{4} \text{ of } \frac{16}{5}}.$$

3. (*a*) Write in a decimal form:

$$\frac{3}{10000},\ \frac{4}{100},\ \frac{516}{100}.$$

(*b*) How do you most easily multiply a decimal by 10?

(*c*) Divide 16 by 4; by 400; by .004.

(*d*) Multiply 41 by .0006.

4. In multiplying $\frac{3}{4}$ by $\frac{5}{8}$ explain why canceling the 3 in the numerator and the denominator will give a correct result.

5. Divide 3.25 by .5, and explain the reason of the rule for pointing off the decimal places in the quotient.

6. What sum of money put at interest for 2 years 9 months and 9 days, at 6 per cent per annum, will produce $2,951.80?

7. A certain cistern can be filled by one pipe in 10 hours, by another in 6 hours, and can be emptied by a third in 5 hours. In how many hours can it be filled if all three pipes are opened at once?

8. An importer bought in France 1,000 pieces of a certain goods at $40 per piece. The duty paid on importing the goods was 50% of the cost, and the freight and other charges were in all $1,500. For how much per piece must he sell the goods so as to make 20%?

9. Two men start from two towns 105 miles apart and walk toward each other. They meet at the end of 15 hours. The first has traveled 3 miles per hour. At what rate has the second traveled?

10. If 10 men, working 8 hours per day, can build a certain wall in 6 days, how many hours a day must 12 men work to build the same wall in 4 days?

11. (*a*) What is meant by the ratio of one quantity to another?

(*b*) What is meant by the Greatest Common Divisor of several numbers? Give an example.

(*c*) What is meant by a minuend? By a quotient?

(*d*) When is a common fraction said to be in its lowest terms?

12. (*a*) Reduce to its simplest form the following expression:

$$2\tfrac{1}{4} \times \tfrac{4}{7} \times \frac{2\frac{1}{2}}{\frac{3}{8}} \text{ of } \tfrac{14}{15}.$$

(*b*) Find the value of the following:

$$\tfrac{3}{4} + \tfrac{2}{3} - \tfrac{3}{5} + \tfrac{5}{6}.$$

(*c*) Reduce $\frac{15}{24}$ to a decimal fraction.

13. (*a*) Find the value of $\frac{3}{440}$ of a mile in the lower denominations.

(*b*) What is the effect upon the value of a fraction if we multiply the denominator by 3?

(*c*) Explain the reason of the last answer.

14. (*a*) Write in decimal form the following:

Ten ten-thousandths.

One thousand twenty-four ten-millionths.

Thirty-two and four thousandths.

(*b*) Multiply .0036 by 1.02.

(*c*) Divide 2.56 by .0016.

15. (*a*) If a cipher is added at the right of the decimal, what effect has this on the value of the decimal?

(*b*) Explain the reason of this.

16. (*a*) What part of $\frac{2}{3}$ is $\frac{4}{7}$?

(*b*) What is the easiest method of dividing a decimal by 10?

(*c*) Reduce 20 square rods to the decimal of an acre.

17. If by selling a house for \$12,600, a builder gains $12\frac{1}{2}$ per cent, what per cent would he have lost by selling it for \$8,400?

18. A, B, and C began a partnership on January 1. A put in \$10,000, B, \$6,000, and C, \$3,000. At the end of six months B put in \$4,000 more, but A withdrew \$2,000. At the end of the year they had on hand \$18,000 in cash, and goods valued at \$12,000. At this time the property was divided and the firm

dissolved, A taking all the goods as part of his share. How much cash did each of the three receive?

19. A traveler going from one town to another walks $\frac{1}{5}$ of the distance on his first day's journey, $\frac{1}{4}$ of the remainder on the second day, 20 miles on the third day, and finds he has $\frac{4}{10}$ of the distance still to go. What is the distance?

20. A dealer bought 1,200 barrels of flour at \$6 per barrel, and shipped them to New York by railroad. An accident on the road destroyed $\frac{1}{8}$ of them. He sold the remainder at \$7.50 per barrel. His freight and cartage were \$312. He sued the railroad company, and recovered some damages, but his legal expenses were \$350. At the close of the transaction, he found he had made, over all expenses, just \$138. How much did he receive from the railroad company?

21. (*a*) What is meant by a decimal fraction?

(*b*) Define a factor; a multiplicand; a root.

(*c*) In every common fraction, what is shown by the denominator? What, by the numerator?

22. (*a*) Reduce to its simplest form the following expression:

$$\frac{3\frac{1}{4}}{4\frac{1}{8}} \text{ of } \frac{12}{7} \times \frac{7}{3}.$$

(*b*) Reduce the following fractions to their least common denominator: $\frac{3}{4}$, $\frac{2}{3}$, $\frac{2}{8}$ of $\frac{15}{16}$.

(*c*) Find the value of $\frac{1}{4}$ of $\frac{3}{8} + \frac{1}{3} - \frac{1}{12}$.

23. (*a*) What part of $\frac{3}{8}$ is $\frac{2}{7}$?

(*b*) Reduce $\frac{3}{800}$ to a decimal fraction.

(*c*) Multiply .05 by 3.2.

24. (*a*) If the numerator of a common fraction is divided by 3, what is the effect upon the value of the fraction?

(*b*) If the denominator is divided by 3, what is the effect upon the value of the fraction?

(*c*) Explain the reason of this last result.

25. (*a*) Write in decimal form:

Ninety millionths; thirty ten-thousandths; ten and twenty-five thousandths.

(*b*) Divide .064 by .000016.

(*c*) Add .003, 12.06, 1.1.

(*d*) Subtract 2.3 from 4.006.

26. (*a*) What is the effect on the value of a decimal of moving the decimal point two places to the right?

(*b*) Explain the reason of this.

(*c*) In the multiplication of two decimals, how many decimal places are to be pointed off in the product?

(*d*) Explain the reason of this rule.

27. How much is $5\frac{4}{5}$ tons of coal worth, if $17\frac{2}{5}$ tons are worth \$100?

28. A person expended 16 per cent of all he was worth in buying 20 per cent of the stock of a mining company. If the entire stock of the company sold for \$160,000, what must the person have been worth?

29. Four men undertook to do a piece of work in 18 days and worked at it 6 hours a day for 10 days, when they found they had finished only $\frac{1}{3}$ of it. How many more men did they have to engage, in order to finish the job by the time agreed upon, provided they all worked thereafter for 10 hours each day?

30. A provision merchant bought 100 barrels of apples at a farm house at \$1 per barrel, and paid 5¢ per barrel to have them taken to the railroad station. Then he paid \$50 freight on them to New York, and \$20 cartage in the city. They were sold at once for \$3 per barrel, but the commission merchant charged him 10% commission on the sale. Also, when some of the barrels were opened, the apples were found to be damaged, and he had to repay the purchasers \$20 on account of this. How much did he gain in all?

31. (*a*) What is an improper fraction?

(*b*) What is a mixed number?

(*c*) What is interest?

(*d*) When are four numbers said to be in proportion?

32. (*a*) Divide $1\frac{2}{3}$ by $\frac{\frac{5}{7}}{3}$.

(*b*) Reduce the following fractions to equivalent fractions having their least common denominator:

$$\tfrac{3}{10},\ \tfrac{3}{8} \text{ of } \tfrac{7}{8} \text{ of } \frac{2\frac{2}{7}}{3},\ \tfrac{2}{5} \text{ of } \tfrac{3}{5}.$$

(*c*) Add together the results obtained in (*b*).

33. (*a*) What fraction of $\frac{5}{7}$ is $\frac{1}{2}$?

(*b*) Find the value of $\frac{1}{14}$ of a mile in rods, yards, feet, and inches.

(*c*) Reduce $\frac{5}{80}$ to a decimal fraction.

34. What is the effect of multiplying the numerator and the denominator of the same fraction by 4? Explain the reason.

35. (*a*) Write in decimal form the following:

Ten and ten hundredths; nine millionths; thirty thousandths; thirteen hundred and forty-two hundredths.

(*b*) Add .003, 1.25, 20,006.

(*c*) Divide .048 by 1,600.

36. (*a*) Multiply .26 by .0035.

(*b*) Divide .006 by 100, by the shortest method.

(*c*) Explain the reason of this method.

37. If 4 men, working 8 hours per day, can mow a meadow in 3 days, how many men, working 9 hours per day, can mow a meadow three times as large in 4 days?

38. A, B, and C entered into partnership for one year. A put in \$5,000, B, \$6,000, and C, \$4,000. At the end of six

months, A withdrew \$2,000, and C put in \$8,000 more. The profits at the end of the year were \$6,000. What was each man's share?

39. A bought merchandise from B for \$10,000, and gave his note for six months, without grace, with interest at 6%. Just when the note was due, he sold the goods to C for \$12,000, taking C's note at 3 months, without interest, which his bank discounted for him the same day. After paying his note to B, how much money had he remaining?

40. A drover bought a drove of 50 cattle for \$2,000. He sold $\frac{1}{5}$ of them at a gain of 10% on the average price, and $\frac{1}{5}$ of them at a gain of 15%. Half of the remainder, however, were so injured in a railroad accident that he could obtain only \$100 for them. For what price apiece must he sell the rest so that his total loss shall be \$100?

CUBE ROOT.

1162. Find the cube root of 9,938,375.

When the root contains more than two figures, continue, as shown in the accompanying example, taking for divisor three times the square of the first two figures considered as tens, plus three times the product of the first two figures considered as tens by the third figure, plus the square of the third figure.

		2 1 5
		9'938'375
		8
$3 \times 20^2 =$	1 200	1 938
$3 \times 20 \times 1 =$	60	
$1^2 =$	1	1 261
$3 \times 210^2 =$	132 300	677 375
$3 \times 210 \times 5 =$	3 150	
$5^2 =$	25	
	135 475	677 375

1163. Find the value of the following:

1. $\sqrt[3]{1,442,897}$
2. $\sqrt[3]{1,906,624}$
3. $\sqrt[3]{3,723,875}$
4. $\sqrt[3]{39,651,821}$
5. $\sqrt[3]{12.977875}$
6. $\sqrt[3]{66.923416}$

1164. Wilmington, Del., Public Schools. Examination Questions.

1. What is the interest on $500, for 3 years 7 months and 18 days, at 6 per cent?

What is the exact interest on $400, from March 1 to December 17, at 5 per cent?

2. At what rate will a principal double itself in 12 years?

In what time will a principal amount to 2½ times itself, at 10 per cent?

3. What is the face of a note that will give $597.60 as the proceeds when discounted at a bank for 24 days?

A ninety-days note, dated May 7, was discounted May 10. For how many days was it discounted?

4. I bought July 5th, goods to the amount of $2,400. $630 was to be paid at once, $820 in 8 mo., and $950 in 9 mo. What is the equated time for the payment of the whole?

5. A and B in partnership have together a capital of $7,500, and gain $1,200. A's share of the gain is $250. What is B's share of the gain? What is B's share of the capital?

6. The par of Union National Bank stock is $25, and the present value $85; and the par of Delaware Railroad stock $25, and the present value $31. The first pays a semi-annual dividend of 7½ per cent, and the second of 3 per cent. What rate of interest semi-annually does each dividend pay?

7. The diagonal of a rectangle is 100 feet, and the base 80 feet. What is the altitude?

What is the cube root of 12,326,391?

8. The circumference of a circle is 15.708 feet. What is the radius of it?

The radius of a circle is 42. What is the circumference of it?

9. How many square inches in the entire surface of a triangular prism, if the edges at the ends of the prism are each 5 inches long, and the prism is 10 inches long?

10. Find the entire surface of a cylinder 10 inches long, and 8 inches in diameter.

Find the number of cubic inches in the same cylinder.

1165. New York Public Schools. Admission to Normal College.

1. Define the following terms employed in arithmetic:

Quantity, number, abstract number, multiplication, proportion.

Explain the reason for multiplying the second and third terms together and dividing by the first term in solving an example in simple proportion.

2. $\frac{\frac{2}{3} \text{ of } \frac{11}{12}}{\frac{1}{81} \text{ of } 5\frac{1}{2}}$ is what part of $\frac{\frac{2}{3} \text{ of } 3\frac{2}{3}}{\frac{1}{6} \text{ of } 5\frac{1}{2}}$?

3. Columbus is 83° 3′ west longitude; and when it is 37 min. 33 sec. past 1 P.M., it is 11 o'clock A.M. in San Francisco. What is the longitude of the latter city?

4. Divide thirty-two hundred-millionths by sixty-four ten-thousandths.

5. A, B, and C gained by speculation $11,480, of which A's share was twice as much as C's, and B's five times as much as C's. How much did each gain?

6. A man owes $600, of which one-third is to be paid in one year, and the remainder in two years. What is the present value, money being worth 6 per cent? (Art. 1017.)

7. I bought a watch for $120 and set such a price on it that, after falling $12, I still made 15 per cent on the purchase. What per cent did I abate from the asking price?

8. A pole was broken 52 feet from the bottom, and it fell so that the end struck 39 feet from the foot. Required the length of the pole.

9. Extract the cube root of $\frac{243}{24}$ to two places of decimals.

10. Sold a horse so that $\frac{4}{5}$ of the gain equaled $\frac{2}{15}$ of the cost. What was the gain per cent?

1166. Public Schools of British Columbia. Examination Questions. Oral Problems.

1. If $\frac{2}{3}$ yd. costs $4.50, what will $\frac{4}{5}$ yd. cost?

2. If 3 men can do a piece of work in 4 days, how long will it take 24 men to do it?

3. What principal at interest for 5 years, at 6 per cent, will produce $12, simple interest?

4. A stack of hay will keep a cow 20 weeks, and a horse 15 weeks. How long will it keep them both?

5. Divide $7\frac{1}{2}$ lb. of tea into two parcels, one of which is $1\frac{3}{4}$ lb. heavier than the other.

6. Sold a cow for $24, losing thereby 40% of the cost price. Had I sold her for $33\frac{1}{3}$% advance on the cost, what would I have received for her?

7. What will 460 lb. of tea cost at $.48 per lb.?

8. If 12 oz. of bread are destroyed in making a gill of whiskey, how much will be destroyed in making a gallon?

9. If the weight of air is 15 lb. on the square inch, what is it on the square foot?

10. Seven is three-fifths of what number?

11. What is the value of 960 lb. of wheat at $1.25 per bu. of 60 lb.?

12. At what rate % will $400 make $37.50, simple interest, in 1 yr. 3 mo.?

13. What is the brokerage on $10,400, at $1\frac{3}{4}$%?

14. What will 3,280 feet of lumber cost @ $25 per M.?

15. A and B are partners; A puts in $\frac{5}{12}$ of the stock, and B the remainder; B's gain is $1,400. Find A's gain.

1167. Public Schools of British Columbia. Examination Questions. Slate Problems.

1. Find the value in U. S. money of 2.5 of $3\frac{1}{2}$ guineas.

(1 guinea = 21 shillings. £1 = $4.88.)

2. Reduce to its simplest form:

$$\frac{\frac{5}{7} \text{ of } \frac{3}{10} + \frac{1}{2} \text{ of } \frac{8}{21}}{\frac{2}{3} \text{ of } \frac{9}{14} - \frac{5}{6} \text{ of } \frac{2}{15}}.$$

3. What is the difference between the compound interest on $5,000, for 3 years, at 5%, and on $10,000, for $1\frac{1}{2}$ years, at the same rate?

4. A trader gets 500 barrels of flour insured for 75% of the cost, at $2\frac{1}{5}$ per cent, paying $80.85 premium. At what price per barrel did he purchase the flour?

5. A can do a piece of work in 27 days, and B in 15 days; A works at it alone for 12 days, B then works alone for 5 days, then C finishes the work in 4 days. In what time could C have done the work by himself?

6. If 72 men dig a ditch 20 yd. long, 1 ft. 6 in. broad, 4 ft. deep, in 3 days of 10 hours each, how many men would be required to dig a ditch 30 yd. long, 2 ft. 3 in. broad, and 5 ft. deep, in 15 days of 9 hours each?

7. A man has been receiving 12% on his capital in Canada. He goes to England to live, and invests it in the 3 per cents, at $94\frac{3}{8}$, and his income in England is £2,400. What was his income in Canada, £1 being equal to $4.86\frac{2}{3}$?

8. At what price must an article that cost $30 be marked so that after deducting 40% from the marked price, 30% profit may be realized?

9. Write a ninety-days promissory note for which you should get $240 at the bank, discount being 10%.

10. A room is 15 feet long, 10 feet broad, and 9 feet 9 inches high. Find the cost of painting the walls and the ceiling, at 1*s.* 9*d.* a square yard.

1168. U. S. Naval Academy. Admission as Cadet Midshipman.

1. Express 6749 thousandths and 397 millionths as decimals, and find the quotient of the first by the second.

Reduce ($\frac{8}{25}$ of $2.45 - \frac{1}{100}$ of $.02) \div 1{,}000$ to a decimal.

Which is the greatest and which the least of the expression, $\frac{2}{3} + \frac{3}{4}$, 1.41421, and $\frac{1}{2} + \frac{1}{5} + \frac{5}{7}$?

What decimal part is .7409375 of 237100?

2. By what fraction must $\frac{1\frac{1}{4}}{1\frac{1}{12}}$ of $\frac{2}{3} + \frac{2\frac{1}{2} - 1\frac{5}{6}}{\frac{1}{4} + 1\frac{5}{6}} - \frac{8\frac{1}{5}}{7\frac{4}{5}}$ be divided, that the quotient may be $\frac{2}{3}$?

The factors of a certain number are $\frac{7}{12}$, $2\frac{5}{6}$, $\frac{11}{14}$, and $5\frac{1}{4}$. What is $\frac{22}{9}$ of $\frac{8}{21}$ of $\frac{3}{5}$ of this number?

The sum of two fractions is $\frac{11}{126}$, and their difference is $\frac{7}{18}$. Find the fractions.

3. Reduce $\frac{5}{9}$ of 16*s.* $4\frac{1}{3}$*d.* to the decimal of £1.

One kilometer = 1,000 meters = $\frac{5}{8}$ of a mile. Reduce 17 miles 6 furlongs 82 yards $1\frac{1}{2}$ feet to meters.

(8 furlongs = 1 mile.)

Find the value of 27 yards 2 feet 9 inches of gold lace, $\frac{7}{8}$ of an inch wide, if 17 yards 1 foot 11 inches of lace, 1 inch wide, cost $25.40.

4. One-third of A's money is equal to two-fifths of B's, and A has $17.50 more than B. How much money has each?

A and B run a race, their rates of running being as 17 to 18. A runs $2\frac{1}{8}$ miles in 16 minutes 48 seconds, and B runs the whole distance in 34 minutes. What is the distance run?

5. What is the present worth of $5,747, due 9 months hence, interest at $3\frac{1}{2}$ per cent? (Art. 1017.)

A vessel is worth $96,084. For what sum must the owner insure her, so that, if she is lost, he may receive both the value of the vessel and the sum paid for insuring her, the rate of insurance being $1\frac{7}{8}$ per cent?

6. Extract the square root of 900,780,169, and find the value of $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ to four decimal places.

7. A contractor undertook to complete a piece of work in 84 days, and engaged 30 men to do it. After 40 days, he put on 20 more men, and then finished the work 2 days too soon. How many days behindhand would he have been if he had not engaged the additional men?

8. The walls of a room are to be papered. The room is 23 feet 8 inches long, 15 feet 10 inches wide, and 11 feet 9 inches high; there are two windows, each 9 feet 6 inches high and 5 feet wide; a fireplace, 4 feet 6 inches high and 6 feet wide; and a door, 7 feet 6 inches high, 3 feet 6 inches wide. Find the cost of the paper required, at $4.08 per roll of 12 yards, the paper being 26 inches wide.

9. Divide $336 between two persons so that one may have $\frac{7}{9}$ as much as the other.

10. A man bought a horse for $365, giving his note due in 30 days. He sold the horse, taking in payment a note for $435, due in 4 months. What was his gain per cent?

(Interest at 6 per cent per annum.)

11. In what time will the interest on £57 1*s*. 8*d*. amount to £2 11*s*. $4\frac{1}{2}$*d*. at $7\frac{1}{2}$ per cent per annum?

12. Which is the greater, the square root of 281 or the cube root of 4,711?

13. A contractor engages what he considers a sufficient number of men to do a piece of work in 84 days; but he ascertains that 3 of his men can do respectively $\frac{1}{6}$, $\frac{1}{7}$, and $\frac{1}{8}$ less than an

average day's work, and two others $\frac{1}{8}$ and $\frac{1}{10}$ more. In order to complete the work in the given time, he procures the help of 17 additional men for the 84th day. How much less or more than an average day's work on the part of these 17 men is required?

14. The floor of a hall is 260 feet long and 93 feet wide. Find the cost of covering it with carpet and oil-cloth; the oil-cloth to be laid along the sides and ends a yard wide, and the carpet to extend 6 inches over the oil-cloth. Carpet, \$2.09 a yard, 38 inches wide; oil-cloth, 90 cents a square yard.

15. A man can walk from A to B in 1 hour less than a second man, and, when they start from opposite ends of the distance and walk in contrary directions, they meet at a point which is twice as far from A as it is from B. If the first man walks 5 miles per hour, what is the distance between A and B?

16. A train leaves B at 9 A.M., and runs to C at the rate of 15 miles an hour; and another train leaves A at noon, and, running through B to C at 25 miles an hour, reaches C half an hour later than the train from B. Find the distance from A to C, the distance from A to B being 15 miles.

1169. U. S. Naval Academy. Competitive Examination for Appointment as Cadet Engineer.

1. Add $\frac{11}{504}$ to the difference between .014 and $\frac{8}{250}$, and reduce the result to a decimal (six places). What per cent of £31 17*s.* 4*d.* is £5 19*s.* 6*d.*? Reduce $.20\dot{8}\dot{3}$ (the 83 is repeated indefinitely) to a common fraction.

$(.20\dot{8}\dot{3} = .20\frac{83}{99}.)$

2. What will it cost to make a road 104 rods long and 60 feet wide, the land costing \$154 an acre, grading at the rate of \$200 a mile, and fencing one side at 50 cents a yard?

3. Find the square root of 324.00007625, and the value of $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} - \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, each to six places of decimals.

4. The French gram is the weight of a cubic centimeter of water; a kilogram is 1,000 grams. Find the weight in kilograms of an anchor weighing 6,500 pounds, given that a cubic foot of water weighs 1,000 ounces, and the centimeter is .3937 of an inch.

5. In loaning money, banks deduct the interest, including three days of grace, in advance. For what sum shall a person who wishes to use $1,500 give his note for 60 days, interest 8 per cent per annum?—A person having a note for $2,000 due at a bank, pays in $200, and requests that a new note may be made payable at 90 days, the sum paid serving to pay the interest on the new note as well as to reduce his debt. For what sum shall the note be issued, the rate of interest being 8 per cent per annum?

1170. U. S. Military Academy, West Point. Examination for Admission.

1. Which is the greater, and by how much, $\frac{7}{9}$ or $\frac{19}{24}$?

2. What number subtracted 88 times from 80.005 will leave .013 as a remainder?

3. What is the value of a pile of wood 127 feet long, 4 feet wide, and 3 feet 8 inches high, at $7 a cord?

4. Reduce 3 acres 107 square rods 27 square yards 7 square feet 23 square inches to square inches.

5. Divide $4.14 among Thomas, Richard, and Henry in such a way that Henry shall receive 3 cents for every 5 cents that Thomas gets, and Richard shall receive 2 cents for every 5 cents that Henry gets.

6. Reduce 272 liquid quarts to dry quarts.

7. A pipe discharging 3 gallons 1 pint a minute fills a tub in 4 minutes 20 seconds. Another pipe discharges 83 quarts a minute. If both pipes discharge together into the tub, how long will they take to fill it?

8. A, B, and C start together and walk around a circle in the same direction. It takes A $\frac{5}{86}$ hour, B $\frac{2}{9}$ hour, and C $\frac{35}{99}$ hour to walk once around the circle. How many times will each go around the circle before they will all be together at the starting-point?

ANNUAL INTEREST.

1171. Slate Problems.

DETROIT, MICH., June 1, 1890.

Four years after date, without days of grace, I promise to pay to the order of Daniel W. Lawler, Six Hundred Dollars, value received, with annual interest at six per cent.

$600.$\frac{00}{100}$. GEORGE OXNARD.

1. Find the amount due June 1, 1894, no payments of principal or interest having been made.

1172. When the maker of a note fails to keep his contract to pay interest annually, the laws of some states, including Michigan, permit the collection of *simple interest* on the deferred payments of interest.

Principal,	$600.00
Interest, 4 years, at 6%,	144.00
3 years' interest, at 6%, on the 1st year's interest, $36,	6.48
2 " " " " " " 2d " " "	
1 " " " " " " 3d " " "	
Amount due June 1, 1894,	$

2. Find the amount due, at 5%, for 5 years, on a note for $1,200, annual interest being unpaid.

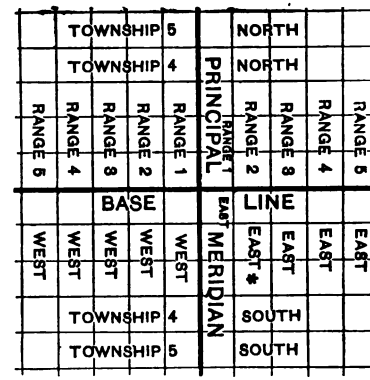
3. What is the amount of a note for $720, at 4 years, at $4\frac{1}{2}$%, annual interest unpaid after the first year?

4. The maker of a note for $900, with annual interest at 7%, makes the first and the second interest payments when due. How much will he owe at settlement, 6 years after the date of the note?

U. S. GOVERNMENT LANDS.

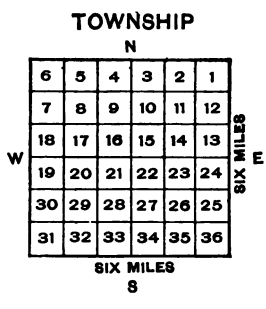
1173. In surveying government lands, a line is run east and west, called the *base line*, and one perpendicular to it, called the *principal meridian*.

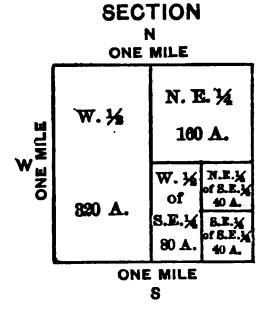
Parallel lines are run north and south, and east and west, 6 miles apart, forming squares, called *townships*. The row of townships adjoining the principal meridian is called *Range* 1 East or West, according to its location. The row of townships north of the base line is called *Township* 1, *North;* the row above, *Township* 2, *North*, etc.



The township in the diagram marked by a star (*) is designated 3 T. S., R. 2 E. (third township south of base line, in the second range east of the principal meridian).

1174. A township, which contains 36 square miles, is divided into sections one mile square, numbered as in the diagram, No. 1 being found at the northeast corner. Each section contains 640 acres.





1175. Sections are divided into half-sections (320 A.) and quarter-sections (160 A.), and the latter are subdivided into half quarter-sections (80 A.) and quarter quarter-sections (40 A.).

1176. Slate Problems.

1. Find the cost of the S.W. $\frac{1}{4}$ of the N. $\frac{1}{2}$ of sec. 13, T. 7 N., R. 4 E., at \$1.87$\frac{1}{2}$ per acre.

2. What will be the cost of fencing, at 75¢ per rod, the W. $\frac{1}{2}$ of the N.W. $\frac{1}{4}$ of sec. 36?

3. Mr. Thompson owns sec. 1, and his brother owns sec. 30 of the same township. What is the length of the shortest line between the boundaries of the two farms?

4. A road runs east and west between townships 4 and 5, south. Another road runs north and south between R. 7 and 8 east. How far is it by road from the north-east corner of T. 5 S., R. 10 W., to the north-west corner of T. 7 N., R. 8 E.?

5. How many feet of boards, 6 inches wide, would be needed to build an open fence, 4 boards high, around the N. $\frac{1}{2}$ of the S.W. $\frac{1}{4}$ of sec. 16?

6. The owner of secs. 19 and 20 has sold the W. $\frac{1}{2}$ of N.W. $\frac{1}{4}$ of sec. 19; also the N. $\frac{1}{2}$, the N. $\frac{1}{2}$ of S.E. $\frac{1}{4}$, and the S.E. $\frac{1}{4}$ of the S.E. $\frac{1}{4}$ of sec. 20. Draw a map of the land he still owns, and calculate its area.

METRIC SYSTEM.

1177. The metric system, which is used in nearly all the countries of continental Europe, is based upon the meter. The length of the meter is one ten-millionth part of the length of the meridian from the equator to the poles — about 39.37 inches.

1178. The subdivisions of the meter are denoted by the Latin prefixes milli ($\frac{1}{1000}$), centi ($\frac{1}{100}$), deci ($\frac{1}{10}$). For the multiples, the Greek prefixes deka (10), hecto (100), kilo (1,000), and myria (10,000) are used.

1179. It will be noticed, in the table below, that small letters are used for the abbreviations of the Latin prefixes of the

subdivisions, and capital letters for the Greek prefixes of the multiples.

The following is the table of

1180. Measures of Length.

10 millimeters (mm.)	1 centimeter (cm.)
10 centimeters	1 decimeter (dm.)
10 decimeters	1 *meter* (m.)
10 *meters*	1 dekameter (Dm.)
10 dekameters	1 hectometer (Hm.)
10 hectometers	1 kilometer (Km.)
10 kilometers	1 myriameter (Mm.)

1181. The units of this table in common use are the centimeter, the meter, and the kilometer.

1182. A person who wishes to buy 124 meters of cloth, would not ask for 1 hectometer 2 dekameters 4 meters, any more than a New York merchant would tell a person who owes him $38.75 that his bill is 3 eagles 8 dollars 7 dimes 5 cents.

1183. Long distances are expressed in kilometers. The thickness of wire is given in millimeters.

1184. Problems.

1. What will be the cost in francs of 380 m. 75 of dress goods at 2 f. 60 per meter? (380.75 meters @ 2.60 francs.)

2. How many square meters in a piece of carpet 26 m. 50 long, 85 cm. wide?

3. How many square meters in a circle whose diameter is 15 meters?

4. An *are* is a surface 10 meters long, 10 meters wide. How many ares in a field 135 meters long, 69 meters wide?

5. Find the area in ares of a right-angled triangle whose base is 245 meters, hypotenuse 875 meters.

6. A *stere* is a cubic meter. What will be the cost, at 8 f. 50 per stere, of a pile of wood 10 meters long, 1 meter wide, 3 m. 25 high?

7. A cube one decimeter each way contains a liter (l.), which is the principal unit of dry and liquid measure.

How many liters' capacity has a tank 10 m. 50 long, 8 m. wide, 6 m. 50 high?

8. How many bottles, each containing 0 l. 75, can be filled from a hogshead containing 222 l.?

9. How much will be received for 36 bags of beans, each containing 68 liters, at 1 mark 25 per dekaliter?

10. A liter of water weighs a kilogram (1,000 grams). How many kilos of oil would a tank contain, its dimensions being 5 meters $\times$ 4 meters $\times$ 3 meters, the weight of the oil being 92% of the weight of water?

11. Assuming the length of the meter as 39.37 inches, what is the length of the kilometer?

1185. Greater accuracy is assured in operations requiring multiplication and division by indicating the operations beforehand, and performing the division last.

Length of meter in yards $\frac{39.37}{36}$. 1 mile = 1,760 yd. 1 km. = 1,000 m.

$$Ans. = \frac{39.37 \times 1{,}000}{36 \times 1{,}760} = \frac{3{,}937}{36 \times 176} \text{ mile.}$$

12. Mt. Blanc is 4800 m. high. How many feet high is it?

1186. In the following ten problems call the meter 40 inches. Give answer in two decimal places.

13. How many cubic inches in a liter? (See problem 7.) How many quarts?

14. How many bushels in a hectoliter? How many gallons?

15. How many pounds in a kilo, when a cubic foot of water weighs 1,000 oz.? (See problem 10.)

16. What would be the circumference of the earth in miles if the meter measured 40 inches?

(The meter is $\frac{1}{10000000}$ of what part of circumference?)

17. How many square yards in a square meter?

18. How many acres in a hectare? (See problem 4.)

19. How many rods in a hectometer?

20. How many cubic feet in a stere? (See problem 6.)

21. How many troy grains (7,000 to av. lb.) in a gram? (See problem 15.)

22. How many kilometers in a mile?

1187. Measures of Surface.

100 sq. mm. = 1 sq. cm.
100 sq. cm. = 1 sq. dm.
100 sq. dm. = 1 sq. m. = 1.196 sq. yd.

1188. The square meter is the principal unit of surfaces, such as walls, ceilings, floors, etc.

100 centiares (ca.) = 1 are (a.) = 119.6 sq. yd.
100 ares = 1 hectare (Ha.) = 2.47 acres.

1189. The are is the principal unit of surface of small plots of land. The area of a farm is expressed in hectares, of a country in square kilometers.

1190. Measures of Volume.

1,000 cu. mm. = 1 cu. cm.
1,000 cu. cm. = 1 cu. dm.
1,000 cu. dm. = 1 cu. m. = 35.316 cu. ft.

1191. The principal unit is the cubic meter.

1192. The *stere* (cubic meter) is used for measuring wood.

10 decisteres (dst.) = 1 *stere* (st.) = 35.316 cu. ft.
10 *steres* = 1 dekastere (Dst.)

The stere is the only unit used.

1193. Dry and Liquid Measures.

		Dry.	Liquid.
10 milliliters	= 1 centiliter		
10 centiliters	= 1 deciliter		
10 deciliters	= 1 *liter* (l.)	= .908 qt.	= 1.057 qt.
10 *liters*	= 1 dekaliter	1.135 pk.	= 2.642 gal.
10 dekaliters	= 1 hectoliter	2.837 bu.	= 26.417 gal.
10 hectoliters	= 1 kiloliter		
10 kiloliters	= 1 myrialiter		

1194. The liter and the hectoliter are the principal units.

1195. Table of Weight.

10 milligrams (mg.)	1 centigram
10 centigrams	1 decigram
10 decigrams	1 *gram* (gr.)
10 *grams*	1 dekagram
10 dekagrams	1 hectogram
10 hectograms	1 kilogram (kilo) 2.2046 lb.
10 kilograms (Kg.)	1 myriagram
10 myriagrams	1 quintal
10 quintals	1 tonneau (ton)

1196. The kilo is the ordinary unit. Heavy articles are sold by the tonneau.

23. What will be the duty on 150 liters of wine at 50¢ per gallon?

24. Paris is 466 Km. from Lyons. How many miles apart are the two cities?

25. Find the duty, at 8¢ per sq. yd., on 360 meters of dress goods 72 centimeters wide. (Cancel.)

Kansas City Public Schools. Examination Questions.

1197. Oral Problems.

1. A man has two suits of clothes and an overcoat. The first suit is worth $12. The first suit and the overcoat are worth twice as much as the second suit. The second suit and the overcoat are worth three times as much as the first suit. What is the value of the overcoat? Of the second suit?

2. A lost $22. He then found $\frac{1}{4}$ as much as he had remaining, and then he had $\frac{1}{3}$ as much as he had at first. How much had he at first?

3. What time is it, if $\frac{1}{5}$ of the time past midnight equals the time past noon?

4. At what time between 3 and 4 is the hour hand 5 minute spaces in advance of the minute hand?

5. $\frac{2}{10}$ of A's age equals $\frac{4}{5}$ of B's age. The sum of their ages is 30 years. How old is each?

6. A and B are partners. They gain $720. A owns $\frac{3}{4}$ of the stock, lacking $30, and gains $525. What was the whole stock? What was the share of each?

7. A bought sheep for $80. 5 of them died. He then sold $\frac{2}{3}$ of the remainder at cost, and received $40. How many did he buy?

8. $\frac{1}{2}$ of A's age equals $\frac{1}{3}$ of B's age. B is 10 years older than A. Give the ages of each.

1198. Slate Problems.

1. Add 24 lb. 10 oz. 17 pwt. 22 gr.; 16 lb. 19 pwt.; 10 oz. 15 pwt. 21 gr.; 45 lb. 9 oz. 18 gr.; and 13 lb. 11 oz. 18 pwt. 23 gr.

2. A man was born Aug. 31, 1855. How old is he to-day?

3. Divide 19 mi. 180 rd. 2 yd. 9 in. by 7, and multiply the quotient by 12.

4. If two places are 18° 24′ 12″ apart, what is the difference of time? When it is 9 o'clock A.M. at Chicago, how far away is a place whose sun time is 10 hr. 45 min. A.M.? Which way is the place, and why?

5. A merchant bought 160 meters of cloth at $3.25 per meter, and sold it at $3.85 per meter. What was his gain? Give the meaning of the prefixes used in the metric system.

6. Find the circumference of a circle whose diameter is 6 ft. Find its area. Find the solid contents of a cylinder 2 inches in diameter and 4 inches high.

7. What will be the cost of a board 16 ft. long, 9 in. wide, and $\frac{1}{2}$ inch thick, at $40 per M.? If the thickness is $2\frac{1}{2}$ inches, what will a board of similar width and length cost, at the same price per M.?

(Boards less than 1 inch thick are considered as an inch thick in calculating the number of board feet.)

8. Draw a diagram showing how to locate the N.E. $\frac{1}{4}$ of the N.E. $\frac{1}{4}$ of a section of land. How many acres in the part marked off?

9. A bin is 8 ft. long, $7\frac{1}{2}$ ft. wide, and 4 ft. deep. How many bushels of grain will it hold? How many cubic inches in a gallon liquid measure?

10. How many yards of carpeting, $\frac{3}{4}$ of a yard wide, will carpet a room 27 feet long and 24 feet wide, if the strips run lengthwise, with no loss for matching?

CHAPTER XV.

ALGEBRAIC EQUATIONS.—TWO UNKNOWN QUANTITIES.—THREE UNKNOWN QUANTITIES.—PURE QUADRATICS.—AFFECTED QUADRATICS.

ADDITION OF ALGEBRAIC QUANTITIES.

1199. Sight Exercises.

Add:

1.	2.	3.	4.	5.
2 fours	6 hundredths	\$4	3¢	$7x$
3 fours	8 hundredths	\$5	5¢	$4x$
4 fours	10 hundredths	\$7	8¢	$2x$
5 fours	12 hundredths	\$8	9¢	$5x$
? fours	? hundredths	\$?	?¢	$?x$

6.	7.	8.	9.	10.
$-2a$	$+3x$	$-5xy$	$9abc$	$-24xyz$
$-4a$	$+4x$	$-4xy$	$15abc$	$-5xyz$
$-6a$	$+5x$	$-xy$	$6abc$	$-xyz$
$-7a$	$+10x$	$-2xy$	abc	$-15xyz$
$-19a$	$+?x$	$-?xy$	$?abc$	$-?xyz$

1200. In the quantities $2a$, $3x$, $5xy$, $15abc$, the numbers 2, 3, 5, 15, are called coefficients. When no coefficient is expressed, 1 is understood. Thus, $abc = 1abc$.

Where no sign is expressed, + is understood.

1201. What a person has may be represented with a plus sign (+) placed before the amount; debts may be shown by a minus sign (−) placed before the amount.

A has \$500; B owes \$300. If they unite their fortunes, what will they be worth together?

$$\begin{array}{r} +\$500 \\ -\$300 \\ \hline +\$200 \end{array}$$

Both together are worth \$200.

The sum of $+500$ and -300 is $+200$.

1202. If A had \$300 and B owed \$500, the firm would be \$200 in debt.

$$(+\$300)+(-\$500)=-\$200.$$

1203. Add:

1.	2.	3.	4.	5.
$-2a$	$7x$	$-5xy$	$-9abc$	$-24xyz$
$-4a$	$-4x$	$-4xy$	$15abc$	$5xyz$
$-6a$	$-2x$	xy	$6abc$	xyz
$7a$	$5x$	$2xy$	$-abc$	$15xyz$
$-5a$	$6x$	$-?xy$	?	?

1204. Can you give the rule for addition where the quantities have different signs? Which sign does the sum take?

1205. Add:

6. $3x+14$, $-7x+9$, -23, $4x-5$, $-2x$, and $3x+11$.

$$\begin{array}{r} 3x+14 \\ -7x+9 \\ -23 \\ 4x-5 \\ -2x \\ \hline 3x+11 \end{array}$$

7. $4a+3x$, $-2a$, $-7x-3a$, $-5x$, $-9a+x$.

8. $-3b+c$, $4a+6b$, $5b-9c$, $-3a$, $-2a-3b+4c$.

9. $\frac{1}{2}x-8$, $-x+4$, $-\frac{1}{4}x-3$, $7x+16$, $-5x-10$.

10. $4x+23$, $-8x+2\frac{1}{2}$, $-\frac{1}{8}x+11$, $-x+5$, $9x-3$.

SUBTRACTION OF ALGEBRAIC QUANTITIES.

1206. Oral Problems.

1. The thermometer in the morning was 33 degrees, at noon it was 52 degrees. What was the difference in temperature?

2. In December the thermometer was 10 degrees below zero. In July it was 90 degrees above. What was the difference in temperature?

3. Two cities are in the same latitude. One is in 34° east longitude, and the other in 17° west longitude. What is their difference in longitude?

4. What is the difference in longitude between two cities on the equator, one being in 56° west longitude, and the other in 47° west longitude?

5. A boy makes 40¢ one day and 50¢ the next. How does he stand at the end of the two days?

6. How would he stand if he made 40¢ one day and lost 50¢ the next day?

7. A man traveled from the town M, 60 miles due north, and then traveled 50 miles due north. How far is he from his starting-point?

8. One day a man goes 50 miles due north; the next day he travels 70 miles due south. How far is he then from his starting-point?

9. On Monday A is worth $250; on Tuesday he is worth $150. What has he lost in a day?

10. A man has $150 Jan. 1. Feb. 1 he owes $250. What has he lost in a month?

1207. The degrees above zero on a thermometer may be indicated by a plus sign (+); those below, by a minus sign (−).

What is the difference between $+52°$ and $+33°$? Between $+90°$ and $-10°$?

Show by a diagram.

1208. A has \$600, B owes \$400. What are they worth together?

$$(+\$600)+(-\$400)=?$$

How much better off is A than B?

$$(+\$600)-(-\$400)=?$$

1209. In subtracting algebraic quantities, change the signs of the subtrahend, and proceed as in addition.

1. From $8a$ take $2a$.

$$\begin{array}{r} 8a \\ -2a \\ \hline \textit{Ans.}\quad 6a \end{array}$$

2. From $2a$ take $8a$.

$$\begin{array}{r} 2a \\ -8a \\ \hline \textit{Ans.}\quad -6a \end{array}$$

3. From $-8a$ take $2a$.

$$\begin{array}{r} -8a \\ -2a \\ \hline \textit{Ans.}\quad -10a \end{array}$$

4. From $8a$ take $-2a$.

$$\begin{array}{r} 8a \\ +2a \\ \hline \textit{Ans.}\quad 10a \end{array}$$

5. From $-8a$ take $-2a$.

6. From $-2a$ take $8a$.

7. From $-2a$ take $-8a$.

8. From $2a$ take $-8a$.

9. From $3x+14$ take $x+10$.

$$\begin{array}{r} 3x+14 \\ -x-10 \\ \hline \end{array}$$

10. From $5x-8$ take $-3x-9$.

11. From $x-28$ take $5x-37$.

12. From $7x+16$ take $9x-4$.

13. From $6x$ take $2x-5$.

14. From $8x$ take $9x+3$.

15. From $3x+2a-5$ take $x-a-9$.

16. From $7y-2z+b$ take $-8y+6b-z$.

17. From $c-d+e$ take $c+d-f$.

REMOVING PARENTHESES.

1210. From 84 take the difference between 49 and 25.

$$84 - (49 - 25) = \text{what?}$$

Would the result be the same if we should write the above

$$84 - 49 - 25?$$

What sign must be changed?

1211. Write the following without parentheses:

1. $57 + (33 - 16) = 74$
2. $92 - (63 + 25) = 4$
3. $(43 - 10) + (24 - 5) = 52$
4. $(17 - 8) - (16 - 14) = 7$
5. $75 + 4 \times (15 - 10) = 95$
6. $75 - 4 \times (15 - 10) = 55$

1212. Is there any change made in the signs of the first? In the signs of the second? Of the third? Of the fourth? Of the fifth? Of the sixth?

1213. Solve the following equations. Prove the correctness of your answers.

1. $6(2x - 5) = 5x + 12$

NOTE. $6(2x - 5)$ means 6 times $(2x - 5)$, or $12x - 30$.

2. $7(x + 2) = 3x + 50$
3. $5(3 + x) + 16 = 61$
4. $3(16 - x) = 4(13 - x)$
5. $15(x - 3) = 2(189 - 16x)$
6. $38 - (11 - 9x) = 10x$

Removing the parenthesis, we have

$$38 - 11 + 9x = 10x$$

Transposing, $$9x - 10x = -38 + 11$$

or, $$-x = -27$$

Bringing $-x$ to the right side of the equation, and -27 to the left side, we have

$$(+)\,27 = (+)\,x$$

In practice, however, when the result is such as the above, $-x = -27$, the signs of both members are changed, and the result becomes

$$x = 27$$

7. $2(x-1)-2(2x-19)=3(x-3)$

8. $6(2x-5)-5x=12$

9. $5x-6(2x-5)=-12$

10. $\frac{11-3x}{2}+5x=19.$

1214. $\frac{18-6}{2}-\frac{24-4}{5}=2$

Clear of fractions by multiplying both members of the equation by 10, and observe which sign must be changed to preserve the equality.

When $x=6$, the above may be written

$$\frac{3x-6}{2}-\frac{4x-4}{5}=2$$

Clearing of fractions,

$$15x-30-(8x-8)=20$$

Removing the parenthesis,

$$15x-30-8x+8=20$$

Transposing, $$15x-8x=20+30-8$$

or, $$7x=42$$

$$x=6$$

NOTE. — The horizontal line between the numerator and the denominator of the foregoing fractions has the effect of a parenthesis, the *entire* quantity above the line being divided by the number below.

$\frac{18-6}{2}=(18-6)\div 2$ $\qquad$ $\frac{24-4}{5}=\frac{1}{5}$ of $(24-4)$

$\frac{3x-6}{2}=\frac{1}{2}$ of $(3x-6)$ $\qquad$ $\frac{4x-4}{5}=(4x-4)\div 5$

1215. Solve:

11. $\frac{x-1}{2}+\frac{x-2}{3}=8$

12. $\frac{x-1}{2}-\frac{x-2}{3}=2$

13. $\frac{x-1}{2}-\frac{x-2}{3}-\frac{x-3}{4}+2=0$

14. $\frac{2x-5}{2}+\frac{x-7}{4}=\frac{5x-3}{6}$

15. $\frac{7x-8}{9}-(x+2)=\frac{4x+5}{6}-\frac{x+2}{3}$

16. $\frac{40-5x}{3}=\frac{52+9x}{7}$

17. $9\frac{3}{4}-\left(\frac{5}{4}x-\frac{x}{2}\right)=\frac{3}{8}x+3\frac{3}{4}x$

18. $2x=3+2\frac{1}{4}x-(5+\frac{2}{5}x)+2\frac{3}{5}$

19. $\frac{3}{4}x+9=2x+(\frac{3}{5}x-\frac{1}{2}x)$

20. $\frac{x}{4}+\frac{x}{5}+\frac{x}{6}+\frac{x}{8}+31=x$

21. $\frac{5}{4}x-120=\frac{x}{6}+10$

22. $x-20=\left(\frac{x}{7}+15\right)4$

23. $x+\frac{x}{3}+\frac{x}{4}=19$

24. $9(8x+1)-4=4(9x+5)+3$

25. $2x+3=\frac{5x-6}{2}$

1216. Slate Problems.

1. A certain number is multiplied by $3\frac{2}{3}$; 7 is subtracted from the product; the remainder is divided by 16, giving a quotient of 3. What is the number?

2. Three-eighths of what number is 60 less than the number itself?

3. Four persons are of the same age. If the first were $\frac{1}{8}$ of his age older, the second $\frac{1}{4}$ of his age older, the third $\frac{1}{5}$ of his age older, and the fourth $\frac{1}{8}$ of his age older, the sum of their ages would be 99 years. What is the age of each?

4. A man spends $\frac{1}{2}$ of his earnings on board and lodging, $\frac{1}{8}$ on clothing and repairs, and $\frac{1}{5}$ on sundries. At the end of the year he has \$280 left. What are his yearly earnings?

$$\left(x = \frac{x}{2} + \frac{x}{8} + \frac{x}{5} + 280.\right)$$

5. A boy gave $\frac{1}{8}$ of his marbles to one companion, and $\frac{1}{8}$ of them to another. He then bought $\frac{1}{8}$ as many as he originally had, and had 4 marbles more than he had at first. How many did he have at first?

6. A father's age and a son's age added together amount to 138 years. Twelve years ago the father was twice as old as the son. How old is each now?

Let x = son's age 12 years ago. $2x$ = father's age then.

7. John has 80 cents, and William has 60 cents. How many cents will William have to give John so that the latter shall have $2\frac{1}{2}$ times as much money as the former?

After William gives John x cents, the former has $(60 - x)$ cents, and the latter has $(80 + x)$ cents.

8. In how many years will a man, now 25, be double the age of his 11-year-old brother?

Let x = number of years. $25 + x$ and $11 + x$ = ages after x years.

9. A man has a cask of 60 gallons' capacity. He draws off one-fourth of its contents, and then fills it. If it takes 24 gallons to fill it, how many gallons did the cask originally contain?

10. A number is divided by 3, and 40 is subtracted from the quotient, leaving a remainder of 104. What is the number?

11. The difference between two numbers is 430. When the greater is divided by the less, the quotient is 4, and the remainder is 76. What are the numbers?

$$\text{Let } x = \text{less.} \quad \frac{\text{greater}}{\text{less}} = 4 + \frac{76}{\text{less}}.$$

12. A person pays \$103 with 29 \$2 and \$5 bills. How many are there of each denomination?

13. A father is 30 years older than his daughter. In 4 years, his age will be four times her age. What are their present ages?

x and $x + 30$ = present ages. $x + 4$ and $x + 34$ = ages 4 years later.

14. The product of two numbers is 180. If the smaller number be increased by 3, the product of the two numbers will be 225. What are the numbers?

$$\text{smaller} = x; \quad \frac{180}{x} = \text{greater}.$$

15. A man's wages are \$1 per day more than his son's. For 33 days' work, the father receives \$12 more than the son earns in 40 days. Find the wages of each.

16. The sum of two numbers is 47; their difference is 17. What are the numbers?

17. A mother is 41 years old. Her son's age is 5. In how many years will the son's age be $\frac{1}{8}$ of his mother's?

TWO UNKNOWN QUANTITIES.

1217. Preliminary Problems.

1. I paid a dollar for two 25¢ balls and five bats. How much did I pay apiece for the latter?

2. When three times one number is added to five times another, the sum is 84. If the second number is 12, what is the first number?

3. A girl paid 75¢ for $\frac{1}{2}$ pound of tea and $2\frac{1}{2}$ pounds of coffee. The coffee cost 20¢ per pound. What was the price of the tea per pound?

4. A man sold pigs at \$5 each and lambs at \$8 each, receiving \$42. He sold 4 lambs. How many pigs did he sell?

5. Four times a father's age added to twice his daughter's age amounts to 180 years. The girl is 10 years old. What is the father's age?

6. Eight peaches and seven pears cost 44¢. The peaches cost 2¢ each. What is the cost of a pear?

7. Two pieces of cloth and eleven pieces of silk contain 152 yards. There are 10 yards in each piece of cloth. How many yards in each piece of silk?

8. Two-thirds of a yard of linen and three-fourths of a yard of lace cost 40¢. The price of the lace is 32¢ a yard. Find the price of the linen.

9. Three and one-half times one number added to four and one-third times a second number equals 60. The second number is 9. What is the first number?

1218. Slate Exercises.

Find the value of the unknown quantity:

1. $8x + 7y, = 44$. When $x = 2$, find the value of y.
2. $3y + 5z = 34$. Find the value of z; $y = 3$.
3. $2x + 11z = 152$. $x = 10$; $z = ?$
4. $14x + 7y = 98$. $x = 3\frac{1}{2}$; $y = ?$
5. $\frac{2}{3}x + \frac{3}{4}z = 40$. $z = 32$.

6. $9x - 25y = 8.$ $x = 12.$

7. $3\frac{1}{2}y + 4\frac{1}{8}z = 60.$ $z = 9.$

8. $16x - 19z = 49.$ $z = 5.$

9. $7y - 3z = 18.$ $y = 6\frac{1}{7}.$

10. $32x + 50y = 2{,}600.$ $y = 20.$

1219. A boy gave 17¢ for 3 lemons and 4 oranges, another boy paid 25¢ for 3 lemons and 8 oranges. How much did the lemons cost apiece?

$x =$ cost of lemons	$3x + 4y = 17$	(1)
$y =$ cost of oranges	$3x + 8y = 25$	(2)
Subtracting (1) from (2)	$4y = 8$	
The oranges cost 2¢ each	$y = 2$	

How much apiece was paid for the lemons?

11. If 3 coats and 14 vests cost \$78, and 2 coats and 14 vests, at the same rate, cost \$66, how much does 1 coat cost? What is the price of a vest?

12. Given $4x + 7y = 53$ (1)

$2x + 3y = 25$ (2)

to find the value of y.

First multiply (2) by 2, making it $4x + 6y = 50$. Why?

13. What is the value of x in equation (1), when the value found for y is substituted therein? Substitute the same value for y in equation (2) and find the value of x.

1220. Find the values of x and y in the following equations:

14. $x + y = 15.$ $2x + 3y = 38.$

15. $2x + 2y = 30.$ $x + 3y = 27.$

16. $2x + 3y = 18.$ $4x + 3y = 24.$

17. $2x + 3y = 40.$ $3x + 2y = 35.$

18. $7x+5y=82.$ $2x+2y=28.$

19. $5x+9y=14.$ $9x+5y=14.$

20. $3x+5y=17.$ $8x+2y=17.$

1221. Given $\left.\begin{matrix}(1)\ x+3y=46\\(2)\ 7x-4y=22\end{matrix}\right\}$ To find values of x and y.

Multiply (1) by 7, $7x+21y=322$

(2) $7x-4y=22$ Subtract.

$25y=300$

$y=12$

Substituting this value of y in (1), we have

$x+36=46$

$x=46-36=10$

Answers. $x=10,\ y=12.$

21. $x+y=18$ Add or subtract.
$x-y=4$

22. $4x+3y=17$ (1) Multiply (2) by 2 and subtract.
$2x-y=1$ (2)

23. $3x+4y=48$ Add.
$x-4y=0$

24. $3x+5y=13$ (1) Multiply (1) by 7 and (2) by 3.
$7x+3y=13$ (2) Subtract.

25. $4x+5y=32$ Add.
$6x-5y=-2$

26. $3x+4y=3$ (1) Multiply (2) by 2. Add.
$12x-2y=3$ (2)

27. $5x=6y+5$ Transpose.
$3x=5y-4$

28. $3x+5y+8=0$
$2x-y-12=0$

29. $y-2x=8x-1$
$2y-4x=y+x+9$

30. $\frac{x}{4}+\frac{2y}{3}=17$ Clear of fractions.

$\frac{5x}{4}+\frac{5y}{8}=20$

31. $\frac{1}{2}x+\frac{1}{3}y=42$
$\frac{1}{5}x+\frac{1}{7}y=17\frac{2}{5}$

32. $23x-7y=3x+51$
$11y=15x+2$

33. $x+y=100{,}000$
$\frac{5x}{100}+\frac{4y}{100}=4{,}640$

34. $\frac{3x+7}{3y-4}=5$
$\frac{7x-6}{5y+3}=2$

35. $4\frac{1}{4}x+3\frac{2}{3}y=67$
$7\frac{1}{2}x-5\frac{1}{3}y=12$

36. $3(x+7)=9(y-9)$
$4(3x-8)=17y-155$

37. $2(x-11)-2(y-9)=6$
$\frac{x+9}{y-3}=\frac{32}{15}$

38. $\frac{x-4}{3}+\frac{y-1}{4}=5$
$\frac{x-4}{3}-\frac{y-1}{4}=1$

39. $\frac{2x+5y+3}{3x-4y-2}=6$
$\frac{4x-7y+5}{x-2y+2}=5$

1222. Slate Problems.

1. The sum of two numbers is 37. Twice the first added to three times the second is 96. What are the numbers?

(Let x = first number; y = second number.)

2. The difference between two numbers is 28. Five times the first less twice the second is 197. What are the numbers?

($x-y=28$; $5x-2y=197$.)

3. The product of the first of two numbers by 5, added to the product of the second by 3, gives 37. The product of the first by 6, diminished by 5 times the second, equals 10. Find the numbers.

4. Divide 65 into two parts whose difference shall be 19.

(Let x and y = parts. Solve also by one unknown quantity.)

5. A person pays \$103 with 32 bills, some of them \$2 bills, the others \$5 bills. How many of each does he use?

6. For 25 head of pigs and sheep, a farmer received \$145. How many of each did he sell, if he sold the former at \$7 each, the latter at \$5 each?

7. 10 oranges and 4 peaches cost 38¢; 6 oranges and 7 peaches cost 32¢. Find the cost of an orange. Of a peach.

8. 5 pounds of tea and 3 pounds of coffee cost \$3.75; 8 pounds of tea and 1 pound of coffee cost \$5.05. What is each worth per pound?

9. A farmer buys a certain number of horses at \$125 each, four times as many cows at \$45 each, eight times as many sheep at \$10 each, and half as many pigs at \$5 each, spending \$1,550 for all. How many of each does he buy?

10. A man paid 75¢ for 2 pounds of raisins and 3 pounds of cheese. 5 pounds of raisins and 2 pounds of cheese at the same price would have cost 94¢. What did each cost per pound?

11. The sum of two numbers is 19. The sum of the second number and ten times the first, minus the sum of the first and ten times the second, equals 45. What are the numbers?

12. Reduce $\frac{5}{13}$ to an equivalent fraction, the sum of whose numerator and denominator shall be 126.

x = numerator; y = denominator.

$$\frac{x}{y} = \frac{5}{13};\ x + y = 126.$$

13. What fraction equivalent to $\frac{8}{15}$ has 147 for the difference between its numerator and denominator?

$(x - y = -147.$ Why?)

14. 10 pounds of coffee at 30¢ per pound are mixed with x pounds of coffee at 25¢ per pound. What is x equal to, when the mixture is worth 26¢ per pound?

$$25x + (10 \times 30) = 26(10 + x).$$

15. A grocer mixes green tea costing 60¢ per pound with black tea costing 40¢ per pound. He uses 100 pounds in all, and the mixed tea costs him 48¢ per pound. How many pounds of each does he use?

Let x = number of pounds of black tea; y = number of green. Then $x + y$ = number of pounds of mixed tea.

$$x + y = 100;\ 40x + 60y = 48(x + y).$$

THREE UNKNOWN QUANTITIES.

1223. 1. Given the following:

$$3x + 2y - z = 12 \ (a)$$
$$5x - 4y + 3z = 16 \ (b)$$
$$2x + 3y + 2z = 35 \ (c)$$

to find the values of x, y, and z.

(a) multiplied by 5,	$15x + 10y - 5z = 60$	
(b) " " 3,	$15x - 12y + 9z = 48$	
Subtract,	$22y - 14z = 12$	(d)

an equation containing only two unknown quantities.

(b) multiplied by 2,	$10x - 8y + 6z = 32$	
(c) " " 5,	$10x + 15y + 10z = 175$	
Subtract,	$-23y - 4z = -143$	(e)

an equation containing only two unknown quantities.

Compare the two equations (d) and (e), which contain the same two unknown quantities.

(d) multiplied by 2,	$44y - 28z = 24$	
(e) " " 7,	$-161y - 28z = -1{,}001$	
Subtract,	$205y = 1{,}025$	
	$y = 5$	

Substituting this value of y in (d), we have

$$110 - 14z = 12, \quad -14z = -98, \quad z = 7.$$

Substituting values of y and z in (a), we have

$$3x + 10 - 7 = 12, \quad 3x = 9, \quad x = 3.$$

$$\textit{Ans.} \quad \left.\begin{aligned} x &= 3, \\ y &= 5, \\ z &= 7. \end{aligned}\right\}$$

2. Find the values of the unknown quantities in the following equations:

$$x - 3y + 2z = 3 \quad (a)$$
$$2x + y + 3z = 22 \quad (b)$$
$$5x + 2y + 7z = 51 \quad (c)$$

Multiply (a) by 2, and subtract from (b). Multiply (a) by 5, and subtract from (c). This gives two equations, each of which contains two unknown quantities.

Compare these two resulting equations, and eliminate y.

3.
$$5x - 2y + z = 10 \quad (a)$$
$$3x + 8y - 5z = 120 \quad (b)$$
$$7x - 3y - 2z = 8 \quad (c)$$

Eliminate z by comparing (a) and (b), multiplying the former by 5. Compare (a) and (c), multiplying the former by 2.

4.
$$13x - 4y + 15z = 317$$
$$7x + 2y - 3z = 89$$
$$21x - 17y + 9z = -104$$

5.
$$-8x + y - 12z = -259$$
$$7x - 4y + 25z = 418$$
$$13x + 2y - 41z = -500$$

6. $\frac{x}{3}+\frac{x+y}{3}=14$

$\frac{x+y}{2}-\frac{x-y}{6}=16$

7. $\frac{3x-5y}{2}+3=\frac{2x+y}{5}$

$8-\frac{x-2y}{4}=\frac{x}{2}+\frac{y}{3}$

8. $2+\frac{5x-6y}{13}=4y-3x$

$12+\frac{5x-6y}{6}=2y+\frac{3x-2y}{4}$

9. $\frac{5x-3}{4}-\frac{3x-19}{4}=2-\frac{3y-x}{6}$

$\frac{2x+y}{2}-\frac{9x-7}{8}=\frac{3y+9}{4}-\frac{4x+5y}{16}$

1224. Slate Problems.

1. A man placed $\frac{2}{3}$ of his capital at 5% and the other third at 6%. At the end of a year, capital and interest amounted to $31,600. What was his capital?

$\frac{2x}{3}\times\frac{5}{100}$ and $\frac{x}{3}\times\frac{6}{100}$ = interest.

2. A has 18 chestnuts more than B. If each finds 4 more, A will have four times as many as B. How many chestnuts has each?

3. Two mechanics earn together $8 per day. One works 23 days and the other 17 days, for which they receive together $166. What does each earn per day?

4. The sum of the first and the second of three numbers is 55, of the first and the third 62, of the second and the third 83. What are the numbers?

5. The sum of two numbers is 53. Four times the first is 20 more than twice the second. Find the numbers.

6. A certain sum of money is divided among four persons. The first takes $\frac{1}{5}$ of it, the second takes $\frac{4}{5}$ of the remainder, the third takes $\frac{2}{5}$ of what then remains, the fourth receives the balance, $24. What is the share of each of the other three?

7. A merchant sold a lot of goods for $510, thereby losing $\frac{3}{20}$ of their cost. What did the goods cost?

8. A man collected a bill for a physician and deducted $\frac{1}{50}$ of the amount for his services. If he gave the physician $147, what was the amount collected?

9. Divide $130\frac{1}{4}$ acres of land among three persons, giving the first $27\frac{1}{2}$ acres more than the second, and the second $13\frac{3}{4}$ acres more than the third.

10. A merchant has sold $\frac{4}{7}$ of a piece of cloth, and has remaining 16 yards more than $\frac{1}{5}$ of the piece. How many yards did the piece contain originally?

11. A servant is engaged for a year for $280 and a suit of clothes; he leaves at the end of six months, and receives $130 and the suit. What is the value of the clothes?

MULTIPLICATION OF ALGEBRAIC QUANTITIES.

1225. Multiply $x+3$ by $x+4$.

The product is equal to x times $(x+3)+4$ times $(x+3)$.

$$x(x+3) = x^2+3x$$
$$4(x+3) = \quad 4x+12$$
$$(x+3)(x+4) = x^2+7x+12 \quad \textit{Ans.}$$

NOTE. — x^2 is read x square. The 2 is called an *exponent*.

Multiply $(x+7)$ by $(x+8)$.

$$\begin{array}{rl} & x+7 \\ & x+8 \\ \hline \text{Product by } x, & x^2+7x \\ \text{Product by } 8, & \quad 8x+56 \\ \hline & x^2+15x+56 \quad \textit{Ans.} \end{array}$$

1226. Multiply:

1. $(x+5)$ by $(x+2)$
2. $(x+8)$ by $(x+9)$
3. $(2x+5)$ by $(x+2)$
4. $(2x+8)$ by $(x+9)$
5. $(3x+1)$ by $(x+7)$
6. $(2x+1)$ by $(2x+1)$

1227. $(x-5)\times(x+4)=?$

$$\begin{array}{rl} & x-5 \\ & x+4 \\ \hline x(x-5) & x^2-5x \\ 4(x-5) & \quad 4x-20 \\ \hline & x^2-x-20 \quad \textit{Ans.} \end{array}$$

1228. Find products:

NOTE. — $(x-3)(x+9)$ means $x-3$ multiplied by $x+9$.

7. $(x-3)(x+9)$
8. $(x-6)(x+7)$
9. $(x-5)(x+5)$
10. $(x+5)(x-5)$
11. $(2x-6)(x+1)$
12. $(x-6)(2x+1)$
13. $(2x-6)(3x+3)$
14. $(3x+6)(2x-3)$
15. $(2x+3)(2x-3)$

1229. $(x-5)(x-4)=?$

The product is equal to $x(x-5)-4(x-5)$; that is, that $4(x-5)$ is to be subtracted from $x(x-5)$.

$$x(x-5)=x^2-5x;\ 4(x-5)=4x-20.$$

Placing the subtrahend under the minuend, and changing the signs of the former (Art. 1209), we have

$$\begin{array}{rl} & x^2-5x \\ & \quad -4x+20 \\ \hline (x-5)(x-4)= & x^2-9x+20 \quad \textit{Ans.} \end{array}$$

1230. $(x-7)(x-9)=?$

Using either as a multiplier, place one under the other. Commencing with x, say $x \times x = x^2$, $-9 \times x = -9x$. Taking -7 as a multiplier, say $x \times (-7) = -7x$, $(-9) \times (-7) = 63$. Combining, we get the product.

$$\begin{array}{r} x - 9 \\ x - 7 \\ \hline x^2 - 9x \\ \quad - 7x + 63 \\ \hline \textit{Ans.}\ x^2 - 16x + 63 \end{array}$$

1231. Note that the multiplication of a + (positive) quantity by a + (positive) quantity gives a + (positive) product; that $(+) \times (-)$ or $(-) \times (+)$ gives a − (negative) product; and that $(-) \times (-)$ gives a + (positive) product. This is usually stated as follows:

1232. *Like signs produce +, and unlike signs produce −.*

1233. Give results:

16. $(x-7)(x-7)$ **20.** $(x+7)(x-6)$ **24.** $(2x+7)(3x+3)$

17. $(x-5)(x-9)$ **21.** $(x-4)(x-7)$ **25.** $(2x-3)(3x-2)$

18. $(x+5)(x+5)$ **22.** $(2x-4)(3x-6)$ **26.** $(2x-3)(2x+3)$

19. $(x-3)(x+8)$ **23.** $(2x+6)(3x-7)$ **27.** $(2x+9)(4x-6)$

PURE QUADRATICS.

1234. Given $\frac{x^2+6}{5} = \frac{3x^2-66}{9}$, to find the value of x.

Clearing of fractions, $9x^2 + 54 = 15x^2 - 330$

Transposing and combining, $-6x^2 = -384$

Dividing by 6, and changing signs, $x^2 = 64$

Extracting square root, $x = \pm 8$.

1235. Since $(-8) \times (-8) = 64$, the square root of 64 may be either $+8$ or -8. It is written ± 8, and is read "*positive or negative* 8." (It is sometimes less correctly called *plus or minus* 8.)

1236. Slate Exercises.

Find value of x, y, z, etc.:

1. $x^2 - 13 = 36$
2. $3y^2 + 25 = 100$
3. $5z^2 - 13 = 3z^2 + 37$
4. $5(x^2+17) - 3x^2 + 63 = 198$
5. $5(x^2+17) - 3(x^2-21) = 198$
6. $y^2 + 2y + 1 - y^2 = 49$
7. $(x+1)^2 - x^2 = 49$
8. $\frac{y^2+5}{3} - \frac{2y^2-18}{4} = 2$
9. $\frac{z+7}{z-3} = \frac{z-5}{z-9}$
10. $\frac{20x}{x-1} = \frac{30x}{x+1}$
11. $(x-3)(x+3) = 40$
12. $(x+5)(x+5) = 10x + 26$
13. $(x+4)^2 = 8x + 80$
14. $z^2 + 64 = 5z^2$
15. $3x^2 + 18 = 2\frac{1}{2}x^2 + 36$
16. $(x-3)^2 - (x-5)^2 = 12$
17. $(x+7)(x-9) = (x-3)(x-5)$
18. $\frac{x}{4} + \frac{4}{x} = \frac{x}{9} + \frac{9}{x}$
19. $\frac{x+7}{x-5} = \frac{x-3}{x-9}$
20. $\frac{y-9}{y-5} = \frac{y-3}{y+7}$

1237. Slate Problems.

1. Find the dimensions of a field, the length of which is twice its breadth, its area being 1,800 square rods.

2. The surface of the six equal faces of a cube contains 96 square inches. Find the length of one edge.

3. One number is fourth-fifths of another, and their product is 80. What are the numbers?

4. One-third of a number multiplied by two-fifths of the same number gives a product of 270. Find the number.

5. Thirty per cent of a number multiplied by forty per cent of the same number gives a product of 300. What is the number?

6. Thirty per cent of forty per cent of a number is 300. What is the number?

7. The base of a right-angled triangle is $\frac{3}{4}$ as long as the perpendicular, and the area of the triangle is 96 square rods. Find the length of the base. What is the length of the hypotenuse?

8. The base of a right-angled triangle measures x yd., the perpendicular measures $\frac{3x}{4}$ yd. What is the length of the hypotenuse? If the hypotenuse measures 15 yd., find the length of the base.

9. The base of a right-angled triangle measures x ft., the perpendicular measures $(x+7)$ ft., the hypotenuse measures 17 ft. What is the length of the base?

10. The difference between the squares of two consecutive numbers is 49. What are the numbers?

AFFECTED QUADRATICS.

1238. Preliminary Exercises.

$$(x+1)(x+1)=x^2+2x+1$$

The square of the sum of two quantities is equal to the square of the first + twice the product of the first and the second + the square of the second.

$$(x-1)(x-1)=x^2-2x+1$$

The square of the difference of two quantities is equal to the square of the first − twice the product of the first and the second + the square of the second.

$$(a+b)^2 = a^2+2ab+b^2$$
$$(m-n)^2 = m^2-2mn+n^2$$
$$(10+5)^2 = 10^2+2\times 10\times 5+5^2$$
$$(10-3)^2 = 10^2-2\times 10\times 3+3^2$$

1239. Oral Exercises.

Square:

1. $x+3$	4. $x+10$	7. $30-1$	10. $x-y$
2. $x-7$	5. $a-b$	8. $40-1$	11. $80+5$
3. $x-9$	6. $x+y$	9. $m+n$	12. $60-5$

1240. Sight Exercises.

Extract the square root of

1. x^2+6x+9	6. $x^2+2xy+y^2$
2. $x^2-14x+49$	7. $x^2-2xy+y^2$
3. $x^2-18x+81$	8. $a^2-2ab+b^2$
4. $x^2+20x+100$	9. $x^2-24x+144$
5. $a^2+2ab+b^2$	10. $x^2+22x+121$

1241. The square of $(x+3)$ consists of how many terms? Of how many terms does $(x+4)^2$ consist? $(x+5)^2$?

1242. Supply term necessary to make a complete square:

1. $x^2+6x+?$	6. $x^2+2x+?$
2. $x^2-12x+?$	7. $x^2-4x+?$
3. $x^2-8x+?$	8. $x^2-10x+?$
4. $x^2-16x+?$	9. $x^2+14x+?$
5. $x^2+18x+?$	10. $x^2-22x+?$

1243. Slate Exercises.

Given $x^2+6x=27$

What number must be added to the first member of the equation to make it a "complete" square?

If a number is added to one member of an equation, what must be done to the other member to preserve the equality?

1244. Extract the square root of both members of the following equations, adding to both, where necessary, such a number as will make the first member a complete square.

1. $x^2+6x+9=40+9$ 2. $x^2-12x+36=28+36$

Remember that $(+7)\times(+7)=49$, and that $(-7)\times(-7)=49$.
$\therefore \sqrt{49}=+7$ or -7, written ± 7.

3. $x^2-8x+16=20+16$
4. $x^2-16x+64=-39+64$
5. $x^2+18x+?=19+?$
6. $x^2+2x+?=24+?$
7. $x^2-14x=15$
8. $x^2-22x=23$
9. $x^2+14x=51$
10. $x^2-22x=48$

1245. Given $x^2-10x=24$.

Completing the square, we have $x^2-10x+25=24+25=49$.
Extracting the square root of both sides, we have

$$x-5=\pm 7,$$
$$x=7+5=12, \text{ or } -7+5=-2.$$

Ans. 5 or -2.

1246. Find values of x:

1. $x^2-6x=7$
2. $x^2-12x=108$
3. $x^2+2x=48$
4. $x^2+18x=115$
5. $x^2-14x=-13$
6. $x^2-10x=0$
7. $x^2+20x=125$
8. $x^2+26x=56$
9. $x^2-24x=0$
10. $x^2-8x=384$
11. $x^2-4x=-3$
12. $x^2+30x=175$
13. $x^2+28x=29$
14. $x^2+22x=104$
15. $x^2-16x=-64$
16. $x^2+36x=76$

1247. To make the first member a complete square, you added the square of what part of the coefficient of x?

1248. Find values of x:

1. $x^2 + x = 12$
   $x^2 + x + (\frac{1}{2})^2 = 12 + (\frac{1}{2})^2$
2. $x^2 - 3x = 10$
   $x^2 - 3x + (\frac{3}{2})^2 = 10 + (\frac{3}{2})^2$
3. $x^2 + 5x = -4$
4. $x^2 - 7x = 8$
5. $x^2 + 9x = -20$
6. $x^2 - 11x = -28$
7. $x^2 + 13x = -42$
8. $x^2 - 15x = 76$
9. $x^2 - 17x = 18$
10. $x^2 + 19x = -18$

1249. When x^2 has a coefficient, divide both members by the coefficient.

$$3x^2 + 9x = 84.$$

Dividing by 3, $x^2 + 3x = 28.$

Completing the square,

$$x^2 + 3x + (\tfrac{3}{2})^2 = 28 + \tfrac{9}{4} = \frac{112 + 9}{4} = \frac{121}{4}.$$

Extracting square root, $x + \frac{3}{2} = \pm \frac{11}{2}$.

$$\therefore\ x = \tfrac{11}{2} - \tfrac{3}{2} = \tfrac{8}{2} = 4;\ \text{or } -\tfrac{11}{2} - \tfrac{3}{2} = -\tfrac{14}{2} = -7.$$

Ans. 4 or -7.

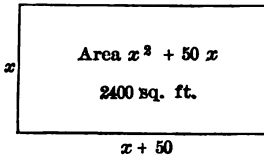
1. $6x^2 - 6x = 36$
2. $9x^2 + 9x = 180$
3. $7x^2 + 28x = 147$
4. $4x^2 - 40x = -64$
5. $8x^2 - 16x = 504$
6. $3x^2 + 9x = 54$
7. $8x^2 - 72x = -160$
8. $7x^2 + 49x = 56$
9. $3x^2 + 21x = 54$
10. $5x^2 - 25x = -20$

1250. Slate Problems.

1. The sum of two numbers is 12; their product is 32. What are the numbers?

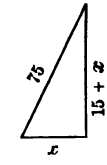
x and $12 - x$ = numbers. $(12 - x)x$ = product.

2. The base of a rectangle is 50 feet longer than its altitude. Its area is 2,400 square feet. How long is the base?



3. The perpendicular of a right-angled triangle measures 15 yards more than the base. The hypotenuse is 75 yards. Find the length of the perpendicular.

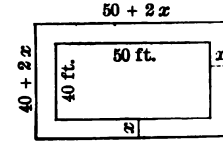
$[x^2 + (15 + x)^2 = 75^2.]$



4. The hypotenuse of a right-angled triangle is $1\frac{1}{4}$ times as long as the base. The area of the triangle is 150 square yards. How long is the hypotenuse?

[Perpendicular $= \sqrt{(\frac{5}{4}x)^2 - x^2}$; area $= \frac{1}{2}$ base $\times$ perpendicular.]

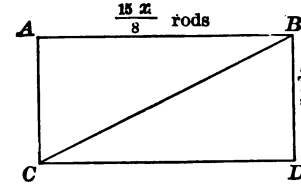
5. The entire surface of a square prism is 170 square feet. Its altitude is 6 feet, and one side of its base is x feet. Find the value of x.



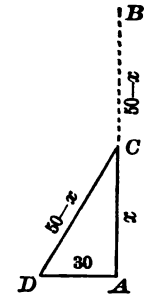
6. A garden 50 feet long, 40 feet wide, has a walk just outside it x feet wide. Find the area of the walk.

If the area of the walk is 784 square feet, what is its width?

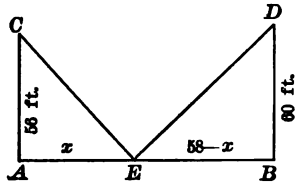
7. A field, $ABCD$, contains 12 acres. Its length is $1\frac{7}{8}$ times its breadth. How many rods long is the diagonal BC?



8. A flag-staff, AB, 50 feet high, was broken off at the point C. The broken part, resting on C, reached the ground D, 30 feet from the base of the staff. Find the length of the part broken off.

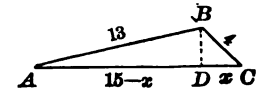


9. A ladder, CE or DE, placed at a point E, in a street 58 feet wide between the opposite houses, just touches the top of a house, DB, 60 feet high on one side of the street, or the top of a house, CA, 56 feet high on the other side. Find the length of the ladder.



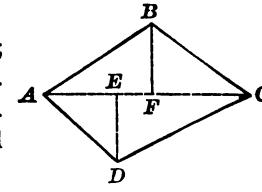
$$\overline{DE}^2 = 60^2 + (58 - x)^2 = \overline{CE}^2 = 56^2 + x^2.$$

10. ABC is a triangle. The side AB measures 13 feet; the side BC, 4 feet; AC, 15 feet. Find the altitude BD.



$$\overline{BD}^2 = \overline{AB}^2 - \overline{AD}^2 = \overline{BC}^2 - \overline{CD}^2.$$

11. $ABCD$ is a trapezium. $AB = 34$ ft.; $BC = 20$ ft.; $CD = 40$ ft.; $DA = 26$ ft. The perpendicular BF measures 16 ft. Find the length of the diagonal AC and of the perpendicular ED.



CHAPTER XVI.

ELEMENTARY GEOMETRY. — PROBLEMS IN CONSTRUCTION. — PRACTICAL APPLICATIONS. — CALCULATION OF HEIGHTS AND DISTANCES. — MENSURATION.

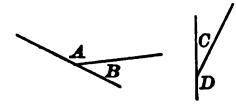
ELEMENTARY GEOMETRY.

1251. Angles.

When two straight lines meet at a point, they are said to form an *angle*.

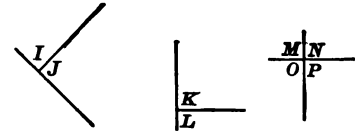
The point at which the lines meet is called the vertex of the angle.

When two angles are formed by the meeting of two straight lines, they are called *adjacent* angles. A and B are adjacent angles. C and D are adjacent angles.

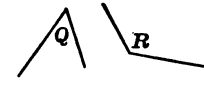


The angles E and G, formed by the intersection of two straight lines, are called *vertical*, or *opposite* angles. F and H are vertical angles. E and F, F and G, G and H, H and E, are adjacent angles.

When two adjacent angles are equal to each other, each is said to be a *right* angle. The angles I, J, K, L, M, N, O, P are right angles.



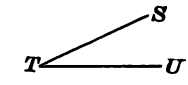
An angle that is smaller than a right angle is called an *acute* angle; one larger than a right angle is called an *obtuse* angle. Q is an acute angle; R is an obtuse angle.



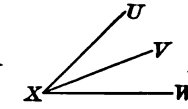
Angles that are not right angles are called, without regard to their size, *oblique* angles.

1252. Designation of Angles.

The angle formed by the lines ST and TU may be called the angle T. It is frequently better to call it the angle STU or UTS, the letter at the vertex being placed between the two others.

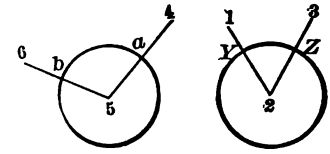


The use of the three letters is necessary where there is more than one angle having its vertex at the same point, as in the accompanying figure, where UX, VX, and WX meet at the point X.



1253. Measurement of Angles.

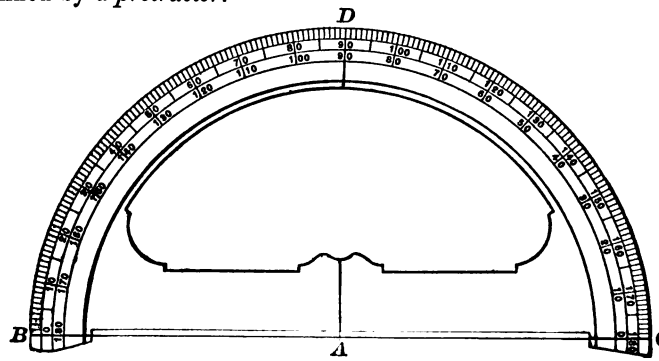
An angle is measured by the arc of a circle, the center of the circle being at the vertex of the angle. The angle 1 2 3 is measured by the arc YZ; the angle 4 5 6 by the arc ab.



1254. Circular Measure.

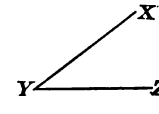
60 seconds (″)	1 minute,
60 minutes (′)	1 degree,
360 degrees (°)	1 circle.

1255. The number of degrees in an arc or an angle is determined by a *protractor*.

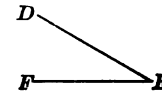


SEMI-CIRCULAR PROTRACTOR

To measure an angle, XYZ, for instance, produce the lines YX and YZ. Place the point A of the protractor on the vertex (Y) of the angle, and the edge AC on the line YZ produced. Using the lower line of figures, read off from the protractor the number of degrees at the point where the line YX produced cuts the semi-circle.



In measuring the angle DEF, the line AB is placed on EF, the point A on the vertex E. The number of degrees in this case is read from the upper row of figures.



EXERCISES IN CONSTRUCTION.

1256. NOTE. — In the following 100 exercises, the ruler, the compasses, and the protractor may be used.

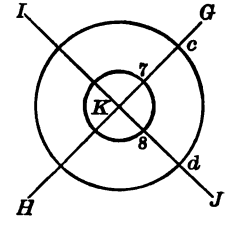
The drawing should be carefully done with a sharp, hard pencil.

1. Draw an obtuse angle formed by two lines, each one inch long. Draw an acute angle formed by two lines, each six inches long. Which is the larger?

2. Fold a piece of paper twice, so that the lines made by the creases will form four right angles.

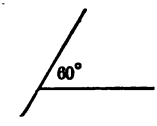
Fold a piece of paper so as to make four angles that are not right angles.

3. The lines GH and IJ intersect at K, making four right angles. Which arc is longer, 7 8, or cd? Which contains the greater number of degrees?



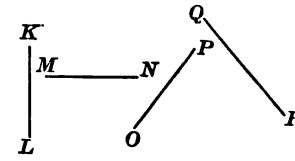
4. Draw two lines meeting at an angle of 45°. Two lines meeting at an angle of 90°. Two meeting at an angle of 135°.

5. Draw two lines making two angles, one of which measures 60°. How many degrees does the other angle contain?



1257. NOTE. — A line parallel to the right or the left side of the paper is called a *vertical* line; one parallel to the top or the bottom of the paper is called a *horizontal* line; others are called *oblique* lines.

KL is a vertical line, *MN* is a horizontal line, *OP* and *QR* are oblique lines.



6. To a horizontal line draw a line making two equal adjacent angles. How many degrees does each angle contain?

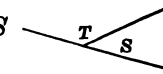
To a vertical line draw a line making two equal adjacent angles. How many degrees does each angle contain?

To an oblique line draw a line making two equal adjacent angles. How many degrees does each angle contain?

7. How many degrees are there in a right angle?

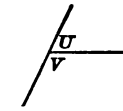
8. To an oblique line draw a line making two unequal adjacent angles. How many degrees are there in the sum of the two angles?

9. How many degrees in the angle *T*, if *S* contains 75°?



V measures 110°. How many degrees does *U* measure?

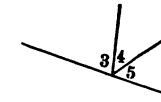
If one of two adjacent angles measures $63\frac{1}{2}°$, how many degrees are there in the other angle?



How many degrees are there in an angle adjacent to one of 47° 45′?

10. Construct angle 5, 60°; angle 4, 50°. Measure angle 3.

How many degrees and minutes will there be in angle 5, when 3 contains $49\frac{1}{2}°$ and 4 contains $83\frac{3}{4}°$?



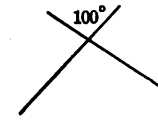
When angle 3 contains 36° 30′ and angle 5 contains 79° 45′, how many degrees and minutes will angle 4 contain?

11. Erect a perpendicular at each extremity of a horizontal line. At each extremity of a vertical line. At each extremity of an oblique line.

1258. NOTE. — A line making a right angle with another line is said to be *perpendicular* to it.

12. Construct a square upon a horizontal line. Upon an oblique line.

13. Draw two lines intersecting at an angle of 100°. Mark in each of the other three angles the number of degrees it contains.



14. If one of the four angles formed by two intersecting lines measures 90°, what does each of the other three measure?

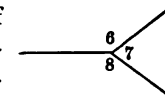
If one measures 60°, what does each of the others measure?

15. At each extremity of a horizontal line draw a line making an angle of 40° with the first line.

16. At each extremity of a vertical line draw a line making an angle of 100° with the first line.

17. At one extremity of an oblique line draw a line making with the first line an acute angle; at the other extremity draw a line making an obtuse angle with the first line.

18. Draw two lines making an angle (6) of 150°. Construct an adjoining angle (7) containing 80°. How many degrees will angle 8 contain?



19. How many degrees will there be in the sum of five angles having the same vertex?

20. Draw five equal angles having a common vertex.

Is any one of these five angles adjacent to any other?

21. Draw six equal angles having a common vertex. Is any angle adjacent to the angle next it? Why?

Are any of the angles vertical? Why?

22. Draw two angles, one of 65° and the other of 25°. Draw a third angle equal to the sum of both.

Draw an angle equal to their difference.

23. Draw an angle equal to the sum of three angles measuring, respectively, 40°, 50°, and 60°.

24. How many degrees are contained in the angle made by the hour and the minute hand of a watch at 1 o'clock? How many degrees in the angle made by the hands of a church-tower clock at the same hour?

25. How many degrees are passed over by the minute hand in a quarter of an hour? How many are passed over by the hour hand in an hour? In half an hour? In 15 minutes?

What angle is made by the hands at 12:15? At 6:30? At 8:20?

1259. NOTE. — The pupils should, of course, understand that the angles in questions 24 and 25 are those formed by imaginary lines passing through the centers of the hands. They should know, too, that a geometrical line has neither breadth nor thickness

1260. Parallels.

Lines lying in the same plane that will not meet, no matter how far they are produced, are said to be *parallel.*

26. Draw two or more lines that shall be perpendicular to a horizontal line. Where will they meet?

Draw two or more that shall be perpendicular to a vertical line. Where will they meet?

Draw two or more that shall be perpendicular to an oblique line. Where will they meet?

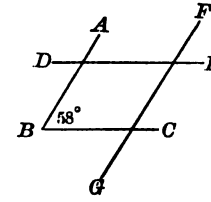
27. To a horizontal line draw two or more lines running in the same direction, and each making an angle of 35° with the first line. Will the oblique lines meet?

Draw two or more lines running in the same direction, and each making an angle of 125° with a vertical line. Will the oblique lines meet if produced very far?

Draw two or more lines running in the same direction, and each making an angle of 74° with an oblique line? Will the former lines meet?

28. Draw two lines making angles of 30° and 60°, respectively, with a third line. Will the two former lines meet if produced in either direction?

29. Draw a line, AB, meeting a horizontal line BC at an angle of 58°. Draw a third line, DE, parallel to the horizontal line, and cutting the oblique line. What angles does it make with the oblique line?

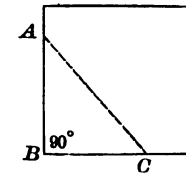


Draw a fourth line, FG, parallel to the oblique line, and cutting both horizontal lines.

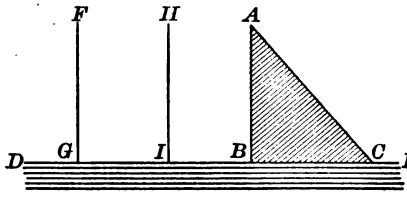
Mark in each of the twelve angles the number of degrees it contains.

Take a stiff card, having a square corner, and cut off the corner, making the side AB equal to the side BC, and each about 4 or 5 inches long.

Measure the angle at A and the angle at C.

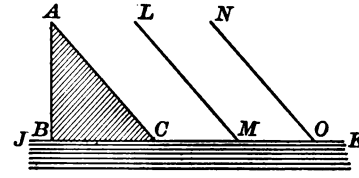


30. With a ruler, draw a line DE; and without moving the ruler, draw lines FG, HI, AB, by placing the side BC of the triangle firmly against the edge of the ruler.



FG, HI, and AB are parallel. Why?

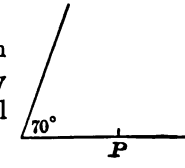
31. Holding the triangle against the edge of ruler JK, draw oblique lines AC, LM, NO.



If the angle ACB contains 45°, how many degrees will LMJ contain? How many will NOJ contain?

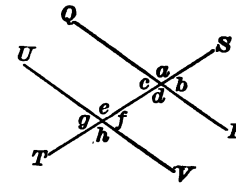
Are AC, LM, and NO parallel? Why?

32. Draw a line making an angle of 70° with a horizontal line. At the point P, using only the ruler and the triangle, draw a line parallel to the oblique line.



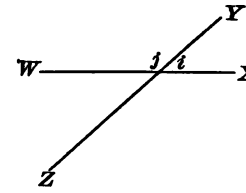
33. QR and UV are parallel lines, cut by a line ST. If the angle b measures 50°, how many degrees does a measure?

Find the number of degrees in each of the other six angles.



34. Draw WX and YZ intersecting at any angle. With the ruler and the triangle draw a line parallel to WX.

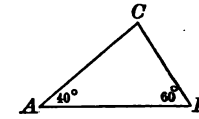
Mark all the angles that are equal to i. Mark all those that are equal to j.



1261. Triangles.

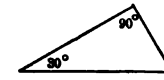
35. From the extremities of the line AB, draw lines that shall make angles of 60° and 40°, respectively, with AB. Prolong the lines until they meet at C, forming a triangle.

Measure the angle at C. How many degrees does it contain? How many degrees are there in the sum of the three angles of the triangle?



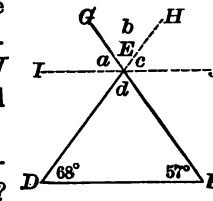
36. Construct a triangle having one angle of 90° and one of 30°. Measure the third angle.

How many degrees are there in the sum of the three angles?



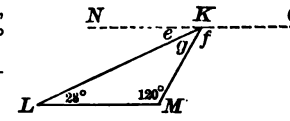
37. Draw a triangle having angles at the base measuring, respectively, 57° and 68°. With the ruler and the triangle draw IJ parallel to DF. Produce DE to H and FE to G.

How many degrees will the angle a contain? How many degrees in the angle c? Calculate the number of degrees in the angle b when you know



the number of degrees in a and c, respectively. What angle is vertical to b? How many degrees in d?

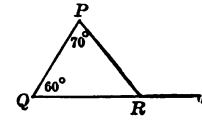
38. Construct a triangle, KLM, making the angles at the base 28° and 120°, respectively. Draw, as before, NO, parallel to LM.



Is the angle e equal to any angle of the triangle? How many degrees does it contain? Is the angle f equal to any angle of the triangle? How many degrees does it contain?

How many degrees are there in the sum of the angles e, g, and f? How many degrees are there in the angle g?

39. The angle at P is 70°, the angle at Q is 60°. Can you tell how many degrees there are in the angle PRS, formed by producing the base RS?



The angle PRS is called an *exterior* angle of the triangle PQR.

40. How many degrees are there in the three angles of any triangle?

41. Two angles of a triangle measure 36° and 65°, respectively. How many degrees does the third angle contain?

1262. A triangle containing a right angle is called a *right-angled* triangle.

A triangle containing an obtuse angle is called an *obtuse-angled* triangle.

A triangle all of whose angles are acute, is called an *acute-angled* triangle.

Obtuse-angled triangles and acute-angled triangles are also called *oblique-angled* triangles.

42. Draw a triangle that shall contain an acute angle. Mark the acute angle.

Draw one that shall contain two acute angles. One that shall contain three acute angles.

43. See if you can draw a triangle containing three right angles.

A triangle containing two right angles.

A triangle containing one right angle.

44. Try to draw a triangle that shall contain three obtuse angles.

A triangle that shall contain two obtuse angles.

A triangle containing one obtuse angle.

45. Draw a triangle containing two angles of 50° and 70°, respectively. How many degrees are there in the third angle?

Measure each side, and mark on the side its length.

Opposite which angle is found the longest side? Opposite which, the shortest side?

46. Draw a triangle having two angles of 75° each. Are any two of its sides equal?

Draw a triangle having two angles of 50° each. Are any of its sides equal?

47. Draw a triangle having two angles of 60° each. How many degrees does the third angle contain?

Are any of its sides equal?

1263. A triangle having all its sides equal, is called an *equilateral* triangle.

A triangle having two equal sides, is called an *isosceles* triangle.

A triangle having all its sides unequal, is called a *scalene* triangle.

48. Fold evenly on the line AC (Fig. 1), a rectangular sheet of paper. Cut off, on the line AB, one of the folded corners. When this piece is opened out (Fig. 2), it makes what kind of a triangle? Which are the equal sides? How does the crease line, AC, divide the base, BB?

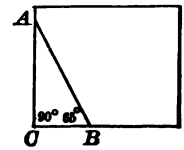


FIG. 1.

If the angle ABC, Fig. 1, is 65°, and the angle at C is a right angle, what is the angle BAC?

How many degrees in the angle BAB', Fig. 2? How does a perpendicular let fall upon the base of an isosceles triangle from the opposite angle divide the angle? How does it divide the base? How do the angles at the base of an isosceles triangle compare with each other as to size?

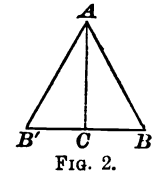


FIG. 2.

1264. The *unequal* side of an isosceles triangle is called the base.

49. Draw an isosceles triangle having the base a vertical line.

An isosceles triangle having the vertex below the base.

One having an oblique line for the base.

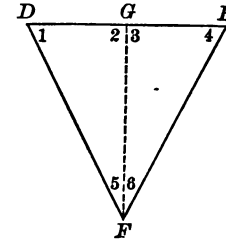
50. Draw a right-angled isosceles triangle. How many degrees will there be in each of the other angles?

Draw an obtuse-angled isosceles triangle.

51. How many degrees will there be in each angle of an equilateral triangle?

Draw an equilateral triangle having one side vertical.

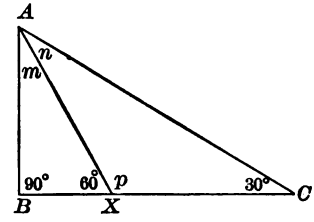
Draw an equilateral triangle having its vertex below the base.



52. DEF is an isosceles triangle, DF and EF being the equal sides. If the angle 1 measures 50°, how many degrees are there in each of the other five angles, when the line FG bisects the base?

53. If the angle DFE in the above triangle measures 45°, how many degrees will there be in the angles 1, 2, 3, 4, 5, and 6?

54. ABC is a right-angled triangle, the angle at B measuring 90°, and the angle at C measuring 30°. If the line AX is so drawn as to make the angle AXB equal to 60°, find the number of degrees in the angles m, n, and p, respectively?



1265. Quadrilaterals.

A plane figure of four sides is called a *quadrilateral.*

When the opposite sides are parallel, the quadrilateral is called a *parallelogram.* (Fig. 1 to 8.)

A *rectangle* is a parallelogram all of whose angles are right angles. (Fig. 1 to 4.)

When the four sides of a rectangle are equal to each other, it is called a *square.* (Fig. 1 and 2.)

The term *oblong* is frequently applied to rectangles whose adjacent sides are unequal. (Fig. 3 and 4.)

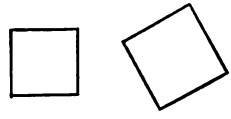


FIG. 1. FIG. 2.

FIG. 3. FIG. 4.

A *rhombus* is a parallelogram all of whose sides are equal, but whose angles are oblique. (Fig. 5 and 6.)

When the adjacent sides of a parallelogram are unequal and the angles are oblique, it is called a *rhomboid.* (Fig. 7 and 8.)

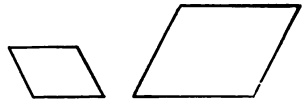


FIG. 5. FIG. 6.

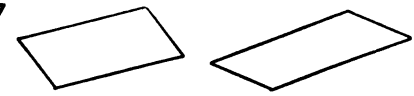


FIG. 7. FIG. 8.

A *trapezoid* is a quadrilateral having only two of its sides parallel. (Fig. 9 and 10.)

A *trapezium* is a quadrilateral having no two sides parallel. (Fig. 11 and 12.)

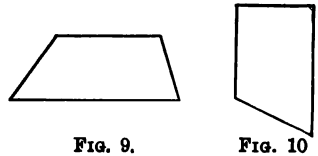


FIG. 9. FIG. 10

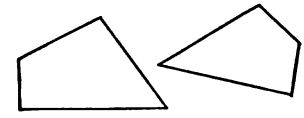
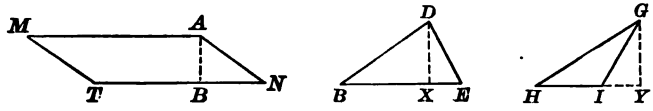


FIG. 11. FIG. 12.

1266. The *altitude* of a *parallelogram* is the perpendicular distance between its base and the side opposite.



The *altitude* of a *triangle* is the perpendicular distance between the vertex and the base, or between the vertex and the base produced.

AB is the altitude of $MANT$; DX is the altitude of DBE; GY of GHI.

55. Draw a parallelogram. How many angles does it contain? Into how few triangles can you divide a parallelogram? How many degrees are there in the sum of the angles of each triangle? How many degrees are there in the sum of the angles of a parallelogram?

56. Construct a parallelogram, the adjacent sides of which shall measure 2 inches and 3 inches, respectively, and the angle between them 60°. How long will each of the other two sides be? Measure each of the other angles. How many degrees are there in the sum of the four angles?

57. Construct a trapezoid having a base of 5 inches, altitude 3 inches, the angles at the base measuring 90° and 60°, respectively. Measure the remaining angles, and find the sum of the four angles. How long is each of the remaining sides?

58. Fold a piece of paper twice at right angles, and cut off the folded corner, making a rhombus when the part cut off is opened out.

Can you cut out a rhombus having two angles of 60° each? A rhombus having two angles of 80° each?

59. Can you so cut a piece of paper, folded twice at right angles, that the part cut off will be a square?

60. We have seen that an equilateral triangle is *equiangular;* that is, that all its angles are equal. Are the four angles of a square equal? Are the four angles of a rhombus equal?

61. A triangle that has three equal angles is *equilateral;* that is, all its sides are equal. Can you draw an equiangular parallelogram that will be equilateral? Can you draw an equiangular parallelogram that will not be equilateral?

62. Draw a rectangle, base $2\frac{1}{2}$ inches, altitude 2 inches.
A rhomboid, base $2\frac{1}{2}$ inches, altitude 2 inches.

63. Make, out of paper, a rectangle and a rhomboid, each having the above dimensions, and endeavor to ascertain, by cutting, whether or not they are equal to each other in area.

64. Show, by cutting, that any parallelogram can be divided into two equal triangles.

65. Make three rhomboids of different shapes, the base of each to measure 3 inches, and the altitude 2 inches. Are they equal to each other in area?

66. Make three rhomboids of different shapes, the base of each to measure 3 inches, and the adjacent side 2 inches. Are they equal to each other in area?

67. How do we calculate the area of a rectangle 3 inches by 2 inches? Of a rhombus whose base is 3 inches, altitude 2 inches? Of a rhomboid, base 3 inches, altitude 2 inches? Of a right-angled triangle, base 3 inches, altitude 2 inches? Of an obtuse-angled triangle, base 3 inches, altitude 2 inches? Of an acute-angled triangle, base 3 inches, altitude 2 inches? Draw each of the foregoing.

68. Draw a rectangle having two adjacent sides measuring 3 inches and 2 inches, respectively. A rhomboid having two adjacent sides measuring 3 inches and 2 inches, respectively. A trapezium having two adjacent sides measuring 3 inches and 2 inches, respectively. A trapezoid having two adjacent sides

measuring 3 inches and 2 inches, respectively. A right-angled triangle, an obtuse-angled triangle, and an acute-angled triangle, each having two adjacent sides measuring 3 inches and 2 inches, respectively.

Calculate the area of each, making such measurements as may be necessary.

69. Draw three trapezoids of different shapes, whose parallel sides shall measure 3 inches and $2\frac{1}{2}$ inches, respectively, and whose altitude shall be 2 inches.

Draw a rectangle 2 inches high that shall be equal in area to each of the foregoing. How long will be its base? What will be the relation of the length of its base to the lengths of the parallel sides of the trapezoids?

70. Draw three trapeziums of different shapes, each having one diagonal of 3 inches, the perpendiculars let fall on this diagonal from the opposite corners measuring $2\frac{1}{2}$ and 2 inches, respectively.

What is the altitude of a rectangle that is equal in area to each of the foregoing, when the base of the rectangle measures 3 inches?

1267. The Circle.

71. With the compass points one inch apart, draw a circle.

Draw a radius. How long is it?

The line made by the compass pencil is called the *circumference*. A line drawn from the center to the circumference is called a *radius*.

72. Draw a circle, radius $1\frac{1}{2}$ inches. From a point on the circumference, draw a line through the center to an opposite point on the circumference.

This line is called a *diameter*. How long is the diameter?

73. Draw a circle, radius $1\frac{3}{4}$ inches. Draw a diameter and several radii.

Write *diameter*, *circumference*, *radius*, each in its proper place.

74. Draw a circle. Between two points on the circumference draw a line that does not pass through the center.

This line is called a *chord.*

75. Draw a circle. In it draw two diameters, a radius, and three chords. Write on each line its name.

76. Draw a part of the circumference of a circle greater than one-half of it. Draw the chord.

A part of the circumference is called an *arc.*

77. Draw an arc less than a semi-circumference. Draw a chord. Write the name on each.

Can you make a chord that will be longer than the diameter?

78. Draw two equal circles. In the first draw the chord of an arc of 120°. In the second, the chord of an arc of 240°. What is the ratio between the two chords you have drawn?

79. In a circle draw a chord equal in length to the radius. How many degrees are there in the arc whose chord has been drawn?

80. Draw an arc of 72°. To its extremities draw two radii.

The part of the surface of a circle enclosed by two radii and the intercepted arc is called a *sector.*

81. Draw a sector of 60° (a *sextant*). A sector of 90° (a *quadrant*).

82. Draw an arc of 120 degrees. Draw the chord.

The part of the surface of a circle bounded by an arc and its chord is called a *segment.*

83. Draw several circles having the same center, but of unequal radii (*concentric* circles).

84. Draw two equal circles just touching each other (*tangent*). Draw two unequal circles tangent to each other.

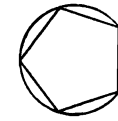
Within a large circle draw a smaller one tangent to it.

85. Draw circles of equal radii cutting each other. Draw intersecting circles of unequal radii.

1268. Pentagons, Hexagons, Octagons.

86. Divide the circumference of a circle into four equal arcs. Draw the chords, forming an inscribed square.

87. If you wish to inscribe in a circle a figure of five equal sides, into how many equal arcs must the circumference be divided? How many degrees will each arc contain?



1269. A plane figure bounded by straight lines is called a *polygon.*

A five-sided polygon is called a *pentagon;* one of six sides, a *hexagon;* of seven, a *heptagon;* of eight, an *octagon;* of nine, a *nonagon;* of ten, a *decagon;* etc.

A *regular* polygon is one that is both equilateral and equiangular.

88. Inscribe a regular pentagon in a circle. Use the protractor.

89. Inscribe in a circle a regular hexagon. A regular octagon. An equilateral triangle.

90. Inscribe in a circle a regular hexagon. Connect the opposite corners by lines passing through the center of the circle, forming six triangles.

How many degrees are there in each of the six angles about the center of the circle? In each of the twelve angles at the circumference? How many degrees are there in the sum of angles 1 and 2?

Is each of the six triangles scalene, equilateral, or isosceles?

91. Divide a regular inscribed pentagon into five equal triangles by lines drawn from the center of the circle.

What kind of triangles are formed; isosceles, scalene, or equilateral?

How many degrees are there in each angle at the center? In each angle at the circumference? How many degrees are there in the sum of two adjoining angles at the circumference?

92. Circumscribe a square about a circle. An equilateral triangle. A regular pentagon. A regular hexagon. A regular octagon.

93. Draw a pentagon. What is the smallest number of triangles into which a pentagon can be divided? How many degrees are there in all of these triangles? How many degrees are there in each angle of a regular pentagon?

94. Draw a hexagon. What is the smallest number of triangles into which it can be divided? How many degrees do the six angles of a hexagon contain? How many degrees are there in each angle of a regular hexagon?

95. A quadrilateral is divisible into how few triangles? A pentagon into how few? A hexagon? A heptagon? An octagon?

The smallest number of triangles into which a polygon is divisible is how many less than the number of its sides?

96. How many degrees are there in each angle of a regular octagon?

97. Using a protractor, construct a regular pentagon on a line two inches long.

98. On a three-inch line construct an equilateral triangle. On the same line construct a square, a regular pentagon, a regular hexagon, heptagon, octagon, nonagon, etc.

99. Inscribe a square in a circle. Circumscribe a square about the same circle by drawing lines touching the four corners of the inscribed square.

What is the ratio between the areas of the two squares?

100. Inscribe an equilateral triangle in a circle. Circumscribe an equilateral triangle about the same circle by drawing lines touching the vertex of each of the three angles of the inscribed triangle.

What is the ratio between a side of the inscribed triangle and a side of the circumscribed triangle? What is the ratio between the areas of the two triangles?

PROBLEMS IN CONSTRUCTION.

1270. In drawing the following one hundred exercises, only the ruler and the compasses are to be used. Use neither the protractor nor the triangle.

1. Draw a circle, radius an inch and a half. Outside of it, and tangent to it, draw a second circle of an inch radius.

How far apart are the centers?

2. Draw two tangent circles having radii of an inch and a half and an inch, respectively, one within the other.

How long is the line joining the centers?

3. With centers 3 inches apart draw two equal circles tangent to each other. How long is the radius of each?

4. With centers three inches apart draw two equal circles of 2 inches radius. Connect the centers.

Draw a line joining the two points in which the circles intersect. How does this line divide the line connecting the centers?

Draw radii from each center to each point of intersection.

5. Construct an isosceles triangle, base 3 inches, equal sides 2 inches.

NOTE. — Use circles or arcs where necessary.

6. Construct an isosceles triangle, base $3\frac{1}{2}$ inches, equal sides 4 inches.

Divide it into two equal parts. Do not locate the center of the base by measurements.

7. On a vertical line construct an isosceles triangle. Without measuring the length of the base draw a perpendicular to the center of the base.

8. Bisect a vertical line. An oblique line.

Do not measure the length of the line.

9. Construct an equilateral triangle on a two-inch line.

10. Construct an equilateral triangle on a vertical line. On an oblique line.

11. Construct a scalene triangle.

A triangle having sides measuring 1, $1\frac{1}{2}$, 2 inches, respectively.

One whose sides measure 2, $2\frac{1}{2}$, and 3 inches, respectively.

12. Can you construct an isosceles triangle whose base measures 4 inches, equal sides 2 inches?

Try to construct a scalene triangle with sides measuring 1, 2, and 3 inches, respectively.

13. Draw a circle. In it draw a chord.

Bisect the chord, using as few lines and as short ones as you can.

NOTE. — Do not use the ruler to ascertain the length of the chord before bisecting it.

14. Divide a sector into two equal parts.

15. Draw a circle. Draw a chord. Draw a radius through the center of the chord.

Is the radius perpendicular to the chord? Why?

16. Bisect the arc of a circle and its chord.

Bisect the arc of a circle without drawing the chord.

17. Draw a perpendicular to the middle point of a horizontal line. To the middle point of a vertical line. To the middle point of an oblique line.

18. Draw in a circle two diameters perpendicular to each other.

19. Divide the circumference of a circle into four equal parts. Into eight equal parts.

Inscribe a square in a circle.

20. Inscribe a regular octagon in a circle.

21. Connect the opposite vertices of a regular octagon inscribed in a circle by lines passing through the center of the circle.

Lines connecting the opposite vertices of a polygon are called *diagonals*.

22. Inscribe a square in a circle. Circumscribe a square whose sides shall be perpendicular to the diagonals of the inscribed square.

23. Construct an equilateral triangle on a horizontal line 1 inch long. On the right side as a base, construct a second equilateral triangle. On the left side of the first triangle, construct a third. Construct three more, completing the hexagon.

24. Can you circumscribe a circle about the above hexagon? What is the radius of the circle?

25. Inscribe a regular hexagon in a circle whose radius is $1\frac{1}{2}$ inch. What is the length of each side of the hexagon?

26. Inscribe in a circle an equilateral triangle. On each of its three sides construct an equilateral triangle.

27. Construct an arc of 60°. Draw two lines meeting at an angle of 60°.

28. Bisect an arc of 60°. Draw two lines meeting at an angle of 30°.

29. Construct an angle of 60° and an angle of 30°. Draw two lines making an angle equal to the sum of the two angles first constructed.

30. Erect a perpendicular at the end of a horizontal line. At the end of a vertical line. At the end of an oblique line.

31. Construct an angle of 45°. An angle of $22\frac{1}{2}$°. An angle of 135°. An angle of 15°. An angle of 75°.

32. Draw a circle, radius 1 inch. Draw a diameter, and produce it an inch beyond the circumference. At the center of the circle erect a perpendicular to the diameter.

33. An inch from one end of a 3-inch line, erect a perpendicular, using as few and as short lines as possible.

34. Erect a perpendicular at the center of an oblique line. Erect a perpendicular at one end of an oblique line. Erect a

perpendicular at a point in a line between one end and the center.

35. Construct a right-angled triangle, base 2 inches, altitude 2 inches.

A right-angled triangle, base 3 inches, altitude $2\frac{1}{2}$ inches.

36. On a line 2 inches long, construct a square. Construct a rectangle, base 3 inches, altitude 2 inches.

37. On an oblique line 3 inches long, construct a square. On an oblique line 3 inches long, construct a rectangle, altitude $2\frac{1}{2}$ inches.

38. Construct a rhombus whose side is 3 inches, altitude $2\frac{1}{2}$ inches.

A rhombus whose side is 3 inches, and which contains an angle of 60°.

39. Construct a rhomboid whose adjacent sides measure 4 inches and 3 inches, respectively, altitude $2\frac{1}{2}$ inches.

40. Construct a right-angled isosceles triangle. An isosceles triangle containing an angle of 120°. One containing an angle of 135°.

41. Construct an isosceles triangle, base 3 inches, altitude 3 inches. Can you construct an equilateral triangle whose altitude shall be 3 inches?

42. Construct a scalene triangle, base 3 inches, altitude 3 inches. An obtuse-angled triangle having a 3-inch base, and the altitude 3 inches.

43. Construct a triangle whose base measures 5 inches, the other sides being 3 inches and 4 inches, respectively. Draw its altitude geometrically.

44. Construct a triangle whose base measures 4 inches, the angles at the base measuring 120° and 30°, respectively. Draw its altitude geometrically.

45. Draw a line. From a point above the line, let fall a perpendicular to the line.

46. Inscribe a regular hexagon in a circle. Divide it into six equilateral triangles by diagonals.

How many degrees are there in each of the six angles at the center of the circle? How many degrees are there in the arc on which each angle at the center stands?

47. Inscribe an equilateral triangle in a circle. Mark in each angle the number of degrees it contains. Mark on each arc the number of degrees it contains.

An angle at the circumference of a circle is measured by what part of the arc on which it stands?

48. Do you know how many degrees there are in each angle of a regular hexagon?

Construct a regular hexagon without drawing a circle and without using triangles.

49. Construct a regular octagon, each side one inch. Do not draw a circle.

50. If an angle at the circumference is measured by one-half of the arc on which it stands, how many degrees are contained between two lines forming a right angle whose vertex is at the circumference?

51. Inscribe several right angles in a circle, their sides intercepting the same arc.

52. Inscribe in a circle a right-angled isosceles triangle.

53. Construct a square whose diagonal measures 4 inches.

A rhombus whose side is 3 inches and which has one diagonal 3 inches long.

54. Inscribe in a circle a scalene triangle. How many degrees are there in the sum of the arcs intercepted by the sides forming the three angles?

How many degrees are there in the sum of the three angles?

55. Draw two equilateral triangles having one side common to both triangles. What figure have you drawn?

56. Construct two triangles, each having two sides measuring 2 inches and $2\frac{1}{2}$ inches, respectively, and the angle made by these sides (the included angle) measuring 60°.

Cut out the triangles. Place one upon the other, and ascertain if the third side of one triangle is equal to the third side of the other. Are the two remaining angles of the first triangle equal to the corresponding angles of the second?

57. Construct a triangle having a base of 3 inches, and angles at the base containing 60° and 45°, respectively. Construct a second triangle having angles of 60° and 45°, respectively, and the included side 3 inches.

Cut both out of paper, and test the equality of the corresponding parts of each.

58. Construct a triangle whose angles measure 30°, 60°, and 90°, respectively. Can you construct another triangle having angles equal to those in the first triangle, but with sides not equal to those of the first triangle?

59. Construct a triangle having sides of 2, $2\frac{1}{2}$, and 3 inches, respectively.

Try to construct a second triangle having sides equal to those of the foregoing triangle, but having the corresponding angles unequal.

60. Can you construct a triangle containing three angles of 75° each?

61. In a circle, inscribe a right-angled triangle whose hypotenuse is 3 inches.

One whose hypotenuse is 3 inches, base 2 inches.

Hypotenuse 3 inches, perpendicular $1\frac{1}{2}$ inches.

What is the diameter of the circle in each case?

62. Draw two equal circles. Mark off an arc in one. Mark off an equal arc in the other.

63. Draw two lines meeting at any angle. Construct a second angle equal to the first.

64. Draw a horizontal line. From each end, and from two points between, draw four oblique lines that shall be parallel to each other.

From one end of a three-inch horizontal line, draw an oblique line running upwards; at the other end, draw a line running downwards that shall be parallel to the first oblique line.

65. On a three-inch line as a diagonal, construct a rhomboid having two opposite sides each $2\frac{1}{2}$ inches long. Mark off on each of these two sides half-inch divisions. Draw lines through the diagonal, dividing the rhomboid into 5 equal parts. How do these lines divide the diagonal?

66. Divide a 3-inch line into 5 equal parts.

Draw a line exactly $\frac{3}{5}$ inch in length. One exactly $\frac{5}{7}$ inch.

67. Draw a right-angled triangle whose base is 4 inches, perpendicular 3 inches. How long is the hypotenuse?

One whose base is 3 inches, perpendicular 2 inches. How long is the hypotenuse?

One whose hypotenuse is 4 inches, base 3 inches. How long is the perpendicular?

68. Can you draw a line measuring exactly $\sqrt{13}$ inches? One measuring exactly $\sqrt{7}$ inches? One measuring exactly $\sqrt{5}$ inches?

69. Divide a 2-inch equilateral triangle into two right-angled triangles. Mark the number of degrees in each of the six angles.

In each right-angled triangle, find the ratio between the length of a side opposite an angle of 30° and the length of a side opposite an angle of 90°.

70. Construct a right-angled triangle, hypotenuse 4 inches, angle at base 30°. Measure the length of the perpendicular.

Construct a right-angled triangle whose perpendicular is $1\frac{1}{2}$

inches, the angle formed by the perpendicular and the hypotenuse being 60°. How long is the hypotenuse?

71. Draw in a circle whose radius is two inches, the chord of an arc of 60°. Draw the chord of an arc of 180°. What is the ratio between their respective lengths?

What is the ratio between the chord of an arc of 60° and the chord of an arc of 300°?

72. In a circle of 2 inches radius, draw a chord. At one extremity erect a perpendicular. From the other extremity draw a diameter. Where will this diameter meet the perpendicular? Why?

73. Make a triangle of splints, fastening them by one tack at each angle. Will the triangle retain its shape under pressure?

Make a rectangle of splints. Fasten each corner by a single tack. Will the rectangle retain its shape under pressure?

74. In a circle draw three parallel chords the same distance apart.

Draw three parallel chords not equidistant.

75. At the circumference of a circle, draw a line perpendicular to a radius. This perpendicular is tangent to the circumference.

A line that touches the circumference in a single point is called a *tangent*. A line that cuts the circumference is called a *secant*.

76. Inscribe a circle in a square.

77. In a circle draw two chords not parallel. Bisect each by a perpendicular. Where will these perpendiculars intersect?

78. Inscribe a circle in an equilateral triangle. In an isosceles triangle. In a scalene triangle.

79. Circumscribe a circle about an equilateral triangle. About an isosceles triangle. About a scalene triangle.

80. Draw a circle, using a coin or something similar. Can you find the center?

81. Draw an obtuse-angled triangle. Circumscribe a circle about it.

82. Without using the compasses, draw an arc. Can you find the center of the circle of which the arc forms a part?

83. Construct a rhombus whose sides are 4 inches, the acute angle being 30°. What is its altitude?

84. Construct a square on a line 3 inches long, using only arcs of 3 inches radius.

85. Can you construct a triangle with sides 2, 4, and 6 inches?

86. Draw a circle, radius 3 inches. Draw an equilateral triangle around it.

87. Construct an equilateral triangle, altitude $1\frac{1}{2}$ inches. Construct one having an altitude twice as great.

88. Construct a triangle whose sides measure 1, $1\frac{1}{2}$, and 2 inches, respectively. Construct a second having its corresponding sides twice as large as those of the first triangle.

Construct a triangle whose angles measure 30°, 60°, and 90°, respectively. Try to construct a second triangle having its corresponding angles twice as large as those of the first triangle.

89. Construct a square which shall be equal in area to two squares, one having a side of 2 inches, the other having a side of 3 inches.

90. Can you construct a square whose area will be 13 square inches?

91. Construct a square whose area will be equal to the difference of area between two squares, one having a side of 3 inches, the other having a side of 2 inches.

92. Construct a square whose side is 3 inches. Construct another having double the area.

93. Construct an equilateral triangle, side 2 inches. How many 1-inch equilateral triangles can be made from it?

94. How many 1-inch equilateral triangles can be made from an equilateral triangle whose sides are 3 inches?

From one whose sides are 4 inches? 5 inches?

95. Construct a triangle, sides 2, 3, 4 inches. Divide it into four equal triangles. Give the dimensions of each of the latter.

96. Construct a triangle 1, $1\frac{1}{2}$, 2 inches. Make a triangle nine times as large as the first by producing two of the sides and drawing a fourth line. What are the dimensions of the second triangle?

97. Draw a circle of 1 inch radius. Tangent to it, and enclosing it, draw one having four times the area of the first.

98. Can you draw a circle having half the area of a circle whose radius is 2 inches?

99. Draw an equilateral triangle, side 2 inches. Construct another of half the area of the first.

100. Can you tell the ratio between the area of an inscribed and that of a circumscribed hexagon?

EQUAL TRIANGLES. EQUIVALENT TRIANGLES.

1271. NOTE. — The protractor and the triangle may be used in the following exercises.

1. Draw a rectangle, base $2\frac{1}{2}$ inches, altitude 2 inches. Draw a rhomboid, base $2\frac{1}{2}$ inches, altitude 2 inches. Find the area of each.

2. With a base $2\frac{1}{2}$ inches, altitude 2 inches, draw

(*a*) A right-angled triangle.

(*b*) An isosceles triangle.

(*c*) One or more acute-angled scalene triangles.

(*d*) One or more obtuse-angled triangles.

Calculate the area of each.

3. Can you show, by cutting from paper, that a right-angled triangle having its base and perpendicular 4 inches and 3 inches, respectively, has the same surface as an acute-angled triangle whose base and altitude are 4 inches and 3 inches, respectively, and an obtuse-angled triangle whose base and altitude are 4 inches and 3 inches, respectively?

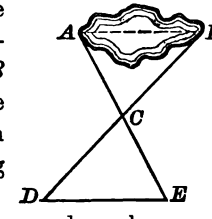
Two triangles that have the same area are called *equivalent* triangles; those having their corresponding sides and angles equal, each to each, are called *equal* triangles.

4. Construct a triangle whose sides measure $1\frac{1}{2}$, 2, and $2\frac{1}{2}$ inches, respectively. Construct another triangle having its sides of the same lengths. Are the angles of the second equal to the angles of the first? Are the triangles equal?

5. Draw two triangles each of which has two sides measuring $1\frac{1}{2}$ and 3 inches, respectively, and the included angle 60 degrees. Is the third side of one triangle equal to the third side of the other? Are the remaining angles of the first triangle equal to the remaining angles of the second?

6. With a base 2 inches long, and with angles at the base measuring 50° and 60°, respectively, construct a triangle. Construct a second triangle having a base measuring 2 inches, and angles at the base measuring 60° and 50°, respectively. Are the two triangles equal?

7. A person wishing to ascertain the length, AB, of a pond, places a pole at a convenient point, C, visible from A and B. The distance BC is measured, and a pole is set up, on a line with B and C, at D, the distance CD being made equal to BC. A pole is also placed at E, on a line with A and C, the distance CE being made equal to AC.

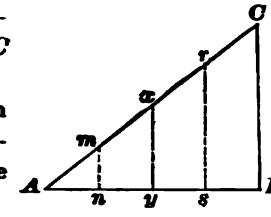


Can you show that the length, AB, of the pond can be ascertained by measuring the distance DE?

SIMILAR TRIANGLES.

1272. Construct a right-angled triangle ABC. Make AB 4 inches, AC 5 inches, BC 3 inches.

At the points n, y, and s, distant from A one, two, and three inches, respectively, erect perpendiculars. Measure the length of each.



8. An is one-fourth of AB; ascertain the ratio between mn and CB.

Compare xy and CB; see if $\frac{CB}{xy} = \frac{AB}{Ay} = 2$.

As is three-fourths of AB; is $rs = \frac{3}{4}BC$?

Measure Am, Ax, and Ar. How does each compare with AC?

Would the relations as found above, exist if mn were not perpendicular to AB?

How do the angles in the triangle Amn compare with the angles in the triangle ABC?

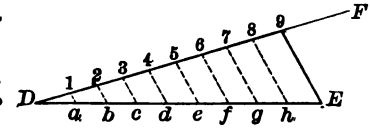
9. Draw a triangle whose sides measure 4, 5, and 6 inches, respectively. Cut out of paper a triangle whose sides measure 2, 2½, and 3 inches, respectively. Place an angle of the small triangle on the corresponding angle of the large triangle, and compare their respective sizes.

How does the area of the small triangle compare with the area of the large triangle?

10. Two angles of a triangle measure about 37° and 53°, respectively. The sides opposite those angles measure 3 inches and 4 inches, respectively. How many degrees does the third angle contain? Calculate the length of the third side.

What will be the dimensions of a similar triangle whose area is one-fourth that of the given triangle? Give the approximate number of degrees in each angle of the small triangle.

11. Draw a line DE, $2\frac{7}{8}$ inches long. At any angle draw DF. Commencing at D, mark off on DF quarter-inch portions, 1, 2, 3, etc., to 9. Join $9E$, and with a triangle and a ruler draw $8h$, $7g$, $6f$, etc., parallel to $9E$. Measure Eh. Is it equal to hg, gf, fe, etc.? Why?



Into how many parts is the line ED divided? What fraction of an inch does each part contain?

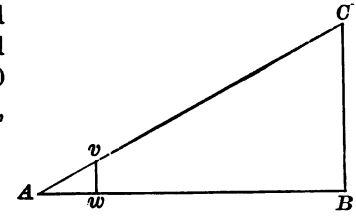
In locating the points 1, 2, 3, 4, etc., is it necessary to make the divisions $\frac{1}{4}$ inch? Would it be sufficient to use the compasses with the points any convenient distance apart?

12. Divide a line $2\frac{7}{8}$ inches long into 5 equal parts.

CALCULATING HEIGHTS AND DISTANCES.

1273. To verify the results obtained by calculation, the pupil should make diagrams, drawing the figures to a convenient scale.

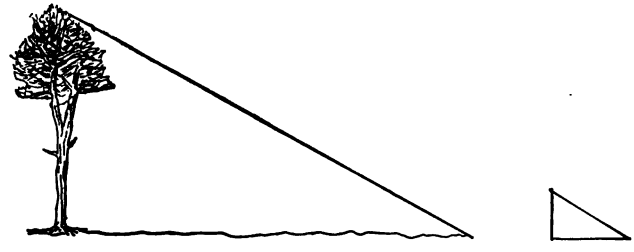
1. If AB in a right-angled triangle measures 120 feet, and a perpendicular, vw, erected 10 feet from A measures 5 feet, calculate the length of BC.



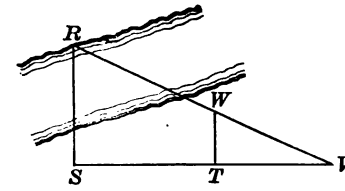
$$Aw : AB :: wv : BC;$$

i.e. $10 : 120 :: 5 : BC.$

2. A post 6 feet above ground throws a shadow of $7\frac{1}{2}$ feet. How high is a tree whose shadow measures 60 feet?

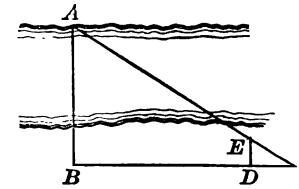


3. Wishing to ascertain the distance between two houses, R and S, on opposite sides of a stream, I measure a line SV, at right angles to SR, 200 feet. At T, 90 feet from S, the perpendicular TW measures 60 feet. Required the distance SR.



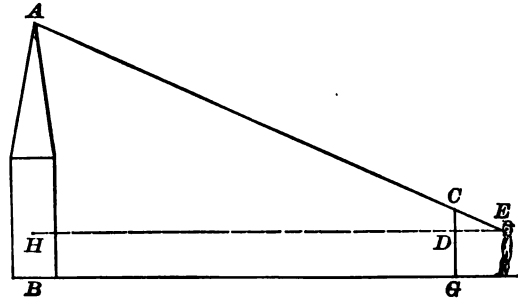
$$VT : TW :: VS : SR \qquad VT = VS - ST.$$

4. Beginning at B, 100 feet from the bank of a river, a line, BC, is measured 1,200 feet long. At D, distant from C 50 feet, the perpendicular DE is found to measure 90 feet. What is the distance from B to A, a tree on the opposite bank? How wide is the river?

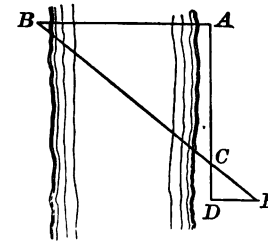


5. A boy, whose eye (E) is 4 feet from the ground, can just see the top (A) of a steeple when he stands back 3 feet from a fence (CG) 6 feet high. The distance from the foot of the fence to the center of the base of the steeple is 177 feet. Find the height of the steeple AB.

$$CD = ? \qquad EH = ? \qquad ED : CD :: EH : AH$$

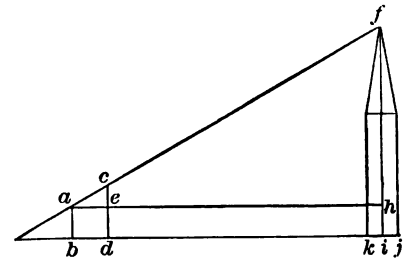


6. Wishing to ascertain the distance AB, I measure a line, AD, at right angles to AB, 12 chains; DE, at right angles to AD, 5 chains; and find that a line sighted from E to B intersects AD at C, distant from D 3.25 chains. What is the distance from A to B?



NOTE. — The triangles DCE and ACB are similar. Why?

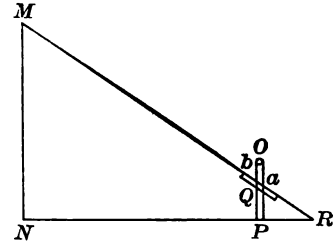
7. Wishing to find the height of a tower fi, I set up a pole, cd, 12 feet long above the ground. Another pole ab, $4\frac{1}{2}$ feet above ground, is set up at such a distance that the tops of the two poles and of the tower are in a line. The distance between the poles (ae or db) is $10\frac{1}{2}$ feet. The distance from d to the foot of the tower is 195 feet. The width of the tower (kj) is 30 feet.



The similar triangles aec and ahf give us the proportion $ae : ah :: ec : hf$.

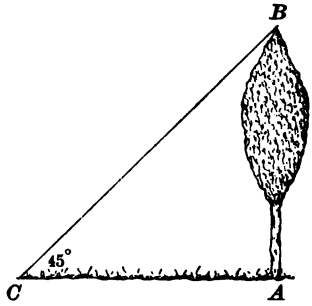
What is the distance ec? $ah = bi = bd + dk + ki$. $ki = \frac{1}{2}\ kj$. When fh is found, what must be added to get the height of the tower?

8. To determine the height of a building, MN, a person attached a strip of wood, ab (a tin tube or a piece of narrow pipe would be better), to a post, OP, in such a manner that sighting from a, he could just see M, the top of the building. He then sighted down from b, and marked on the ground the point R, on a line with ab.



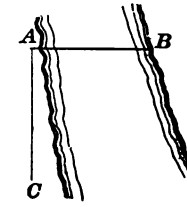
PQ was found by measurement to be 4 feet, RP 6 feet, PN 120 feet. Required MN.

9. Wood-choppers, desiring to know the height of a tree before cutting it, sometimes make an isosceles right-angled triangle of wood or paper, and "step off" the distance on level ground from the point at which they find they can just see the top of the tree looking along the hypotenuse of the triangle, the base being parallel to the ground.



How high is the tree AB, if AC is 36 paces of 3 feet each, and the angle ACB is 45°?

10. B is a point on the bank of a stream due east of A on the other bank. A boy walks due south of A until he reaches a point at which he finds, from his pocket compass, that he is directly south-west of B. If the distance AC measures 119 yards, how wide is the stream?

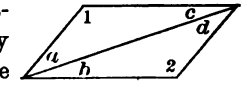


1274. Miscellaneous Exercises.

1. Calculate the length in inches of an arc of 60°, the radius of the circle being two inches. Calculate the length of an arc of 120°. Of 180°. Of 240°. Of 300°.

2. Calculate the length in inches of a chord of 60° in the above circle. Of a chord of 120°. Of a chord of 180°. Of a chord of 240°. Of a chord of 300°.

3. In the parallelogram shown in the accompanying diagram, the angle a measures 40°, and the angle b 35°. How many degrees does the angle c contain? The angle d? The angle 1? The angle 2?



4. Inscribe a regular nonagon in a circle of 2 inches radius, using the protractor.

5. How many degrees does each angle of a regular nonagon contain? Draw a regular nonagon, each side measuring two inches. (Use the protractor.)

6. The distance around a polygon is called its *perimeter*. What is the perimeter of a regular hexagon, inscribed in a circle whose radius is 1 inch?

What is the circumference of the circle?

7. The distance from the center of a regular polygon to the middle point of one side is called the *apothem*.

Draw the apothem of a regular hexagon inscribed in a circle of 1 inch radius. About how long is it?

8. Cut a regular hexagon, side one inch, into six triangles. Place three in a line, and fit in the other three so as to make a rectangle. (Divide one of the triangles into two equal parts.)

How long is the base of the rectangle? What part of the perimeter? About how long is the perpendicular of the rectangle?

9. In a circle, radius 1 inch, inscribe a regular octagon. Divide it into eight triangles, and make out of them a rectangle.

About what is the half perimeter of the octagon? Its apothem? Its area?

Which has the greater perimeter, apothem, area, the hexagon or the octagon?

10. Find the approximate area of a regular hexagon, side 1 inch, apothem about $\frac{7}{8}$ inch.

11. Find the approximate area of a regular octagon, side about $\frac{3}{4}$ inch, apothem about $1\frac{5}{16}$ inch.

12. If we inscribe in a circle a regular polygon of 16 sides, will its perimeter be greater or less than that of the octagon? Which polygon will have the greater apothem?

13. If we inscribe polygons of 32, 64, 128, etc., sides, what will be the greatest perimeter we can have in a circle of 1 inch radius? What will be the greatest apothem?

14. Draw a rectangle that will be about equal to a polygon of a million sides inscribed in a circle whose radius is 1 inch.

Mark upon it the dimensions. Calculate the area.

15. What is the area of a circle whose radius is 2 inches?

16. Find the area of a circle whose diameter is 10 inches.

17. Find the area of a circle whose circumference is 6.2832 inches.

18. Calculate the area of a sector of a circle whose radius is 10 inches, the arc of the sector being 60°.

19. How many square inches are there between the circumferences of two concentric circles whose radii measure 3 and 6 inches, respectively?

SURFACES OF SOLIDS.

1275. Prisms, Cylinders, Pyramids, Cones.

NOTE. — The pupils should first examine a number of prisms and pyramids, right and oblique, regular and irregular, triangular, quadrangular, pentagonal, etc. Right and oblique cylinders and cones should also be at hand.

A **prism** is a body bounded by plane faces, two of which are equal and parallel polygons, the remaining faces being parallelograms.

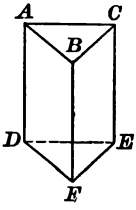


FIG. 1.

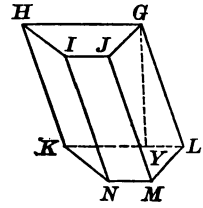


FIG. 2.

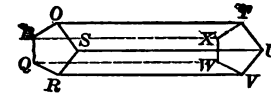


FIG. 3.

The two parallel faces of a prism are called its *bases*. The remaining faces taken together constitute its *convex surface*.

In Fig. 1, *ABC* and *DEF* are the bases; in Fig. 2, the bases are *GHIJ* and *KLMN*; in Fig. 3, *OPQRS* and *TUVWX*.

The sides *AB*, *CE*, etc., *GH*, *IN*, etc., *QR*, *OT*, etc., are called *edges*.

1276. Prisms may be either *right* or *oblique*. The convex surface of a right prism consists of rectangles.

Fig. 1 is a right prism; Fig. 2 is an oblique prism.

NOTE. — When a prism is spoken of, a right prism is meant unless the word *oblique* is used.

The *altitude* of a prism is the perpendicular distance between the bases.

AD, *BF*, or *CE* is the altitude in Fig. 1. *GY* is the altitude in Fig. 2.

1277. The number of sides in each base determines the name, as *triangular* (Fig. 1), *quadrangular* (Fig. 2), *pentagonal* (Fig. 3), etc.

A quadrangular prism whose bases are parallelograms is called a *parallelopipedon*. Fig. 4 is an oblique parallelopipedon. Fig. 5 is a right parallelopipedon. Any two opposite faces of a parallelopipedon may be considered the bases.

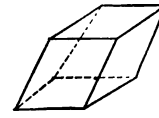


FIG. 4.

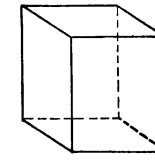


FIG. 5.

1278. When the bases are regular polygons, the prism is said to be *regular*.

Fig. 1 is a right regular triangular prism; Fig. 2 is an oblique irregular quadrangular prism.

1279. A cylinder is a body having two circular parallel plane faces, and one curved face. The plane faces are the bases. The curved face constitutes the convex surface.

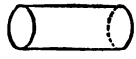


FIG. 6.

Cylinders, like prisms, are either *right* or *oblique*. The altitude of a cylinder is the perpendicular distance between the bases.

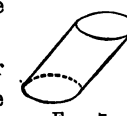


FIG. 7.

FIG. 8.

1280. A **pyramid** is a body whose convex surface is made up of triangles having a common vertex, the base of the pyramid being a polygon.

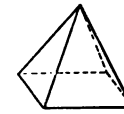


FIG. 9.

FIG. 10.

Pyramids are either *right* or *oblique; regular* or *irregular; triangular, quadrangular, pentagonal*, etc.

In a right pyramid, each of the triangles that make up the convex surface, is isosceles. When, in addition, the pyramid is a regular one, these triangles will be equal to each other.

The altitude of any of these equal triangles constitutes the *slant height* of a right regular pyramid. The *altitude* of the prism is measured by a line drawn from the apex to the center of the base.

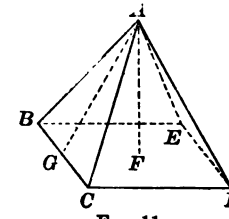


FIG. 11.

AG is the slant height of the square pyramid, Fig. 11. *AF* is its altitude.

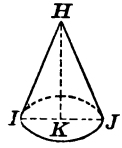


FIG. 12.

1281. The *cone* has a single circular base; its convex surface is curved, sloping to the apex.

In the right cone, Fig. 12, *HI* is the slant height, and *HK* is the altitude. *LO* is the altitude of the oblique cone, Fig. 13.

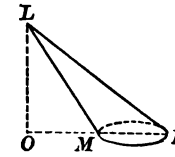


FIG. 13.

1282. Surfaces of Prism, Cylinder, Pyramid, Cone.

1. Draw the developed convex surface of a square prism, height 3 inches, one side of base 1 inch.

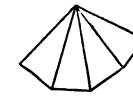
2. Draw the developed (entire) surface of a triangular prism, height 3 inches, each side of base 2 inches.

3. Draw the developed convex surface of a prism 3 inches high, each base being a triangle having sides of 1, $1\frac{1}{2}$, and 2 inches, respectively.

4. Find the convex surface of each prism, and show that the convex surface is found by multiplying the perimeter of the base by the altitude.

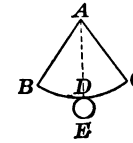
5. Show that the convex surface of a cylinder is found by multiplying the circumference of the base by the altitude.

6. Draw the developed convex surface of a square pyramid, slant height 3 inches, each side of base 2 inches. Cut it out of paper, leaving on one edge a small strip for gumming. Fold into a hollow pyramid, and measure its altitude.



7. Show that the convex surface of a regular pyramid is found by multiplying the perimeter of the base by one-half the slant height.

8. Make out of paper a hollow square pyramid whose altitude shall be 3 inches, each side of base 2 inches.



1283. The developed convex surface of a cone is a sector, *ABDC*.

How many inches does the arc *BDC* measure when the diameter of the base of the cone is 2 inches?

The slant height of the cone is the radius of the circle of which the sector is a part. *AB*, *AD*, or *AC* is the slant height.

9. Cut out of paper a semicircle, radius 3 inches, adding a narrow strip for gumming, and fold into a cone.

What is the slant height of the cone? The diameter of the base? The radius of the base? The circumference of the base?

What is the ratio between the radius of the base and the slant height? Between the diameter of the base and the slant height?

10. Calculate the convex surface of the above cone, and show that it is equal to the circumference of the base multiplied by one-half the slant height.

11. Find the diameter of the base of a cone made by folding a paper sector of 90 degrees (quadrant), the radius of the sector being 3 inches. What is its slant height?

When a sector of 60 degrees is used, what will be the diameter and the slant height, the radius of the sector being 3 inches?

(Make no allowance for overlapping.)

12. If you wish to make a hollow paper cone whose slant height shall be 5 inches, and the diameter of whose base shall be 3 inches, how many degrees should the arc of the sector contain?

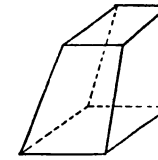
13. Draw the development of a right pyramid 4 inches in altitude, whose base is a rectangle 3 inches by 2 inches. Is the altitude (slant height) of each of the four triangles the same?

14. Calculate the slant height of each convex face of a rectangular pyramid whose altitude is 12 inches, and whose base measures 10 inches by 18 inches.

Find the convex surface.

1284. Surface of Frustum of Pyramid and Cone.

When the upper part of a pyramid or of a cone is cut off by a plane parallel to the base, the remaining part is called a frustum.



15. Draw one face of the frustum of a square pyramid. Of the frustum of a triangular pyramid. What figure have you drawn?

Calculate its surface when the length of the upper side is 4 inches, that of the lower side is 8 inches, and the slant height of the frustum is 10 inches.

16. Draw the developed convex surface of the frustum of a regular triangular pyramid, each side of the upper base measuring 1 inch, of the lower base 2 inches, the slant height being 2 inches.

SUGGESTION. — Locate the apex of the whole pyramid of which the given frustum forms a part.

17. Find the convex surface of the frustum of a square pyramid, one side of the upper base measuring 2 feet, of the lower base 3 feet, and having a slant height of 4 feet.

Find the entire surface.

18. Show that the convex surface of the frustum of a pyramid is equal to one-half the sum of the perimeters of the upper and the lower bases multiplied by the slant height.

19. Draw the pattern of a small shade for a candle. Make the upper opening $1\frac{1}{2}$ inches in diameter, the lower one $2\frac{1}{2}$ inches in diameter, and the slant height 2 inches.

20. How many square inches of tin will be required to make a pan, its upper base being 9 inches in diameter, the lower base 6 inches in diameter, and the slant height 4 inches?

(Do not forget the bottom of the pan.)

1285. The frustum of a cone may be considered the frustum of a pyramid whose bases contain a very great number of sides. The convex surface of the frustum of a cone may, therefore, be found by multiplying the half sum of the circumference of the two bases by the slant height.

21. Find the convex surface of a frustum of a cone, the circumferences of the bases being 15 inches and 20 inches, respectively, and the slant height 10 inches.

22. How many square yards are there in the entire surface of a frustum of a cone, the radius of the upper base (r) being 3 yards, of the lower base (R) 5 yards, and the slant height 6 yards?

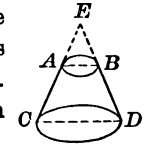
Circumference of upper base $= 2\pi r$; of lower base $= 2\pi R$.

Convex surface $= (2\pi r + 2\pi R) \times \frac{\text{slant height}}{2} = \pi(r + R) \times \text{slant height}$.

Surface of upper base $= \pi r^2$; of lower base $= \pi R^2$.

Entire surface $= \pi \times$ what? Multiply only once by 3.1416.

23. The diameter, AB, of the upper base of the frustum of a cone measures 6 feet, CD measures 8 feet, the slant height AC measures 9 inches. Find the slant height EA of the part cut from the cone in making the frustum.



Let $EA = x$; $EC = x + 9$; $AB = 6$; $CD = 8$.

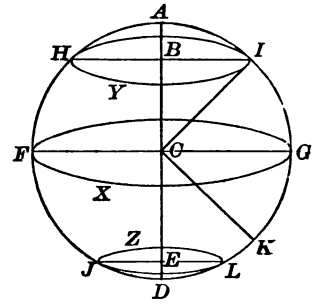
The triangles EAB and ECD are similar.

24. Find the convex surface of the whole cone, ECD, and the convex surface of the part cut off, EAB.

1286. The Sphere.

A *sphere* is a body all points on whose surface are equally distant from the center.

The distance from the center to the surface is called the *radius* of the sphere. The diameter is a line running between two points on the surface and passing through the center.



CG, CK, CD, CF, CA, and CI are radii; AD and FG are diameters.

1287. If a sphere be cut through at any part, the cut surface will be a circle. When the cutting plane passes through the center of the sphere, the circle is called a *great circle;* other circles are called *small circles.*

$FXGC$ is a great circle; $HYIB$ and $JLEZ$ are small circles.

25. Find the length of an arc of 60° of a great circle of a sphere whose circumference is 25,000 miles.

26. Calling the arc AI in the preceding figure, 30°, the angle BCI will measure 30°. Calculate the radius BI of the small circle when the radius CI of the large circle is 4,000 miles. (IAH = arc of 60°; IH = chord of 60°.)

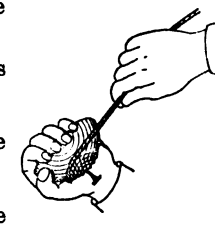
27. If I is 60° from G, a point on the equator, find the length of the circumference of the small circle HYI, assuming the circumference of a great circle to be 25,000 miles.

28. What is the ratio between the length of a degree on the small circle HYI, and the length of a degree of a great circle?

29. Calculate the radius of a small circle formed by passing a plane parallel to $GCXF$ through a point on GA 45 degrees from G.

1288. Surface of Sphere.

We have seen (Art. 1150) that it may be experimentally shown that the surface of a sphere is equal to the surface of four of its great circles.



Calling the radius of the sphere R, its surface is $4\pi R^2$.

30. Find the surface of a sphere whose diameter is 6 inches.

31. How many square inches are there in the convex surface of a hemisphere whose radius is 3 inches?

What is the area of the great circle that forms the base of the hemisphere?

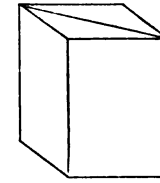
Find the entire surface of the hemisphere.

32. Is there any difference between the convex surface of a sphere and its entire surface? Why?

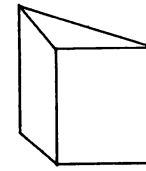
VOLUMES.

1289. Prisms and Cylinders.

1. How many one-inch cubes will cover the base of a box 4 inches by 3 inches? If the box is 2 inches deep, how many one-inch cubes will it contain? How many cubic inches are there in the volume of a right prism whose base is a rectangle measuring 4 inches by 3 inches, and whose altitude is 6 inches?



2. If the above hollow prism were divided into two equal parts by a thin partition extending from a vertical edge to one diagonally opposite, how many cubic inches of sand would each part contain?



3. How many cubic inches are there in the volume of a prism whose base is a right-angled triangle 3 by 4 by 5 inches, and whose altitude is 6 inches?

4. Find the volume of a triangular prism, the area of the base being 6 square inches, and the altitude 6 inches.

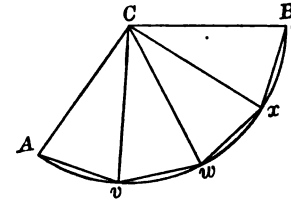
Find the volume of a triangular prism, each side of whose base measures 6 inches, its altitude being 8 inches.

5. What are the solid contents of a pentagonal prism formed by fastening together three triangular prisms whose bases contain, respectively, 12, 16, and 18 square inches, the altitude of each being 15 inches?

6. If a very great number of triangular prisms of the same height are united so as to form a cylinder whose base contains 12.5664 square inches, and whose altitude measures 5 inches, what are the solid contents?

1290. Pyramids and Cones.

With a center at C, and a convenient radius, describe an arc AB. Mark off four equal portions v, w, x, and y; and draw the equal chords. Cutting out $CAvwxy$, with an additional narrow strip along Cy for gumming, and creasing along the lines Cv, Cw, Cx, and Cy, we can fold the paper into a square pyramid.



Measure its altitude, and make a square prism of equal altitude and with an equal base.

Filling the pyramid with sand, and pouring the sand into the prism, it will be found that the latter will contain the contents of the former three times; that is, the volume of a square pyramid is one-third that of a square prism having an equal base and an equal altitude.

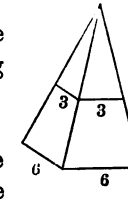
The same ratio will be found true in the case of a triangular, or any other pyramid, as compared with the corresponding prism, and of the cone as compared with a cylinder.

1291. The volume of a pyramid or of a cone is equal, therefore, to the area of the base multiplied by one-third of the altitude.

1292. Frustums of Pyramids and Cones.

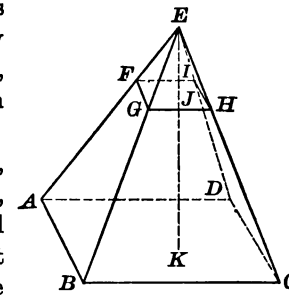
7. Find the volume of a square pyramid whose altitude is 12 inches, one side of the base measuring 6 inches.

Find the volume of a square pyramid whose altitude is 6 inches, one side of the base measuring 3 inches.



8. Find the volume of the frustum of a square pyramid whose altitude is 6 inches, one side of the upper base 3 inches, and one side of the lower base 6 inches.

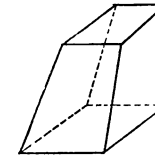
9. A square pyramid whose altitude measures 18 inches, and each side of whose base measures 15 inches, is divided into two parts by a plane, $FGHI$, parallel to the base, the distance, EJ, of the plane from the vertex, E, being 6 inches.



The ratio between the edge, EB, of the whole pyramid and the edge, EG, of the part cut off will be equal to that between EJ and EK; that is, $6 : 18 = 1 : 3$. The same will be the ratio between BC and GH, and the latter will be one-third of 15 inches long, or 5 inches.

Find the volume of the large pyramid and that of the small pyramid.

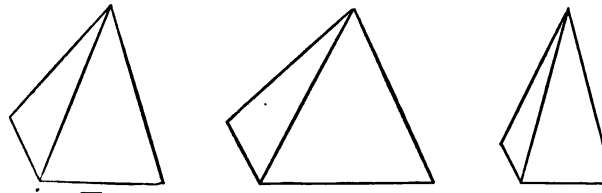
10. Each side of the upper base of the frustum of a square pyramid measures 5 inches; each side of the lower base measures 15 inches; the perpendicular distance between them measures 12 inches. Find the solid contents.



Find the convex surface of the above frustum. Find its entire surface.

NOTE. — What is the slant height?

11. Find the total volume of three square pyramids, the altitude of each being 12 inches, and the areas of their bases being 25 sq. in., 225 sq. in., and 75 sq. in., respectively.



12. Find the number of cubic feet in a block of stone whose shape is that of a frustum of a square pyramid 4 feet high, each side of the upper base measuring 3 feet, and each side of the lower base 5 feet.

1293. The volume of the frustum of a pyramid is equal to the sum of the volumes of three pyramids, each having an altitude equal to that of the frustum; the base of one of them being equal in area to that of the lower base of the frustum, the base of a second being equal in area to that of the upper base of the frustum, and the base of a third being a mean proportional between the area of the other two.

Base of first $= 3 \times 3$ sq. ft.; of second, 5×5 sq. ft.; of third, $\sqrt{9 \times 25}$ sq. ft. $= 15$ sq. ft.

NOTE. — The mean proportional between two numbers is equal to the square root of their product.

13. Find the volume of the frustum of a square pyramid, its upper base containing 64 square inches, and its lower base 196 square inches, its altitude being 18 inches.

1294. Note that the mean proportional between 64 and 196 is 8×14, or 112. Since each is multiplied by one-third of the altitude, the operation is shortened by adding together the three areas, 64, 196, and 112, and multiplying their sum by one-third of 18.

Calling the altitude A, the side of the large square S, of small square s, the volume V, we have

$$V = \frac{A(S^2 + s^2 + Ss)}{3}.$$

1295. The volume of the frustum of a cone is found in the same way as that of the frustum of a pyramid.

$\frac{1}{3}$ altitude (area upper base + area lower base + area mean proportional).

Calling the radius of the upper base r, and that of the lower base R, the area of the upper base will be πr^2, of the lower base πR^2, of mean proportional πrR.

$$V = \tfrac{1}{3} A(\pi r^2 + \pi R^2 + \pi rR)$$

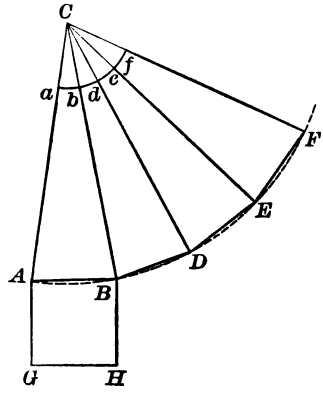
Since π (or 3.1416) is a common factor, we can save time by first adding r^2, R^2, and rR, and then multiplying by π.

$$V = \frac{A\pi(r^2 + R^2 + rR)}{3}.$$

14. The diameters of the bases of the frustum of a cone measure 8 and 15 inches, respectively; the altitude is 18 inches. Find the volume.

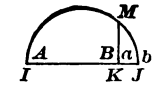
15. How many cubic inches of water will a pan hold, whose lower base is 12 inches in diameter, whose upper base is 16 inches in diameter, and whose depth is 6 inches? How many gallons?

1296. The pupils should make a frustum of a square pyramid of convenient size, and the three corresponding pyramids, as given in the rule. Fill the latter with sand, and pour the contents of all three into the frustum.



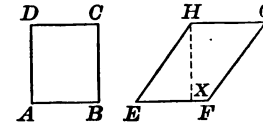
To make the frustum, draw two concentric circles. Lay off equal arcs, *AB*, *BD*, *DE*, *EF*. Draw the chords and radii from the extremities of each chord. Draw the chords *ab*, *bd*, *de*, and *ef*. Cut out, after constructing a square for either the upper or the lower base, and taking care to provide flaps for pasting.

To get a mean proportional between ab and AB for one side of the base of the third pyramid, lay off a line IJ equal in length to $ab + AB$. On this line construct a semicircle. Make IK equal to AB, and at K erect a perpendicular KM. KM is a mean proportional between ab and AB.



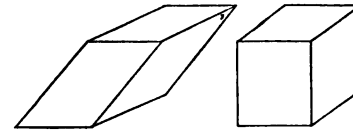
1297. Oblique Prisms.

We have seen that a rectangle, $ABCD$, and a parallelogram, $EFGH$, are equal in area when the bases, AB and EF, and the altitudes, AD and HX, are equal, each to each.



This can be shown by cutting both out of paper, and by shifting the triangle HEX to the right side of the parallelogram.

1298. In a somewhat similar way, we can show that an oblique prism is equal to a right prism that has an equal base and an equal altitude.



Make from a potato or a turnip an oblique prism having rectangular bases, and change it to a right prism of the same height by cutting and shifting a portion.

1299. The volume of *any* prism (or cylinder) is found by multiplying the area of the base by the altitude.

1300. In the same way it can be shown that the volume of any pyramid or cone is equal to the product of the area of the base by one-third the altitude.

1301. The Sphere.

A sphere may be considered as made up of a great number of pyramids whose bases together make the surface of the sphere,

and whose vertices all meet at the center of the sphere, making their altitudes each equal to the radius of the sphere.

The volume of a sphere is equal, therefore, to its surface $\times \frac{1}{3}$ radius.

$$\text{Surface} = 4 \text{ great circles} = 4\pi R^2;$$

$$\text{Volume} = 4\pi R^2 \times \frac{R}{3} \quad = \frac{4\pi R^3}{3}.$$

16. Find the volume of a sphere whose radius is 9 inches.

17. What is the volume of a sphere whose diameter is 9 inches?

Find the volume of a cone whose altitude is 9 inches, diameter of base 9 inches.

How does the volume of the cone compare with the volume of the sphere?

How does the volume of the sphere compare with the volume of a cylinder 9 inches in diameter and 9 inches high?

1302. Take a clay sphere of a convenient size. Make a paper cylinder that will exactly contain it, the height of the cylinder being equal to the diameter of the sphere. Make a hollow cone of the same diameter and altitude.

Place the sphere in the cylinder, carefully fill the cone with water, and pour it into the cylinder, which should then be filled to the top, showing that the volume of the cylinder is equal to that of the sphere and the cone together.

APPENDIX.

TABLES.

1303. Measures of Extension.

Long Measure.

12 inches (in.) . . . 1 foot (ft.)
3 feet . . . . . 1 yard (yd.)
5½ yards . . . . . 1 rod (rd.)
320 rods . . . . . 1 mile (mi.)

A furlong (fur.) = ⅛ mile.

Surveyors' Measure.

7.92 inches . . . . . 1 link (li.)
100 links . . . . . 1 chain (ch.)
80 chains . . . . . 1 mile

1 chain = 4 rods = 66 feet.

Square Measure.

144 sq. in. . . . . 1 square foot
9 sq. ft. . . . . 1 square yard
30¼ sq. yd. . . . . 1 square rod
160 sq. rd. . . . . 1 acre
640 acres . . . . 1 square mile

10 square chains = 1 acre.

Cubic Measure.

1728 cu. in. . . . 1 cubic foot
27 cu. ft. . . . . 1 cubic yard

1 cord = 128 cu. ft.
1 bushel = 2150.4 cu. in.
1 gallon = 231 cu. in.

Measures of Capacity.

Dry Measure.

2 pints (pt.) . . . 1 quart (qt.)
8 quarts . . . . . 1 peck (pk.)
4 pecks . . . . . 1 bushel (bu.)

Liquid Measure.

2 pints (pt.) . . . 1 quart (qt.)
4 quarts . . . . 1 gallon (gal.)

A gill is ¼ pint.

The capacity of tanks, vats, etc., is frequently expressed in barrels of 31½ gallons.

1 qt. dry measure = 67⅕ cu. in.
1 qt. liquid measure = 57¾ cu. in.

Measures of Weight.

Troy Weight.

24 grains (gr.) 1 pennyweight (pwt.)
20 pennyweights, 1 ounce (oz.)
12 ounces . . . 1 pound (lb.)

Troy weight is used in weighing gold, silver, precious stones, etc.

Apothecaries' Weight.

20 grains (gr) . 1 scruple (℈)
3 scruples . . . . . 1 dram (ʒ)
8 drams . . . . . 1 ounce (℥)
12 ounces . . . . 1 pound (℔)

Apothecaries' weight is used in prescriptions. Drugs are bought and sold by avoirdupois weight. The grain, the ounce, and the pound apothecaries' weight are the same as the corresponding denominations of troy weight.

AVOIRDUPOIS WEIGHT.

16 ounces (oz.) . 1 pound (lb.)
2000 pounds . . . . 1 ton (T.)

1 lb. avoirdupois = 7000 grains.
1 lb. troy = 5760 "
1 oz. avoirdupois = 437½ "
1 oz. troy = 480 "

In weighing ores and coal at the mines and in calculating duties at the U. S. custom houses, the following table is used:

28 pounds . . . . . 1 quarter (qr.)
4 quarters, 1 hundredweight (cwt.)
20 hundredweight . . . . 1 ton (T.)
1 cwt. = 112 lb. 1 T. = 2240 lb.

Measures of Value.

U. S. MONEY.

10 mills . . . . . 1 cent
100 cents . . . . . 1 dollar

1 dime = 10c.
1 eagle = $10.

ENGLISH MONEY.

12 pence (d.) . . 1 shilling (s)
20 shillings . . . 1 pound (£)

1 farthing = ¼ penny.
£1 = $4.8665.

The Canadian dollar is equal in value to that of the United States, and is also divided into 100 cents.

The French franc (fr.) = 19.3¢, is divided into 100 centimes (c.).

The German mark (reichsmark) (M., m.) = 23.8¢, is divided into 100 pfennigs (pf.).

Circular Measure.

60 seconds (″) . . 1 minute (′)
60 minutes . . . . 1 degree (°)
360 degrees . 1 circumference

Time Measure.

60 seconds (sec.) 1 minute (min.)
60 minutes . . . . 1 hour (hr.)
24 hours . . . . . 1 day (da.)
7 days . . . . . 1 week (wk.)
365 days 1 common year (yr.)
366 days . . . . . 1 leap year

Years divisible by 4 and not by 100 are leap years.

1892, 1896 are leap years.

Years divisible by 100 but not by 400 are common years.

1700, 1800, 1900 are common years; 1600 and 2000 are leap years.

1304. Time between Dates.

1. In the common method, by compound subtraction, each month is considered one of 30 days, regardless of its length.

EXAMPLE: Find the time between May 13, 1895 and March 2, 1899.

1899	3	2
1895	5	13
3	9	19

Taking 30 days to the month, the difference in time is found to be 3 years, 9 months, 19 days.

2. A more exact method is to take first the number of entire years, then the number of entire months, and lastly the number of days.

May 13, 1895 to May 13, 1898 = 3 years.
May 13, 1898 to Feb. 13, 1899 = 9 months.
Feb. 13, 1899 to Mar. 2, 1899 = 17 days.
Ans. 3 years, 9 months, 17 days.

3. Another method is to find the difference in years and days.

May 13, 1895 to May 13, 1898 = 3 years.
May 13, 1898 to Mar. 2, 1899 = 293 days.
Ans. 3 years, 293 days.

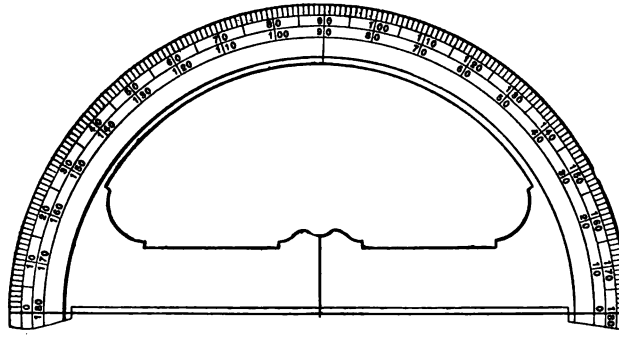
1305. Days of Grace.

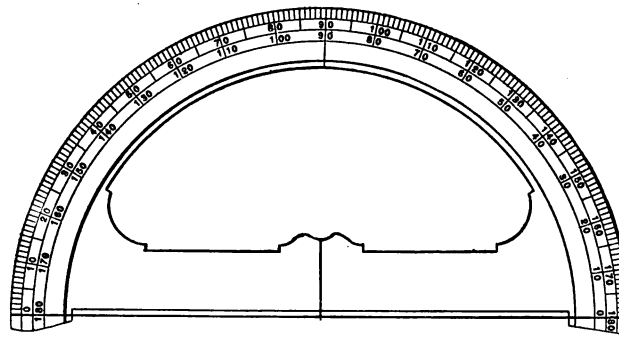
Days of grace are not allowed in California, Idaho, New York, Oregon, Utah, Vermont, and Wisconsin.

Answers in which days of grace are not included, are inclosed in a parenthesis.

When a note falls due on a Sunday or a legal holiday, it is generally payable on the next preceding business day. Some states, however, do not require payment until the next business day following. In these latter cases, banks include the extra days in calculating discount.

In a majority of the states, days of grace are not allowed on sight drafts.





The pupil can make a protractor by pasting one of the above on a stiff piece of paper and carefully cutting it out.

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